A THEORY OF OCCUPATIONAL CHOICE WITH ENDOGENOUS FERTILITY

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Abstract

This paper studies the steady states of a model which combines endogenous fertility with occupational choice. Three sets of results are obtained. (a) There is a negative cross-sectional relationship between parental wages in different occupations and fertility, independent of the relative strength of wealth and substitution effects that determine fertility in the standard framework. (b) The differential fertility across occupational categories creates Intergenerational mobility in steady state. Unlike mobility created by stochastic shocks, such occupational drift has a predictable direction depending on the income-fertility relationship. (c) Steady states are (generically) locally determinate and permit the analysis of various policy changes.

1. Introduction

The connections between economic conditions and fertility have been well recognized since the time of Malthus. These links have been explicitly explored by Becker (1960), Becker and Barro (1986, 1988), and Barro and Becker (1989). The Barro-Becker approach assumes parental preferences over the number and well-being of their offspring, integrated into a representative-agent optimal growth model. A sizable literature has emerged from these papers, including extensions that incorporate human capital, agent heterogeneity and inequality (e.g., Becker, Murphy and Tamura (1990), Sah (1991), Dahan and Tsiddon (1998), Alvarez (1999), Kremer and Chen (1999, 2002), Galor and Weil (2000), De La

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Croix and Doepke (2003), Doepke (2004, 2005), Jones, Schoonbroodt and Tertilt (2008), and Jones and Schoonbroodt (2009)).

Parent-child interaction is not limited to fertility decisions. The theory of occupational choice emphasizes how educational investment decisions made by parents condition the occupational choices of their children. When financial markets are missing or incomplete, such theories generate persistent inequality and history dependence (e.g., Banerjee and Newman (1993), Galor and Zeira (1993), Ljungqvist (1993), Freeman (1996), Aghion and Bolton (1997), Bandopadhyay (1997), Lloyd-Ellis and Bernhardt (2000), Matsuyama (2000, 2003), Ghatak and Jiang (2002) and Mookherjee and Ray (2002, 2003, 2010)). However, this literature is not concerned with fertility.

We combine the theory of endogenous fertility with a theory of occupational choice.[4] As in the endogenous fertility literature, parents decide how many children to have. As in the occupational choice literature, parents make investments in the future of their children, and market forces endogenously pin down the returns to different occupations. The interaction between fertility and human capital investments generates a number of novel results, which we now discuss.

A central goal of the endogenous fertility literature is to explain the demographic transition. Fundamental to that transition is the fall in fertility with economic development. Arguably the most important component of "development" is an improvement in the economic well-being of parents; see Barro and Becker (1989), Alvarez (1999), Kremer and Chen (1999, 2002), Galor and Weil (2000) and Greenwood and Seshadri (2002).[5] This is the cross-sectional relationship that we study in this paper.

[4]A number of the papers on endogenous fertility cited above incorporate human capital and inequality, such as Dahan and Tsiddon (1998), Kremer and Chen (1999, 2002), de la Croix and Doepke (2003) and Doepke (2004). So in a broad sense, we are not the first to do this. But there are distinctive elements of our approach, which include the endogenous determination of steady states as in the occupational choice literature, the use of general equilibrium properties of those steady states as a key source of the main results (rather than restrictions on preferences), and an explicit consideration of the link between fertility and inter-occupational mobility. Section 8 discusses the relation of this paper to the existing literature in more detail.

[5]In related exercises, one might study the effect of a reduction in infant or child mortality on fertility (see, e.g., Sah (1991) and Doepke (2005)), or the effects of *anticipated* improvements in child welfare on fertility choices (see, e.g., Jones and Schoonbroodt (2009)).

Is the *empirical* relationship between parental wages and fertility systematically negative? It is fair to say that the answer is generally in the affirmative. In their excellent overview of the literature, Jones, Schoonbroodt and Tertilt (2008) summarize the empirical cross-sectional correlations between parental income and fertility in developed countries. Most studies find a negative relationship. At the same time, while widespread, the relationship is not universal. In some contexts, e.g., in various agrarian settings, there is systematic evidence of a positive relationship. Studies include Simon (1977) for Poland in 1948; Clark (2005), Clark and Hamilton (2006), and Clark (2007) for England in the 16th and 17th century; Weir (1995) for France in the 18th century; Wrigley (1961) and Haines (1976) for some areas in France and Prussia in the 19th century; and Lee (1987) for the U.S. and Canada. Schultz (1986) reports that empirical studies of fertility have shown a negative relationship between male wages and fertility in high-income urban populations but a positive one in low-income agricultural populations. Even within urban populations in developed countries, a positive relationship has been found within particular segments. For example, in the context of the US, Blau and van der Klaauw (2007) find a negative effect of wages on fertility for white male wage earners, but a positive effect for Blacks and Hispanics. Freedman (1963) found a positive effect of "relative income" of fertility within particular occupational categories. Specifically, within the same occupation, higher income households tend to have more children, whilst average fertility varies negatively with income *across* occupations. This is consistent with views expressed earlier by Spengler (1952) and Easterlin (1973) that fertility growth is positively related to relative incomes within an occupational category.[6]

Motivated by the overall negative relationship between parental wages and fertility, mediated by a possibly different pattern within occupations, this paper isolates and compares different factors that work both within and across occupational categories. Theories of endogenous fertility run into a ubiquitous problem that we, too, must confront: a clear-cut prediction invariably depends on the interaction of wealth and substitution effects in parental preferences. The net effect is generally unclear. The wealth effect captures the

[6]Simon (1969) points out that over the course of the business cycle the correlation of fertility with income tends to be positive, in contrast to the cross-sectional pattern, also possibly for these reasons.

tendency to acquire more children (much like consumer durables) with rising earnings. The substitution effect captures the higher time costs of childrearing: e.g., increased participation in the labor market by women. This effect will work to reduce fertility, in line with observed outcomes. For a large and reasonable class of preferences, one obtains a net positive correlation between parental wages and fertility.[7] To get around this problem, most models impose strong assumptions on preferences that restrict the relative magnitude of wealth effects. Within the family of constant-elasticity utility functions, to obtain a negative relationship, one needs to rule out functions that exhibit at least as much curvature as the logarithmic function. As Jones and Schoonbroodt (2009) point out, calibrated values of intertemporal elasticities of substitution in the utility function that fit central qualitative aspects of the demographic transition, happen to be inconsistent with parameter values used in growth and business-cycle applications.

Our emphasis on occupational choice yields an alternative way of modeling the cross-sectional parental wage-fertility correlation which turns out to be consistent with the empirical findings. We employ the Barro-Becker formulation of parental altruism, in which the strength of parental preferences for children are driven by the lifetimes utilities enjoyed by offspring. We do not impose strong parameter restrictions on parental preferences that restrict the strength of wealth effects relative to substitution effects. When wealth effects are dominant (in the sense of exhibiting at least as much curvature as the logarithmic function), the resulting wage-fertility correlation will indeed be positive, *provided that the parental wage variation induces no change in the occupational choice of progeny*. However, whenever a high parental wage induces a switch to higher occupational choices, there is an associated *robust* drop in fertility which is entirely independent of the relative strength of income and substitution effects. It arises from the way in which an induced trade-off between parental preferences for quality and quantity of children must be resolved, when there is an occupational shift. Our formulation is therefore consistent with a fertility drop across occupations, coupled with ambiguous outcomes within occupations.

[7] Indeed, Becker (1960) claimed that the wealth effect was typically dominant. For Becker and others, the negative cross-sectional relationship was an artifact, caused by ignorance of contraceptive methods on the part of low-income individuals.

The heart of the subsequent analysis is a steady state comparison of overall fertility across skilled and unskilled occupations, which combines the two effects described above. With high curvature of utility the two are in mutual opposition: the "preferences effect" tends to raise fertility; the "occupational shift effect" tends to lower it. Our first main result is that, for a large range of constant elasticity utility functions (including those with arbitrarily large wealth effects), the occupational shift effect outweighs the preference effect, as a consequence of the endogenous determination of skilled and unskilled wages in steady state. In particular, in the leading special case in which when child-rearing costs involve parental time alone, we obtain a declining relationship between fertility and (endogenously determined) parental incomes, *for all constant elasticity utility functions*. Whenever the empirical setting allows substantial variations in occupations or human capital — as in urban settings in modern societies — our theory thus predicts a negative cross-sectional correlation quite generally. But when we look within occupations of modern urban societies, or examine settings (such as traditional agrarian societies) not associated with major variations in occupations or education, this result may get reversed if wealth effects are strong.

In their 2008 survey, Jones, Schoonbroodt and Tertilt describe two other routes to a negative cross-sectional relationship. One of them relies on endogenous human capital investments (e.g., Becker and Tomes (1976) and Moav (2005)), as we do here. But there are fixed rates of return to human capital in these models, and restrictions must be assumed in order for the desired result to work. In contrast, our approach is based on a general equilibrium argument, using the discipline imposed by a steady state. These general equilibrium factors limit the extent to which skilled wages exceed unskilled wages, and thus restrict the scope of a positive (net) correlation arising from preferences alone. A second route reverses the causality: with heterogeneous agents, parents with a greater taste for higher fertility will have more children, adversely affecting their own human capital acquisition in the process. In our approach, the human capital (or occupation) of parents is determined *before* they make fertility and educational decisions. This is particularly pertinent to developing countries with low average levels of educational attainment, in which most of the population do not complete secondary schooling. In any case, the reversal of causality, while interesting in its own right, emphasizes a route, not the one that we might focus on from the perspective of the demographic transition, in which the change in economic circumstances is presumably the factor that drives fertility outcomes.

The second main contribution of this paper is to the theory of intergenerational mobility. Existing theories of mobility rely on stochastic shocks to abilities or incomes, and assume constant, exogenous fertility (e.g. Becker and Tomes (1979), Loury (1981), Banerjee and Newman (1993) and Mookherjee and Napel (2007)). We show that incorporating endogenous fertility induces mobility even in the absence of any stochastic shocks. More generally, mobility levels in steady state depend on fertility patterns. If fertility is higher in unskilled occupations, the proportion of skilled agents in the economy will tend to drift downwards over time. A steady state in which per capita skill in the economy is constant over time therefore requires upward mobility: a fraction of unskilled households must decide to educate their children to prepare them for entry into the skilled occupation. A society with a higher fertility differential between the skilled and unskilled must therefore also involve greater mobility. The possible connection between mobility and fertility patterns has been overlooked in the existing literature on intergenerational mobility (see, e.g. the symposium in the *Journal of Economic Perspectives* 2002, **16**(3)).

This leads to the third contribution of the paper. By the logic sketched above, steady states with differential fertility across occupations must be associated with the indifference of parents in one occupational category between educating and not educating their children. A positive fraction of parents must decide not to educate their children in order to ensure that there will be positive supply of unskilled workers in the next generation. At the same time a positive fraction must also decide to educate their children, in order to ensure steady-state constancy of the skill ratio. This indifference condition ties down relative wages and hence the skill ratio in steady state, ensuring local determinacy of macroeconomic aggregates and the level of inequality. This is in marked contrast to occupational choice models with exogenous and constant fertility, which typically exhibit a continuum of steady states. Hence, endogenizing fertility eliminates the extreme hysteresis of these models, and limits the extent of long-run history dependence: small macro shocks will not generally have permanent effects. Moreover, it permits the analysis of policy questions. Our theory generates predictions about the macroeconomic effects of childcare or education subsidies, redistributive tax-transfer policies or child labor regulations.[8] Specifically, a rise in the non-time component of childcare costs, or a fall in education costs, or stronger child labor regulations, are shown to increase long run human capital investments, raising per-capita income and lowering wage inequality across skilled and unskilled occupations. The same effects obtain with a *reduction* in unconditional transfers to the unskilled that are funded by taxes on earnings of the skilled, or an *increase* in transfers conditioned on school enrollment of children.

The paper is organized as follows. Section 2 introduces the model. Section 3 makes the distinction between partial and general equilibrium effects in the study of fertility decline. Section 4 analyzes household optimal choices in the partial equilibrium setting with given wages and continuation values of children. Following this, Section 5 introduces steady states, and establishes our results concerning mobility in steady state. Section 6 studies conditions under which the general equilibrium of steady states yields a negative wage-fertility correlation. Section 7 then shows steady states are locally determinate and performs comparative static exercises.

2. Model

2.1. **Occupations and Technology.** A single output is produced under competitive conditions, using skilled and unskilled labor. Let λ denote the fraction of skilled labor. The marginal product of skilled labor decreases in λ; the opposite is true of unskilled labor. Both marginal products are smooth functions of λ and satisfy Inada endpoint conditions.[9]

There are two occupations, unskilled (0) and skilled (1). Skilled workers can work as either unskilled or unskilled labor, a choice not available to unskilled workers. Let $\bar{\lambda}$ denote the

[8] It is worth noting the contrast with Barro and Becker (1989) in which long run policies tend not to have any long run effects, owing partly to the assumption of a perfect capital market in their model. In opposing contrast, Mookherjee and Ray (2008) use an occupational choice model with exogenous fertility to study long-run effects of tax-transfer policies, and have to resort to non-steady-state analysis.

[9] That is, they go to ∞ and 0 at either end of their variation.

value of λ for which the marginal products of skilled and unskilled labor are equalized. Then for $\lambda < \bar{\lambda}$, skilled and unskilled wages (w_1 and w_0) equal their respective marginal products, while for higher skill ratios they both equal the common marginal product at $\bar{\lambda}$.

2.2. **Fertility, Childrearing and Education.** There is a continuum of households at every date, with one adult (a single parent) in each household.[10] A parent earns a wage w on the labor market and chooses how many children n to have, where we suppose n to be a continuous variable.[11]

Child-rearing and education are costly activities. We distinguish between the cost incurred in raising an unskilled child — $r_0(w)$ — and the cost of raising a skilled child — $r_1(w)$. We maintain the following assumptions on r_0 and r_1 throughout the paper:

[R.1]. For each category $i = 0, 1$, $r_i(w)$ is smooth and strictly increasing in w, while $\frac{r_i(w)}{w}$ is nonincreasing. There is a positive lower bound $\underline{r}$ to the rate of increase of $r_i, i = 0, 1$.

[R.2]. For every w, $r_1(w) > r_0(w)$.

[R.3]. For every w,

$$\frac{r_1'(w)}{r_1(w)} < \frac{r_0'(w)}{r_0(w)}.$$

Assumption R.1 states that higher parental wages increase child-rearing costs, but less than proportionately. The former arises from the time component to child-rearing which causes the parent to be away from work.[12] Other fixed resource costs of child-bearing and rearing would be independent of parental wages, which would imply that per-child costs would rise less than proportionately with parental wages. Note that R.1 allows such fixed costs to be zero.[13] Assumption R.2 is self-evident: imparting skills to children is costly.

[10]It is possible with no great gain in insight to extend the model to two parents per household.

[11]Conceivably, similar results can be obtained in a model with integer-valued family size and cross-household heterogeneity in parental fertility preferences, where we can interpret the n obtained in the current theory as the average number of children (conditional on parental economic status) that would arise in the richer model. Whether and when such "purification" can be achieved is a question for future research.

[12]This formulation is compatible with the possibility that skilled parents find it easier to educate their children, but we assume that the *net* monetary cost still rises with the wage.

[13]Our result concerning existence of interior steady states, however, does require the assumption of positive fixed costs.

Assumption R.3 states that the *marginal* cost impact of a higher parental wealth (relative to the overall upbringing budget) is lower for skilled children than for unskilled children.

Suppose, for instance, that there is a child-rearing component $k(w)$, and an additional cost $s(w)$ of imparting skills ("k" for kids, "s" for skills). Then $r_0(w) = k(w)$ and $r_1(w) = k(w) + s(w)$, and (R.1)–(R.3) are met provided that k and s are increasing and $k(w)/s(w)$ increases in w. In particular, (R.1)–(R.3) hold if $s(w)$ equals some fixed constant. Call this cost structure *separable*.

An important subcase of the separable structure is one in which rearing each child involves a certain amount of parental time alone. Then $k(w) = \psi w$ for some $\psi > 0$. We will use this for one of the main results.[14]

While the functions r_0 and r_1 are exogenous to the model, they can be influenced by policies pertaining to child-care subsidies, child labor regulations and costs of family planning. Section 5 of the paper will examine these effects.

Unlike most preceding models of fertility, we allow parents to educate some children but not others. Let e be the fraction of children made skilled; then total expenditure on children is given by $r(w, e) \equiv er_1(w) + (1 - e)r_0(w)$, so that the lifetime consumption of the parent is equal to

$$c = w - r(w, e)n,$$

which we constrain throughout to be nonnegative. This reflects the underlying credit constraint common to all occupational choice models, wherein education or child-rearing costs cannot be financed by borrowing and must entail consumption sacrifices made by parents.

2.3. **Preferences.** Each parent possesses, first, a utility indicator defined on lifetime consumption c, given by $u(c)$. The parent also derives utility from the lifetime payoff V to be enjoyed by each child. These latter values will be endogenous to the model and will be

[14]Another relevant subcase is one where $k(w) = f + \psi w$, the sum of a fixed goods cost and maternal time costs.

solved for in equilibrium. We write the overall payoff to a parent as

$$u(c) + \delta n^{\theta} \left[eV_1 + (1-e)V_0 \right]. \tag{1}$$

where δ is the cross-generational discount factor, n^θ is a weighting factor that depends on the total number of children n, e is the proportion of children who are skilled, and V_j is the expected lifetime values accruing to an individual who is placed in skill category j.

As in all the literature, we presume that, *controlling for their own consumption and for child utility,* parents prefer more children to less.[15] This means that θ and the value functions (and consequently the utility function u) must have the same sign. We therefore assume that

[U.1]. u is smooth, increasing, strictly concave, and has unbounded steepness when consumption is zero.

[U.2]. $\theta \neq 0$, and $\theta < 1$.

[U.3]. u is nonnegative throughout when $\theta > 0$, and negative throughout when $\theta < 0$.

[U.4]. $0 < \delta < \underline{r}^{\theta}$.

[U.1] is standard. In [U.2], the restriction that $\theta \neq 0$ means that parents are sensitive to family size, while the assumption that $\theta < 1$ reasonably imposes diminishing marginal returns to family size.[16] [U.3] embodies the discussion that follows equation (1) above. And [U.4] constrains the discount factor so as to ensure that value functions are stationary. In particular, the non-negativity of parental consumption and [R.1] ensure an upper bound $\frac{1}{\underline{r}}$ to fertility.[17] Hence [U.4] ensures that δn^θ always lies between 0 and 1.

Jones and Schoonbroodt (2009) contains more discussion on the joint restrictions that link θ and u, and on the need for u to have a single sign (thereby ruling out, say, the case

[15]We hasten to add that this does not mean that parents will *unconditionally* prefer more children to less! But it does exclude the case in which parents believe that never having been born is a better option than life.

[16]No corresponding restriction on θ needs to be imposed when it is negative.

[17]By [R.1], child-rearing costs are at least $\underline{r}.wn$, so parental consumption is at most $w[1-\underline{r}.n]$.

of logarithmic preferences). They — and most of the contributors to the literature on endogenous fertility — find it convenient to work with the case in which u has constant elasticity:

$$u(c) = \frac{c^{1-\rho}}{1-\rho},$$

where ρ is positive but not equal to 1, this last restriction ensuring that we are always in either the positive or the negative utility case. Accompanying restrictions need to be placed on θ: it must have the same sign as $1-\rho$. This is used in our main result (Proposition 9) concerning steady state characterization, though not for other results concerning the optimal fertility behavior of parents, or for the comparative statics results.

3. The Income-Fertility Relationship: Some Conceptual Issues

Suppose that we study fertility change over the parental cross-section, by examining a variety of parental incomes and associated fertility choices. This examination can be decomposed into two distinct parts.

First, there is the "partial equilibrium" effect, in which wages at different occupations are given, now and for the next generation. As we move over different parental incomes, we would allow parents to re-optimize, not just with respect to fertility but also with respect to occupational choice for their children. This is the exercise that we carry out in the next section, Section 4.

Second, there is the "general equilibrium" effect, in which the wages for different occupations are endogenously determined via market conditions. As far as the model goes, then, we would generate the wage observations at different points on the equilibrium cross-section of occupational categories, and the question is whether these *equilibrium* observations exhibit declining fertility at higher parental incomes. This is the subject of Section 5.

4. The Partial Equilibrium of Fertility Decline

4.1. **The Occupational Shift Effect.** In this section, we identify an effect that invariably works in favor of fertility decline as parents move their children from unskilled to skilled occupations. Because such a shift is also correlated with higher parental income, this creates what we call an "occupational shift effect" that relates parental income negatively to fertility, entirely independent of the specific structure of preferences.

Fix lifetime values V_1 and V_0 for children, with $V_1 > V_0$. Denote by $n(w,e)$ the optimal choice of n when wealth is w and the proportion of skilled children is e.[18] Using the first-order condition with respect to n,

$$u'\left(w - r(w,e)n(w,e)\right) r(w,e) = \theta n(w,e)^{\theta-1}\left[eV_1 + (1-e)V_0\right]. \tag{2}$$

This condition defines the function $n(w,e)$ uniquely at all $w > 0$.[19]

With $n(w,e)$ determined in this way, we can express parental utility as a function of e alone:

$$\begin{aligned} V(w,e) &\equiv u\left(w - r(w,e)n(w,e)\right) + n(w,e)^{\theta}\left[eV_1 + (1-e)V_0\right] \\ &= u\left(w - r(w,e)n(w,e)\right) + \frac{1}{\theta}u'\left(w - r(w,e)n(w,e)\right) r(w,e)n(w,e), \end{aligned} \tag{3}$$

where the second equality invokes the first-order condition (2). This expression allows us to establish the first of two basic propositions that underpin the paper.

Proposition **1.** *Under positive utility, the agent always acts as if she maximizes total expenditure on children, $r(w,e)n(w,e)$, by choosing e, given n.*

Under negative utility, the agent always acts as if she minimizes $r(w,e)n(w,e)$ by choosing e, given n.

Proof. First note that the expression

$$u(w-z) + \frac{u'(w-z)z}{\theta} \tag{4}$$

[18]For now we suppress the dependence of fertility on V_1, V_0 and other parameters. These will be made explicit whenever needed.

[19]Obviously, $n(0,e) = 0$. Also, note that the second-order condition for this maximization problem — given e — is always met. We approach the joint determination of e and n in the main text to follow.

is strictly increasing in z under positive utility, and is strictly decreasing in z under negative utility. To see this, note that the derivative of the expression above equals

$$\left[\frac{1}{\theta} - 1\right] u'(w-z) - \frac{u''(w-z)z}{\theta},$$

which is strictly positive in z under positive utility and strictly negative in z under negative utility.

To complete the proof compare $V(w, e)$ (as in the second line of (3)) with the expression in (4), by setting $z = r(w, e)n(w, e)$. ■

The proposition states that the a parent's choice of occupational mix follows a simple criterion of finding an extremal value for total child-related expenditures. She maximizes such expenditure (by choice of e) when utility is positive and minimizes it when utility is negative.

This property, which plays a key role, should not be misunderstood. It *does not* state that a parent maximizes or minimizes expenditure on children by choosing e *and* n. Rather, it states that a parent maximizes or minimizes expenditure *through the choice of* e *alone*, under the artificial presumption that she "then" chooses fertility n to maximize overall payoff.

Our second proposition states that a parent invariably finds it optimal to educate all or none of her children.

Proposition **2.** *A parent must always set* e *equal to 0 or 1.*

Proof. For any given value of w, set $x \equiv r_1(w) - r_0(w) > 0$. Differentiate the first line of (3) with respect to e and use the envelope theorem to get

$$\frac{\partial V(w,e)}{\partial e} = n(w,e)^{\theta}(V_1 - V_0) - u'(w - r(w,e)n(w,e))xn(w,e),$$

and now use the first-order condition (2) to write this as

$$\frac{\partial V(w,e)}{\partial e} = \frac{n(w,e)^{\theta}x}{r(w,e)} \left[\left\{\frac{r_0(w)}{x} + (1-\theta)e\right\}(V_1 - V_0) - \theta V_0\right]. \tag{5}$$

Of course, if $V_1 \leq V_0$, $e = 0$ is optimal. If $V_1 > V_0$ then (5) proves that V is strictly quasiconvex in e, for given w. It follows that no interior solution to e can ever maximize V. ■

Propositions 1 and 2 immediately imply

Proposition **3.** *Consider any wage w^* at which the parent is indifferent between $e = 0$ and $e = 1$. Then the expenditures on children must be equalized at such a wage:*

$$r_1(w^*)n(w^*, 1) = r_0(w^*)n(w^*, 0). \tag{6}$$

In particular, $n(w^*, 1) < n(w^*, 0)$ at a point of indifference.

Note that this proposition assures us of a fertility decline as soon as a dynasty crosses occupational boundaries towards higher skills. In contrast, in the traditional Barro-Becker setting the question of whether a fertility decline occurs with higher wealth depends on wealth versus substitution effects. We illustrate this further below.

A comparison of the "traditional" effect, which occurs over continuous changes in wage, and the effect in Proposition 2, which addresses the discontinuous crossing of occupational boundaries, is a principal theme of the paper. To set the stage for this analysis, we record the following single-crossing property:

Proposition **4.** *Suppose that both $e = 0$ and $e = 1$ are optimal for some parental wage w^*. Then $e = 1$ (resp. $e = 0$) is the unique optimum choice of e for all higher (resp. lower) wages.*

Proof. Define

$$\Delta(w) = u\left(w - r_1(w)n(w, 1)\right) + n(w, 1)^\theta V_1 - u\left(w - r_0(w)n(w, 0)\right) - n(w, 0)^\theta V_0$$

and differentiate with respect to w, evaluating the result at any point such that $\Delta(w) = 0$. By the envelope theorem applied to $n(w, 1)$ and $n(w, 0)$, we have that

$$\Delta'(w) = u'\left(w - r_1(w)n(w, 1)\right)[1 - r_1'(w)n(w, 1)] - u'\left(w - r_0(w)n(w, 0)\right)[1 - r_0'(w)n(w, 0)].$$

By Proposition 3, the terms within the two u's are exactly equal. It follows that the sign of $\Delta'(w)$ equals the sign of

$$r_0'(w)n(w,0) - r_1'(w)n(w,1) = \frac{r_0'(w)}{r_0(w)}r_0(w)n(w,0) - \frac{r_1'(w)}{r_1(w)}r_1(w)n(w,1).$$

By Proposition 3, we have $r_0(w)n(w,0) = r_1(w)n(w,1)$. Applying (R.3), we see that $\Delta'(w) > 0$ at any point of indifference between $e = 0$ and $e = 1$, which completes the proof. ■

We summarize these propositions. Proposition 2 tells us that parents either leave all their children unskilled, or find it optimal to educate them all; there are no halfway measures. This makes it possible to discuss the notion of an "occupational shift", in which parents switch at some threshold to educating all their children. Proposition 3 proves that at that shift, fertility must undergo a discrete jump downwards. Proposition 4 shows that the switch occurs as parental wages increase. Taken together, these propositions identify a neighborhood of parental wages over which the cross-sectional relation between fertility and parental wages is discretely, and therefore decisively, negative. This is the occupational shift effect.

4.2. **The Traditional Preference-Based Effect.** On either side of the occupational-shift threshold, however, the reaction of fertility to a change in parental wages is given by the conventional conflict between wealth and substitution effects. To describe this tradeoff in the clearest possible way, assume that the utility indicator u displays constant elasticity, so that

$$u(c) = \frac{c^{1-\rho}}{1-\rho},$$

where ρ is positive but not equal to 1. Presume, moreover, that for each occupational category i, rearing/educational costs are of the form

$$r_i = f_i + \psi_i w,$$

where $f_i \geq 0$ and $\psi_i \in (0,1)$. Let $\epsilon_i(w) \equiv \psi_i w/(f_i + \psi_i w)$ be the elasticity of r_i with respect to its argument w.

PROPOSITION 5. *Fix an occupation i with given wage for the children. Then $n_i \equiv n(w, i)$ is locally increasing in parental wage w if*

$$\rho > \frac{\epsilon_i(w) - \psi_i n_i}{1 - \psi_i n_i}, \tag{7}$$

and locally decreasing if the opposite inequality holds. In particular, if $\rho > 1$, then fertility is strictly increasing in parental wage.

Proof. Study the first order condition (2), and note that the right-hand side of this condition is independent of w, while it is strictly decreasing in n.[20] Therefore n_i is locally increasing in w if the derivative of the left-hand side of (2) with respect to w is negative, or equivalently, if

$$u'(c)r'(w) + u''(c)r(w)[1 - nr'(w)] < 0,$$

and is decreasing if the opposite inequality holds. After noting that $1 - \psi_i n_i \geq 1 - (f_i + \psi_i)n_i = c_i/w > 0$, it is easy to see that this expression reduces to condition (7).

Note that $\epsilon_i(w) \leq 1$, so as long as $\rho > 1$, fertility must increase with parental income. ■

Proposition 5 summarizes the familiar tussle between wealth and substitution effects that makes it so difficult to generate an unambiguous prediction of fertility decline in the Barro-Becker model. For instance, in the case in which there are no "material costs" of rearing children, we have $f_i = 0$ and $\epsilon_i(w) = 1$, so that the condition in (7) reduces precisely to $\rho > 1$ for fertility to increase with wages, and to $\rho < 1$ to get declining fertility. This latter condition corresponds to a high elasticity of intertemporal substitution in consumption. But as Jones and Schoonbroodt (2009) have noted, such high elasticities of intertemporal substitution are at odds with what is assumed in the literature on growth and business cycle theory.

When condition (7) is met, the income effects associated with rising parental wages outweigh the associated substitution effects, and fertility must rise with wages on either side of the threshold w^* for an occupational shift. This is in contrast to the discrete drop in fertility at the threshold. In short, the behavior of fertility across parental wages within each

[20]Note that continuation value V and θ have the same sign, and that $\theta < 1$.

occupational choice runs counter to the occupational shift effect. Which effect dominates? This requires an analysis of equilibrium conditions, to which we now turn.

5. The Structure of Steady States

5.1. **Intertemporal Equilibrium.** A study of dynamic equilibrium with dynastic households requires three avenues of closure for the model. First, skilled and unskilled wages in every period must depend on the proportion of skilled labor in that period. Second, the continuation values that parents take as given must be identified with the maximum payoffs to their children as they grow up to be adults. Finally, given the aggregate skill ratio at date t, the skill and fertility choices made by generation t must determine the aggregate skill ratio at date $t+1$.

Formally, a *dynamic competitive equilibrium* for the economy starting with skill ratio λ_0 in generation 0 is described by the following objects, satisfying the restrictions described below:

(a) *Skilled and unskilled wages* (w_{1t}, w_{0t}) and *aggregate skill ratios* λ_t at every date, with $w_{1t} = w_1(\lambda_t)$ and $w_{0t} = w_0(\lambda_t)$;

(b) *Continuation values* V_{1t} and V_{0t} at every date, with every parent in either skill category $i = 0, 1$ seeking to maximize

$$u(w_{it} - r_j(w_{it})n)n + \delta n^{\theta} V_{j,t+1} \tag{8}$$

by choosing fertility n and educational category $j \in \{0, 1\}$ for their children;

(c) *Maximized values* of the payoffs in (8), which must equal the continuation values V_{it} for each category i and date t;

(d) *Fertility choice* by every parent in category i, at every date, conditional on the choice of skill category j for children: $n_{it}(j)$, which are the optimal solutions for n to the maximization of (8), *given* category choice j for children;

(e) *Fractions of parents* in each skill category and at each date, η_{1t} and η_{0t}, that choose the skilled category for their children, with $\eta_{it} \in (0,1)$ only if both category choices are optimal for parents in category i at date t.

(f) *Evolution of aggregate skill ratios*: λ_0 is given, and

$$(9) \qquad \lambda_{t+1} = \frac{\lambda_t \eta_{1t} n_{1t}(1) + (1-\lambda_t)\eta_{0t} n_{0t}(1)}{\lambda_t[\eta_{1t} n_{1t}(1) + (1-\eta_{1t})n_{1t}(0)] + (1-\lambda_t)[\eta_{0t} n_{0t}(1) + (1-\eta_{0t})n_{0t}(0)]}$$

5.2. **Steady States.** A *steady state* has the additional feature that all time subscripts can be dropped from the definition above: wages, continuation values, skill ratios and fertility choices must all be stationary (though, to be sure, the aggregate population might change over time). We also require that output be positive. In a steady state, then,

$$(10) \qquad \lambda = \frac{\lambda \eta_1 n_1(1) + (1-\lambda)\eta_0 n_0(1)}{\lambda[\eta_1 n_1(1) + (1-\eta_1)n_1(0)] + (1-\lambda)[\eta_0 n_0(1) + (1-\eta_0)n_0(0)]} > 0.$$

The positive output requirement means that we ignore the trivial and uninteresting configuration in which there are no skilled people, there is a huge (infinite) skill premium, and yet the unskilled do not acquire any skills because their wages are zero.[21]

A steady-state proportion of skills can be characterized as follows. For each λ, and given the attendant wages $w_1(\lambda)$ and $w_0(\lambda)$, define $V_0(\lambda)$ and $V_1(\lambda)$ as the unique solutions to the following conditions (which by virtue of [U4] generates a contraction mapping from continuation values to current values):

$$(11) \qquad V_i(\lambda) = \max_n [u(w_i(\lambda) - r_i(w_i(\lambda))n) + \delta n^\theta V_i(\lambda)].$$

It is easy to see that $V_1(\lambda)$ decreases in λ while $V_0(\lambda)$ is increasing, that $V_1(\lambda)$ exceeds $V_0(\lambda)$ for low enough values of λ, while the opposite inequality is true at higher values (say for $\lambda \geq \bar{\lambda}$).

Provisionally, think of the $V_i(\lambda)$ defined in this way as the continuation values for children in each skill category. They can't always be the "true" continuation values, as parents may

[21]This configuration is always an equilibrium if $r_0(0) > 0$.

want to switch categories, but in steady state this interpretation will be exactly correct, as non-switching of categories must always be optimal.

Given these values, there exists a parental income threshold $w^*(\lambda)$ at which a parent is just indifferent between imparting skills to all her progeny, or leaving them all unskilled. From Proposition 4, we know that $w^*(\lambda)$ is uniquely defined (it may be infinite). Moreover, if parental wage strictly exceeds $w^*(\lambda)$, the parent has a strict preference for skilled children, while if it is strictly less, she has a strict preference for unskilled children. It follows that a *necessary* condition for $\lambda > 0$ to be a steady-state skill proportion is

$$w_0(\lambda) \leq w^*(\lambda) \leq w_1(\lambda),$$

with, of course, at least one of these inequalities holding strictly.[22]

But this isn't enough. Imagine, for instance, that both inequalities hold strictly. Then $\eta_1 = 1$ and $\eta_0 = 0$, so (10) implies that

$$\lambda = \frac{\lambda n_1(1, \lambda)}{\lambda n_1(1, \lambda) + (1 - \lambda) n_0(0, \lambda)},$$

where $n_i(j, \lambda)$ is the optimally chosen fertility by a parent in category i under the assumption that her children go to category j. But this equality calls for the additional requirement that $n_0(0, \lambda) = n_1(1, \lambda)$.

The following observation contains a full characterization:

OBSERVATION **1.** *A skill proportion $\lambda > 0$ is part of a steady state if and only if*

$$w_0(\lambda) \leq w^*(\lambda) \leq w_1(\lambda), \tag{12}$$

with at least one of these inequalities strict, and:

(a) *If $w_0(\lambda) = w^*(\lambda)$, then $n_0(0, \lambda) \geq n_1(1, \lambda)$.*

(b) *If $w^*(\lambda) = w_1(\lambda)$, then $n_0(0, \lambda) \leq n_1(1, \lambda)$.*

(c) *If $w_0(\lambda) < w^*(\lambda) < w_1(\lambda)$, then $n_0(0, \lambda) = n_1(1, \lambda)$.*

[22] After all, if $w_0(\lambda) = w_1(\lambda)$, then $w^*(\lambda) = \infty$.

Proof. The discussion preceding the Observation already establishes the necessity of (12), as well as part (c). Parts (a) and (b) are established in similar fashion. For instance, to establish (a), suppose that $w_0(\lambda) = w^*(\lambda)$; then $w^*(\lambda) < w_1(\lambda)$. It follows that skilled parents strictly prefer skilled children, so that $\eta_1 = 1$. Therefore — because $n_i(j) = n_i(j, \lambda)$ for every i and j — (10) implies that

$$\begin{aligned}
\lambda &= \frac{\lambda n_1(1,\lambda) + (1-\lambda)\eta_0 n_0(1,\lambda)}{\lambda n_1(1,\lambda) + (1-\lambda)[\eta_0 n_0(1,\lambda) + (1-\eta_0) n_0(0,\lambda)]} \\
&\geq \frac{\lambda n_1(1,\lambda)}{\lambda n_1(1,\lambda) + (1-\lambda)(1-\eta_0) n_0(0,\lambda)} \\
&\geq \frac{\lambda n_1(1,\lambda)}{\lambda n_1(1,\lambda) + (1-\lambda) n_0(0,\lambda)},
\end{aligned}$$

which implies right away that $n_0(0, \lambda) \geq n_1(1, \lambda)$. Part (b) is established in a parallel way.

To establish sufficiency, pick $\lambda > 0$ such that (12) and one of (a)–(c) are satisfied. Let the associated wages be $w_1 = w_1(\lambda)$ and $w_0 = w_0(\lambda)$ and associated continuation values be $V_1 = V_1(\lambda)$ and $V_0 = V_0(\lambda)$, as given by (**??**). Let $n_i(j) = n_i(j, \lambda)$ for every i and j. If case (a) applies, we have $n_0(0, \lambda) \geq n_1(1, \lambda)$, so that

$$\lambda \geq \frac{\lambda n_1(1,\lambda)}{\lambda n_1(1,\lambda) + (1-\lambda) n_0(0,\lambda)}.$$

Of course, $w^*(\bar{\lambda}) = \infty$, which means that $\lambda < \bar{\lambda} < 1$. It is therefore easy to see that there exists $\eta_0 \in [0, 1)$ such that

$$\lambda = \frac{\lambda n_1(1,\lambda) + (1-\lambda)\eta_0 n_0(1,\lambda)}{\lambda n_1(1,\lambda) + (1-\lambda)[\eta_0 n_0(1,\lambda) + (1-\eta_0) n_0(0,\lambda)]}.$$

Choose this value of η_0 and set $\eta_1 = 1$, and now check that all conditions for a steady state are satisfied. In particular, (12) guarantees that it is optimal never to switch categories, so that the V_i's represent the true continuation values.

Similar arguments apply for cases (b) or (c). ∎

Observation 1 allows us to prove the existence of a (non-trivial) steady state. The proof is of interest in its own right, as it says a bit more about the structure of steady states.

To ensure that the steady state has positive output and skill ratio, however we need to impose the assumption of positive fixed costs of child-rearing.[23]

Proposition **6.** *There exists a steady state with* $\lambda > 0$*, provided* $r_0(0) > 0$ *.*

Proof. We display $\lambda > 0$ such that (12) and one of the conditions in (a)–(c) of Observation 1 is met. Observe that $V_1(\lambda)$ is decreasing and continuous in λ, while $V_0(\lambda)$ is increasing and continuous in λ. It is easy to conclude that $w^*(\lambda)$ is continuous in λ (in the extended reals) and that it is strictly increasing as long as it is finite.[24]

On the other hand, $w_1(\lambda)$ is continuous and decreasing in λ, with the assumed end-point conditions.

We must conclude that there exists (unique) $\lambda_1 > 0$ such that $w_1(\lambda_1) = w^*(\lambda_1)$. If at this value, $n_1(1, \lambda_1) \geq n_0(0, \lambda_1)$, we are done (use part (b) of Observation 1).

Otherwise $n_1(1, \lambda_1) < n_0(0, \lambda_1)$. It is obvious that $n_1(1, \lambda)$ is bounded away from 0 as $\lambda \to 0$ (both parental income and $V_1(\lambda)$ go to infinity). On the other hand, given that $r_0(0) > 0$, it must be that $n_0(0, \lambda) \to 0$. It is easy to see that, moreover, that $n_i(i, \lambda)$ is continuous for $i = 0, 1$. It follows that there exists a *largest* value of λ smaller than λ_1 — call it λ_2 — such that $n_1(1, \lambda_2) \geq n_0(0, \lambda_2)$.

Indeed, by continuity of $n_i(i, \lambda)$, we must have $n_1(1, \lambda_2) = n_0(0, \lambda_2)$. Also, $w^*(\lambda_2) \leq w_1(\lambda_2)$.[25] If $w_0(\lambda_2) \leq w^*(\lambda_2)$ as well, then we are again done (use part (c) of Observation 1).

Otherwise $w_0(\lambda_2) > w^*(\lambda_2)$. Define λ_3 to be the smallest value of $\lambda > \lambda_2$ such that $w_0(\lambda_3) = w^*(\lambda_3)$. It is obvious that $\lambda_3 \in (\lambda_2, \lambda_1)$.[26] We claim that $n_1(1, \lambda_3) < n_0(0, \lambda_3)$. This follows right away from our definition of λ_2 as the *largest* value of λ smaller than λ_1,

[23]In the absence of such fixed costs, a steady state exists but may involve zero output and skill ratio. Whether an interior steady state can be shown to exist in the absence of this assumption remains an open question.

[24]That is, $w^*(\lambda)$ is increasing and continuous whenever it is finite, $w^(\lambda') = \infty$ if $\lambda' > \lambda$ and $w^*(\lambda) = \infty$, and $w^*(\lambda_n) \to \infty$ if $\lambda_n \to \lambda$ and $w^*(\lambda) = \infty$.

[25]After all, $w_1(\lambda_1) = w^*(\lambda_1)$, the former function is declining in λ, the latter increasing in λ, and $\lambda_2 < \lambda_1$.

[26]After all, at λ_1 we have $w^*(\lambda_1) = w_1(\lambda_1) > w_0(\lambda_1)$, the strict inequality following from the fact that $w^*(\lambda_1)$ is finite.

such that $n_1(1, \lambda_2) \geq n_0(0, \lambda_2)$. Now the condition in part (a) of Observation 1 is met, and the proof is complete. ■

5.3. **Fertility and Mobility in Steady State.** Observation 1 sets the stage for our study of mobility across occupations when fertility is endogenous. As already discussed, such mobility is necessitated by variations in fertility across occupational categories: in the absence of any mobility (i.e., each parent preparing all her children for the same occupation as hers) such fertility differentials would imply that the skill ratio would not remain steady from one generation to the next.

Proposition **7.** *Only three kinds of steady states are possible:*

(a) *A steady state characterized by* upward mobility, *in which* $n_1(1) < n_0(0)$, *with* $\eta_1 = 1$ *and* $\eta_0 > 0$.

(b) *A steady state characterized by* downward mobility, *in which* $n_1(1) > n_0(0)$, *with* $\eta_1 < 1$ *and* $\eta_0 = 0$.

(c) *A steady state characterized by* no mobility, *in which* $n_1(1) = n_0(0)$, *with* $\eta_1 = 1$ *and* $\eta_0 = 0$.

Proof. A steady state must have either (i) $n_1(1) < n_0(0)$, (ii) $n_1(1) > n_0(0)$, or (iii) $n_1(1) = n_0(0)$. We show that these three cases must respectively correspond to (a)–(c) in the statement of the proposition.

Consider case (i), in which $n_1(1) < n_0(0)$. Then part (a) of Observation 1 is applicable, so that $w_0 = w^*(\lambda) < w_1$. It follows that $\eta_1 = 1$. We claim, moreover, that $\eta_0 > 0$. Suppose not, then $\eta = 0$, and so, using (10) with $\eta_1 = 1$,

$$\lambda = \frac{\lambda n_1(1)}{\lambda n_1(1) + (1 - \lambda) n_0(0)} < \lambda,$$

a contradiction. The proofs of the remaining cases are very similar. ■

Each of the three kinds of steady states unearthed in Proposition 7 is associated with a distinctive relationship between fertility and parental wages:

Proposition 8. *A steady state with upward mobility must involve declining average fertility over equilibrium wealths; that is,*

$$\eta_1 n_1(1) + (1-\eta_1)n_1(0) < \eta_0 n_0(1) + (1-\eta_0)n_0(0). \tag{13}$$

Exactly the opposite is true of a steady state with downward mobility, while fertility is unchanged over equilibrium wealths for a steady state with zero mobility.

Proof. We prove the proposition for steady states with upward mobility; the other parts can be established in very similar ways.

Consider a steady state with upward mobility. By Proposition 7, part (a), we have $\eta_1 = 1$ and $\eta_0 > 0$, so we need to show that

$$n_1(1) < \eta_0 n_0(1) + (1-\eta_0)n_0(0). \tag{14}$$

Using (10) with $\eta_1 = 1$, we have that

$$\begin{aligned} \lambda &= \frac{\lambda n_1(1) + (1-\lambda)\eta_0 n_0(1)}{\lambda n_1(1) + (1-\lambda)[\eta_0 n_0(1) + (1-\eta_0)n_0(0)]} \\ &> \frac{\lambda n_1(1)}{\lambda n_1(1) + (1-\lambda)[\eta_0 n_0(1) + (1-\eta_0)n_0(0)]}, \end{aligned}$$

where the inequality uses the facts that $\lambda \in (0,1)$, $\eta_0 > 0$, and $n_i(j) > 0$ for all i and j. Cross-multiplying and transposing terms, we see that

$$\lambda(1-\lambda)[\eta_0 n_0(1) + (1-\eta_0)n_0(0)] > \lambda(1-\lambda)n_1(1)$$

which establishes (14), and completes the proof. ∎

This proposition establishes a precise connection between the different kinds of steady states and fertility movements across the cross-section of observed incomes.[27] These mobility patterns are of interest *per se*, because they arise in a model with no stochastic shocks to ability or incomes. The key to such patterns in our model is endogenously varying fertility across income (and occupational) categories.

While downward mobility from skilled to unskilled occupations are relatively rare and perhaps explained by bad luck, upward mobility in the reverse direction is usually considered a reflection of determined investments made by the poor and accordingly are held as an indication of equality of opportunity in the society in question. So it may be of interest to provide an equilibrium explanation of this that does not rely on "luck".[28]

The connection between upward mobility and declining fertility over the cross-section of parental wages is of particular interest. Both phenomena are empirically plausible, while sustained downward mobility (or increasing fertility on the cross-section) are not. It is therefore useful to know that the model ties the two phenomena closely together.

To go further, we need to ask the question: to what extent can steady states with downward or zero mobility be ruled out by the theory? We turn to this issue next.

6. The General Equilibrium of Fertility Decline

A steady state generates two observations along the parental cross-section of wages, corresponding to parents in the skilled and unskilled categories. The question is: does fertility decline as we move from the lower-income to higher-income categories, or equivalently — by part (a) of Proposition 6 — from unskilled to skilled categories? This is what we refer

[27]It should be noted that this connection is *not* made in the earlier Proposition 7. That proposition states, for instance, that in a steady state with upward mobility, we have $n_1(1) < n_0(0)$, but while the first of these terms is indeed the average fertility among skilled parents in such a steady state, the second of these terms is not. The reason is that in a steady state with upward mobility, a fraction η_0 of unskilled parents do have skilled children, so that the *average* fertility among unskilled parents is really

$$\eta_0 n_0(1) + (1 - \eta_0) n_0(0),$$

and it is this expression that needs to be compared with $n_1(1)$, which we do in (14).

[28]To be sure, this is not the only explanation. One can derive a general theory of (nonstochastic) mobility by taking recourse to any factor that creates "unbalanced" intertemporal changes in steady state. Differential fertility is one such factor. One might think of others, such as nonhomothetic preferences or skill-biased technical change.

to as the "general equilibrium effect". It incorporates not only parental fertility responses, but also the endogeneity of incomes in steady state. That endogeneity places bounds on the traditional "preference-based" effect relative to the "occupational-shift" effect.

Notice that in either category, fertility is not necessarily the same across parents; it will vary with the occupational choice that those parents make. Recall that average fertility in category i is

$$\eta_i n_i(1) + (1 - \eta_i) n_i(0),$$

where $\eta_i(1)$ is the fraction of parents from category i who put their children into the skilled category 1, with attendant fertility $n_i(1)$, and $1 - \eta_i(1)$ is the remaining fraction who choose the unskilled category for their children, with attendant fertility $n_i(0)$.

For our result, we specialize to the case of constant-elasticity utility functions. Most existing literature on endogenous fertility restricts attention to this class anyway. Thus write:

$$u(c) = \frac{c^{1-\rho}}{1-\rho},$$

where ρ is positive but not equal to 1, this last restriction ensuring that we are always in either the positive or the negative utility case. As before, accompanying restrictions need to be placed on θ: it must have the same sign as $1 - \rho$.

6.1. **Main Result.** We provide conditions under which the occupational-shift effect invariably dominates the traditional preference-based determinants of fertility choice, so that average fertility declines with income:

Proposition 9. *Assume that utility is isoelastic and that childrearing and educational functions satisfy* [R.1]–[R.3]. *Suppose, in addition, that at least one of the following three conditions hold:*

(a) *We are in the positive utility case* $(0 < 1 - \rho < 1)$ *with* $0 < \theta \leq 1 - \rho$.

(b) *We are in the negative utility case* $(1 - \rho < 0)$ *with* $0 > \theta \geq 1 - \rho$.

(c) *The cost structure is separable, and involves time-costs alone; child-rearing costs take the form* $k(w) = kw$.

Then all steady states must involve lower average fertility in the higher occupational category. Equivalently, by Proposition 8, every steady state must exhibit upward mobility.

Before proceeding to a formal proof, some discussion of this result will be useful.

While conditions (a)–(c) do not cover all the possible cases under isoelastic utility, Proposition 9 exhibits a broad range of parameter values for which steady states must involve upward mobility. Range (a) covers a region where the result is perhaps to be expected, in which substitution effects associated with parental wage increases are large relative to wealth effects. That ensures that the "traditional" effect of wage increases on fertility (keeping child quality constant) is negative. This is reinforced by the "occupational shift" or child quality effect to imply that unskilled households must have higher fertility, thereby necessitating upward mobility in every steady state.

More surprising are parts (b) and (c), which allow wealth effects to outweigh substitution effects to imply that the "traditional" wage effect on fertility is positive, as highlighted by condition (7) of Proposition 5. Under the conditions of our proposition, these are outweighed by the occupational-switch effect. The size of the traditional effect depends on the wage differential between skilled and unskilled households, unlike the occupational shift effect. The parameter assumptions in parts (b) and (c) ensure that the wage differentials are not large enough for the traditional effect to dominate. Here the general equilibrium of occupational choice predicts a net fertility decline over observed wages. This is perhaps one of the few cases in which general equilibrium considerations remove ambiguities present under partial equilibrium, instead of adding to them.

To elaborate further, consider any steady state, with wages w_0 and w_1. Even though none of the intermediate wages are observed in this deterministic model, compare fertility by mentally "moving" parental wages from w_0 to w_1 over three (artificial) stages. First, move from w_0, at which unskilled children are preferred, to the threshold w^* at which parents are indifferent between unskilled and skilled children. Fertility changes here are necessarily ambiguous, and follow the traditional theory. The second stage is the abrupt drop in fertility that occurs *at* w^*; this is the "occupational shift" effect. Finally, there is the move from w^* to the skilled wage w_1, over which fertility changes are once again described by the traditional theory.

For steady states with downward or upward mobility, one of these three zones is absent. In a steady state with downward mobility, some skilled parents must want to have unskilled children, which means that $w^* = w_1$. For that steady state to exist, then, the "traditional stretch" between w_0 and w^* must generate enough of an increase in fertility to counteract the downward drop at w^*. In mirror-image fashion, a steady state with upward mobility requires some unskilled parents to have skilled children, which implies that $w^* = w_0$. For this steady state to exist, the abrupt drop in fertility at w^* must dominate any corresponding rise along the "traditional stretch" between w^* and w_1. And for steady states with zero mobility, all three stages are generally nondegenerate, and the overall effect of fertility change along the two "traditional stretches" must exactly offset the drop at w^*.

There are two distinct arguments used in the proof of the proposition. The first runs via Lemma 1 and exploits the steady state condition which requires that unskilled parents must not *strictly* prefer to have skilled children. This places a bound on how different the two wages w_0 and w_1 can be. In particular, Lemma 1 establishes that in a steady state — and under our assumed restrictions — a suitable variant of a condition described by Moav (2005) must *endogenously* be satisfied. This is the condition that child-rearing is relatively more expensive for skilled parents: $\frac{r_1(w_1)}{w_1} > \frac{r_0(w_0)}{w_0}$. (Note well that in our framework the condition must be derived, as wages are not fixed.) Under further restrictions on preferences, this condition suffices to generate declining fertility over the cross-section.

To circumvent some (though, unfortunately, not all) of these restrictions, our proof adopts a different line for some of the cases. This part of the proof directly rules out steady states with downward mobility, instead of proceeding via Moav's condition. Steady states with downward mobility require that skilled parents be indifferent between educating and not educating their children, and this goes towards establishing a different bound on wage ratios, and consequently a complementary and different line of proof.[29]

The question arises whether Proposition 9 extends to both the positive and negative utility models free of charge. Based on our intensive but fruitless search for a counterexample, we tentatively conjecture that the answer to this question is in the affirmative.

6.2. **Proof of Proposition 9.** For ease of notation, denote $n_0(0)$ by n_0 and $n_1(1)$ by n_1. We use the following

LEMMA **1.** *Suppose that at least one of the conditions in Proposition 9 holds. Then in any steady state,*

$$\frac{w_1}{r_1(w_1)} < \frac{w_0}{r_0(w_0)}. \tag{15}$$

Proof. Suppose that (15) is false in some steady state. Then $w_1 - r_1(w_1)n_0 \geq 0$,[30] and so, because n_1 is an optimal choice at parental wage w_1,

$$V_1 = \frac{[w_1 - r_1(w_1)n_1]^{1-\rho}}{1-\rho} + \delta n_1^\theta V_1 \geq \frac{[w_1 - r_1(w_1)n_0]^{1-\rho}}{1-\rho} + \delta n_0^\theta V_1.$$

It follows that

$$V_1 \geq \frac{[w_1 - r_1(w_1)n_0]^{1-\rho}}{(1-\rho)(1-\delta n_0^\theta)} \geq \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} \frac{[w_0 - r_0(w_0)n_0]^{1-\rho}}{(1-\rho)(1-\delta n_0^\theta)} = \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} V_0. \tag{16}$$

Now consider a parent with wage w_0. Suppose that she chooses a fertility of $\tilde{n}$, where

$$\tilde{n} r_1(w_0) = n_0 r_0(w_0), \tag{17}$$

[29]It should be noted that these parametric restrictions are typically not considered in the traditional theory; see, e.g., Barro and Becker (1988, page 483) and Jones and Schoonbroodt (2009, Assumptions AI and AII). The usual grounds for ruling these cases out is that second-order conditions for maximization may not hold. Our arguments do not rely on any second-order conditions and we therefore do not impose these restrictions. Indeed, the appropriate second-order condition is always met; see footnote 19.

[30]Because $w_0 - r_0(w_0)n_0 \geq 0$, we have $\frac{w_0}{r_0(w_0)} \geq n_0$. Now use the negation of (15).

and educates all her children. Then her overall payoff is given by

$$\hat{V}_0 \equiv \frac{[w_0 - r_1(w_0)\tilde{n}]^{1-\rho}}{1-\rho} + \delta \tilde{n}^{\theta} V_1 \tag{18}$$

$$\geq \frac{[w_0 - r_1(w_0)\tilde{n}]^{1-\rho}}{1-\rho} + \delta \tilde{n}^{\theta} \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} V_0 \tag{19}$$

$$= \frac{[w_0 - r_0(w_0)n_0]^{1-\rho}}{1-\rho} + \delta {n_0}^{\theta} \left[\frac{r_0(w_0)}{r_1(w_0)}\right]^{\theta} \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} V_0, \tag{20}$$

where the first inequality follows from (16), and the last equality from (17).

Now, if there is positive utility and $1 - \rho \geq \theta$, as assumed, then

$$\left[\frac{r_0(w_0)}{r_1(w_0)}\right]^{\theta} \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} \geq \left[\frac{r_0(w_0)}{r_1(w_0)}\right]^{1-\rho} \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} = \left[\frac{r_1(w_1)}{r_1(w_0)}\right]^{1-\rho} > 1, \tag{21}$$

where the first inequality uses (R.2), and the last inequality uses (R.1).

On the other hand, if there is negative utility and $1 - \rho \leq \theta$, as assumed, then

$$\left[\frac{r_0(w_0)}{r_1(w_0)}\right]^{\theta} \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} \leq \left[\frac{r_0(w_0)}{r_1(w_0)}\right]^{1-\rho} \left[\frac{r_1(w_1)}{r_0(w_0)}\right]^{1-\rho} = \left[\frac{r_1(w_1)}{r_1(w_0)}\right]^{1-\rho} \leq 1, \tag{22}$$

where (R.1) and (R.2) are used again at exactly the same points.

Noting that $V_0 > 0$ in the positive utility case and $V_0 < 0$ in the negative utility case, we can use (21) or (22) in equation (20) (depending on the case we are in) to conclude that

$$\hat{V}_0 > \frac{[w_0 - r_0(w_0)n_0]^{1-\rho}}{1-\rho} + \delta {n_0}^{\theta} V_0 = V_0,$$

which violates part (b) of Proposition 6 for a steady state.

Finally, if condition (c) of Proposition 9 holds, (15) is obtained free of charge. For

$$\frac{w_1}{r_1(w_1)} = \frac{w_1}{kw_1 + x(w_1)} < \frac{1}{k} = \frac{w_0}{r_0(w_0)}.$$

■

Now we turn to the main proof. The two first-order conditions for the choice of n_0 and n_1 tell us that

$$u'(c_0)r_0(w_0)n_0^{1-\theta} = \frac{\theta\delta u(c_0)}{1-\delta n_0^\theta} \text{ and } u'(c_1)r_1(w_1)n_1^{1-\theta} = \frac{\theta\delta u(c_1)}{1-\delta n_1^\theta}.$$

Using the constant elasticity specification, these equalities imply that

$$\frac{1-\rho}{\theta}\frac{r_0(w_0)}{c_0} = \frac{\delta n_0^{\theta-1}}{1-\delta n_0^\theta} \text{ and } \frac{1-\rho}{\theta}\frac{r_1(w_1)}{c_1} = \frac{\delta n_1^{\theta-1}}{1-\delta n_1^\theta}, \tag{23}$$

and combining,

$$\frac{c_1}{c_0} = \frac{r_1(w_1)n_0^{\theta-1}(1-\delta n_1^\theta)}{r_0(w_0)n_1^{\theta-1}(1-\delta n_0^\theta)}. \tag{24}$$

If there is a steady state with zero mobility, so that $n_0 = n_1$, then (24) immediately implies that

$$\frac{w_1 - r_1(w_1)n_1}{r_1(w_1)n_1} = \frac{c_1}{r_1(w_1)n_1} = \frac{c_0}{r_0(w_0)n_0} = \frac{w_0 - r_0(w_0)n_0}{r_0(w_0)n_0},$$

and using $n_0 = n_1$ once again, we must conclude that

$$\frac{w_1}{r_1(w_1)} = \frac{w_0}{r_0(w_0)},$$

which contradicts the assertion (15) of Lemma 1.

We now eliminate steady states with downward mobility. The following lemma completes part of this task:

Lemma **2.** *If $\rho+\theta \geq 1$ in the positive utility case, and without any further assumptions in the negative utility case, n_i and $w_i/r_i(w_i)$ must co-move over the two occupations $i = 0, 1$.*

Proof. Recall (23). Define $\alpha \equiv (1-\rho)/\theta$ (always a positive number) and $\mu_i \equiv w_i/r_i(w_i)$ for $i = 0, 1$. Then (23) can be written as

$$\delta(\alpha-1)n_i^\theta + \delta\mu_i n_i^{\theta-1} = \alpha \tag{25}$$

for $i = 0, 1$. The left hand side of this expression is strictly increasing in μ_i. By using a standard argument, we establish the co-movement of n_i and μ_i if we can show that the derivative of the left hand side in n_i is strictly negative, evaluated at the equality in (25). To this end, drop the i-subscripts, define

$$D(n) \equiv \delta(\alpha-1)n^{\theta} + \delta\mu n^{\theta-1}$$

and differentiate with respect to n to see that

$$D'(n) = \delta(\alpha-1)\theta n^{\theta-1} + \delta\mu(\theta-1)n^{\theta-2}$$

So we are already done in the positive utility case under the assumption that $\rho+\theta \geq 1$, for then $\alpha \leq 1$ and $\theta < 1$. Otherwise, we are in the negative utility case, and

$$\begin{aligned} nD'(n) &= \theta[\delta(\alpha-1)n^{\theta} + \delta\mu n^{\theta-1}] - \delta\mu n^{\theta-1} \\ &= \theta\alpha - \delta\mu n^{\theta-1}, \end{aligned}$$

where the last equality uses (25). This expression is negative, because $\theta < 0$. ■

Combining Lemmas 1 and 2, the proof of the proposition is complete under all conditions except (a). In the remainder of the proof, then, we concentrate on the positive utility case with $\rho+\theta \leq 1$.

Suppose, on the contrary, that a steady state displays downward mobility in case (a). Then parents in occupation 1 must be indifferent between continuing with occupation 1 and shifting their children (after re-optimizing fertility) to occupation 0. Denote by $\hat{n} = n_1(0)$ the number of children that an occupation-1 parent would choose if she were switching her progeny to occupation 0. Then, by indifference, we have

$$u(c_1) + \frac{\delta n_1^{\theta} u(c_1)}{1-\delta n_1^{\theta}} = u(w_1 - r_0(w_1)\hat{n}) + \frac{\delta \hat{n}^{\theta} u(c_0)}{1-\delta n_0^{\theta}} = u(c_1) + \frac{\delta \hat{n}^{\theta} u(c_0)}{1-\delta n_0^{\theta}},$$

where the second equality follows Proposition 1: total expenditure on children must be equalized at a switch point. Consequently, continuation utilities are equalized, and using the constant-elasticity specification, we obtain

$$\left(\frac{c_1}{c_0}\right)^{1-\rho} = \frac{u(c_1)}{u(c_0)} = \frac{\hat{n}^{\theta}(1-\delta n_1^{\theta})}{n_1^{\theta}(1-\delta n_0^{\theta})} = \frac{r_1(w_1)^{\theta}(1-\delta n_1^{\theta})}{r_0(w_1)^{\theta}(1-\delta n_0^{\theta})}.$$

Combining this equation with (24), we see that

$$\frac{r_1(w_1)n_0^{\theta-1}(1-\delta n_1^\theta)}{r_0(w_0)n_1^{\theta-1}(1-\delta n_0^\theta)} = \frac{r_1(w_1)^{\theta/(1-\rho)}(1-\delta n_1^\theta)^{1/(1-\rho)}}{r_0(w_1)^{\theta/(1-\rho)}(1-\delta n_0^\theta)^{1/(1-\rho)}},$$

or

$$\frac{r_1(w_1)r_0(w_1)^{\theta/(1-\rho)}}{r_0(w_0)r_1(w_1)^{\theta/(1-\rho)}} = \frac{n_0^{1-\theta}(1-\delta n_1^\theta)^{\rho/(1-\rho)}}{n_1^{1-\theta}(1-\delta n_0^\theta)^{\rho/(1-\rho)}}. \tag{26}$$

Because our steady state has downward mobility, we have $n_1 > n_0$, which implies that the right-hand side of (26) is strictly smaller than 1 under condition (a).

On the other hand, the left hand side is given by

$$\frac{r_1(w_1)r_0(w_1)^{\theta/(1-\rho)}}{r_0(w_0)r_1(w_1)^{\theta/(1-\rho)}} = \frac{r_1(w_1)^{(1-\theta-\rho)/(1-\rho)}r_0(w_1)^{\theta/(1-\rho)}}{r_0(w_0)} \geq 1,$$

given the assumptions of the Proposition as well as (R.1) and (R.2). This contradiction completes the proof. ■

7. Steady State Determinacy and Comparative Statics

7.1. **Local Determinacy.** We now turn to the question of local determinacy of steady states. Local determinacy permits us to derive the effects of changed policies, but is of substantive intrinsic interest as well. It bounds the extent of hysteresis or history-dependence that the model permits. In contrast, most models of occupational choice with a discrete set of occupations and exogenous fertility are characterized by a continuum of steady states. We show that incorporation of endogenous fertility into the model eliminates this indeterminacy.

The intuitive basis for this finding is that steady states with either upward or downward mobility can no longer be associated with *strict* incentives for members of *both* occupations: at least one occupation must be indifferent between preparing their children for the same occupation and switching to the alternative occupation. This indifference ties down the steady state skill ratio and per-capita income. And if a steady state involves no mobility, it requires equality of fertility decisions across the two occupations, which also ties down relative wages and hence the equilibrium skill ratio.

To show this formally, introduce a parameter ν of the costs of educating children, and suppose that costs of children prepared for the unskilled occupation $r_0(w)$ is independent of ν, while the costs of children $r_1(w;\nu)$ trained for the skilled occupation is strictly (and smoothly) increasing in ν.

PROPOSITION **10.** *Skill ratios forming a steady state with upward or downward mobility are locally unique and finite in number, for a set of parameter values ν of full Lebesgue measure. The same is true for steady states with zero mobility, provided θ is positive.*

Proof. Recall the characterization of steady states in Observation 1: a steady state with downward mobility satisfies $w^*(\lambda) = w_1(\lambda)$ and with upward mobility satisfies $w^*(\lambda) = w_0(\lambda)$. So for either of these kinds of steady states, it suffices to show (via standard transversality arguments, e.g., see Mas-Colell, Whinston and Green (1995, Proposition 17.D.3)) that an increase in educational cost parameter ν causes $w^*(\lambda)$ to strictly increase, for any λ.

Note initially that at any given λ, V_0 is unchanged, while V_1 must fall as ν rises. This follows from the fact that in steady state it is optimal for unskilled parents to not educate their children, so they must be unaffected by the rise in ν. And it is optimal for skilled people to educate their children, so they must be worse off when ν rises.

Next, manipulate the first order condition (2) for fertility decisions for occupation i to obtain the following equivalent version:

$$r_i^{\theta} u'(w_i - E_i)E_i = \delta E_i^{\theta}.\theta V_i \tag{27}$$

where $E_i \equiv r_i(w_i)n(w_i, i)$ denotes total expenditure on children who are trained for the same occupation, and all variables are evaluated at the given skill ratio. It is evident that all decisions of unskilled parents are unaffected.

Consider first the case where θ is positive. Then θV_1 falls as ν rises. So (2) implies that fertility n_1 of skilled parents must fall (where $n_i \equiv n(w_i, i)$). Now observe that (27) can also be written as

$$u'(w_i - E_i)E_i = \delta n_i^{\theta}.\theta V_i \tag{28}$$

Since n_1 and θV_1 both fall, it follows from (28) that E_1 must fall. Since $\theta > 0$, and E_0 is unaffected, it follows that parents are less inclined to educate their children (as they tend to maximize child expenditures), and w^* must rise.

Now suppose $\theta < 0$. In this case θV_1 rises as ν rises. Equation (27) now implies that E_1 rises. Since parents make human capital decisions for their children on the basis of minimization of total expenditures, they are again less inclined to educate their children, and w^* must rise.

Since the wage functions $w_i(\lambda)$ are unaffected by the change in ν, the result now follows for steady states with either upward or downward mobility. Steady states with zero mobility must satisfy the condition that $n(w_1(\lambda), 1) - n(w_0(\lambda), 0) = 0$, and an increase in ν must cause the left-hand-side of this equation to fall when θ is positive (n_1 must fall while n_0 is unaffected, from (2)). ■

7.2. **Long Run Comparative Statics.** We now explore the long run effects of varying costs of child-rearing and of education, as well as regulations pertaining to child labor and redistributive tax-transfer policies. It will be helpful to restrict attention to a linear formulation of child-rearing cost:

$$r_0(w) = f + k.w, r_1(w) = f + k.w + x \tag{29}$$

where f denotes the fixed 'goods cost' incurred per child, k the parental time away from work, and x a fixed cost of education.

Moreover, we focus attention on steady states with upward mobility, i.e., on the cases covered by Proposition 9. Note that both functions $w^*(\lambda), w_0(\lambda)$ are increasing in λ. w^* tends to a negative number as λ tends to 0, and to ∞ as λ tends to $\bar{\lambda}$. At the same time w_0 tends to 0 and $w(\bar{\lambda})$ respectively. Hence there exists at least one skill ratio where w^* and w_0 are equalized, where the w^* curve cuts the w_0 from below (i.e., has a steeper slope). If $n_0 > n_1$, call this a *locally stable* steady state with downward mobility. Intuitively, if λ falls slightly below the steady state, w^* is smaller than w_0. Then both unskilled and skilled households would want to educate their children so the skill ratio will tend to rise.

Conversely, if λ rises slightly above the steady state, w^* would be higher than w_0, thus eliminating the willingness of some unskilled households to educate their children, and the skill ratio will fall.

The linear formulation of child rearing costs allows us to obtain a closed form expression for the threshold wage:

$$w^*(\lambda) = \frac{1}{k}[x\{(\frac{V_1(\lambda)}{V_0(\lambda)})^{\frac{1}{\theta}} - 1\}^{-1} - f] \tag{30}$$

from the first-order conditions for fertility choice.[31] The threshold wage depends on the skill ratio via the dependence of the wage in occupation i and continuation values V_i on this ratio:

$$V_i(\lambda) = \frac{u(w_i(\lambda) - n_i(w_i(\lambda))(f + kw_i(\lambda) + xi))}{1 - \delta n_i(w_i(\lambda))^\theta} \tag{31}$$

where in addition $n_i(w_i)$ denotes the optimal fertility choice of a parent with wage w_i and selecting the same occupation for her children.

Specifically, small perturbations in child-rearing cost parameters f, x can be shown to be shift the w^* function (and hence the steady state skill ratio) in opposite directions, provided we impose an additional mild assumption on preferences in the case of negative utility):

Proposition 11. *In the case of negative utility, assume in addition to [U1]–[U4] that* $\frac{u'}{-u}$ *is decreasing.*[32] *Consider any steady state skill ratio with upward mobility which is locally stable. A small increase (resp. decrease) in fixed cost component f of child-rearing (resp. education cost x) will cause the steady state skill ratio to fall (resp. rise).*

Proof. Given expression (30) for w^*, it suffices to show that the derivative of $\frac{V_1(\lambda)}{V_0(\lambda)}$ with respect to f and x are respectively positive and negative in the case where $\theta > 0$ (and signs reversed in the case that $\theta < 0$), at any steady state with upward mobility.

[31] Using E to denote total expenditures ($(rw + f + xi)n$) on children, the first order condition implies $E^{1-\theta}U'(w - E) = [\frac{1}{rw+f+xi}]^\theta V_i$ for occupation choice $i = 0, 1$. This generates condition (30).

[32] This condition is satisfied by constant elasticity utility functions, as well as $u(c) = -exp(-ac)$ with $a > 0$.

Recall that

$$V_i(\lambda) = \max_{n_i}[u(w_i(\lambda) - (rw_i(\lambda) + f + xi)n_i) + \delta n_i^\theta V_i(\lambda)]. \tag{32}$$

Applying the Envelope Theorem to this optimization problem we have

$$\frac{\partial V_i(\lambda)}{\partial f} = -\frac{u'(c_i(\lambda))n_i(\lambda)}{1 - \delta n_i(\lambda)^\theta} \tag{33}$$

where $n_i(\lambda)$ denotes the optimal choice of n_i, and $c_i(\lambda) \equiv w_i(\lambda) - (kw_i(\lambda) + f + xi)n_i(\lambda)$. Therefore

$$V_0(\lambda)\frac{\partial V_1(\lambda))}{\partial f} - V_1(\lambda)\frac{\partial V_0(\lambda))}{\partial f} = -V_0(\lambda)\frac{u'(c_1(\lambda))n_1(\lambda)}{1 - \delta n_1(\lambda)^\theta} + V_1(\lambda)\frac{u'(c_0(\lambda))n_0(\lambda)}{1 - \delta n_0(\lambda)^\theta}. \tag{34}$$

Next, note that in any steady state with upward mobility we have $n_1 < n_0$, which in turn implies that $c_1 > c_0$ (in order to ensure that $V_1 > V_0$).

In the case of positive utility where $\theta > 0$, it follows that expression (34) is positive.

In the case of negative utility where $\theta < 0$, use expression (32) for the value function to see that expression (34) reduces to

$$\frac{u(c_1)}{1 - \delta n_1^\theta}\frac{u'(c_0)n_0}{1 - \delta n_0^\theta} - \frac{u(c_0)}{1 - \delta n_0^\theta}\frac{u'(c_1)n_1}{1 - \delta n_1^\theta} \tag{35}$$

Since $n_0 > n_1$, it suffices that $[-u(c_1)].u'(c_0) \geq [-u(c_0)].u'(c_1)$ for (35) to be negative. This is ensured by the property that $\frac{u'}{-u}$ is non-increasing.

It follows that $\frac{\partial w^*}{\partial f} < 0$.

A simpler argument ensures that $\frac{\partial w^*}{\partial x} > 0$. The argument is simpler because the V_0 function is locally unaffected by a small rise in x, while $\frac{\partial V_1}{\partial x} = -\frac{u'(c_1)n_1}{1-\delta n_1^\theta} < 0$.

Hence the w^* curve shifts down following an increase in f or a decrease in x, while the position of the $w_0(\lambda)$ is not affected. If the steady state is locally stable, a small rise in f or fall in x must cause the skill ratio to rise. ■

The model thus allows for local determinacy and comparative statics of long-run effects of changes in key parameters of the child-raising technology. This is in contrast to both the

Barro-Becker theory (which lacks credit market imperfections) as well as most preceding models of occupational choice (which lack endogenous fertility).

The comparative static results follow from the effect of the parametric changes on parental incentives to invest in the education of their children. A rise in the 'goods cost' of child-rearing increases this incentive, just as a fall in education costs does. The former induces a reduction in fertility, which in turn stimulates an increase in desired quality of children. This is for both a direct reason (child-care expenses per child are lower when not investing in education, so a rise in f raises child-care costs by more for non-investors) and an indirect reason (the continuation utility of skilled children falls by less than for unskilled children).

It predicts that societies with a norm where extended family or kinship networks share the burden of child-rearing (so the parents bear a smaller part of the burden) will tend to invest less in education of children. Social changes that cause a shift from joint to nuclear families thus induce higher education. Policies of subsidized childcare undermine skill accumulation incentives, and raise inequality between skilled and unskilled wages.

Effects on aggregate fertility are ambiguous. Consider the case of positive utility. A rise in f tends to lower fertility among both skilled and unskilled households at any given skill ratio.[33] This is further reinforced by the induced rise in the proportion of skilled households, since the skilled tend to have fewer children. If wealth effects dominate substitution effects, there is a counteracting effect: fertility within unskilled households rise as a consequence of the rise in the skill ratio.[34] So the net effect on the fertility of the unskilled is ambiguous, while fertility among the skilled must fall. Since the effects on the fertility differential between skilled and unskilled are ambiguous, so are the effects on mobility. It is therefore possible that lower education costs actually end up lowering mobility, if wages and fertility among the unskilled rise sufficiently. This provides a potential explanation for the empirical

[33]This is both because of the direct effect of rising child-rearing costs, as well as the induced reduction of continuation values of skilled and unskilled (since $\theta > 0$ in the case of positive utility. If utility were negative, $\theta < 0$, and the falling continuation values would induce higher fertility. So the ambiguity is even more pronounced with negative utility.

[34]If the substitution effects dominate instead, fertility among the skilled will rise as a consequence of the fall in their wages. And fertility among the unskilled will fall as their wages rise. In this case the fertility differential will widen, implying a rise in mobility. But the effects on aggregate inequality remain ambiguous.

findings of Chechhi, Ichino and Rustichini (1999) that mobility in Italy appears to be lower than in the US, despite a more extensive public schooling system and a lower skill premium in wages.

7.3. **Child Labor.** Now suppose children that do not go to school can work and augment the incomes of their households. Suppose that children can work as a substitute for unskilled adult labor, and earn a wage of γw_0, where $\gamma \in (0, 1)$ is a parameter that reflects differences in work capacity between adults and children, as well as regulations concerning child labor. Stronger restrictions on child work correspond to a reduction in γ. The preceding model pertains to the case where $\gamma = 0$.

Household consumption corresponding to parental wage w is now $c \equiv w - [rw + f + xi - \gamma w_0(1-i)]n$. This corresponds to our earlier model if we replace f by $f' \equiv f - \gamma w_0$ and x by $x' \equiv x + \gamma w_0$. To ensure $f' > 0$ we must impose the restriction that $\gamma < \frac{f}{w_0(\bar{\lambda})}$.

Stronger restrictions on child labor then correspond to a fall in γ, which is analytically equivalent to a rise in child care costs combined with a fall in education cost. Proposition prcompstat then implies that both of these induce a rise in the long run steady state skill ratio.

There is an additional effect that operates through the effect on wages (stressed in particular by Basu and Van (1998)): a reduction in child labor reduces the supply of unskilled labor in the economy as a whole, which tends to raise unskilled wages. Hence the $w_0(\lambda)$ curve shifts up. This has an additional effect on a steady state with upward mobility, since it is characterized by intersection of the w^* curve and the w_0 curve. If the steady state is locally stable, this effect raises the steady state skill ratio even further.

Hence the net effect of stronger regulations on child labor is to raise the 'level' of long run development: higher per capita skill and income, with lower wage inequality between the skilled and unskilled. Effects on aggregate fertility and mobility are, however, ambiguous for the reasons explained above.

7.4. **Income Redistribution via Taxes and Transfers.** We now show that the results obtained in Mookherjee and Ray (2008) with regard to the effect of different tax-transfer schemes in the occupational choice model with exogenous fertility continue to extend. In their setting there was some arbitrariness associated with selection of a particular skill ratio from the continuum of steady state skill ratios. As shown above this arbitrariness disappears with endogenous fertility.

We provide a brief outline of the analysis here, omitting most details. Consider first an unconditional welfare system paying an income support of σ to unskilled households, which are financed by income taxes levied on skilled households at a constant linear rate τ. In steady state, budget balance requires $(1-\lambda)\sigma = \lambda\tau w_1(\lambda)$, so the size of the income support depends on the skill ratio and tax rate:

$$\sigma = \frac{\lambda}{1-\lambda}\tau w_1(\lambda) \tag{36}$$

Steady states have similar properties as established in preceding sections, except that the values of being unskilled and skilled are now given by

$$V_0 = \max_{n_0}[u(w_0 + \sigma - (kw_0 + f)n_0) + \delta n_0^\theta V_0] \tag{37}$$

$$V_1 = \max_{n_1}[u((1-\tau)w_1 - (kw_1(1-\tau) + f + x)n_1) + \delta n_1^\theta V_1]. \tag{38}$$

These policies lower the value of being skilled, and raise the value of being unskilled. Investment in education is thereby discouraged: the w^* curve shifts up.[35] The long run effect will be to lower per capita skill in the economy. This will raise the skill premium in wages, so the market will undo some of the redistribution.[36]

The adverse long-run effects of unconditional income supports to the poor can be avoided with conditional transfers. An example is an education subsidy which is funded by income taxes on the skilled. In this case the continuation value of the unskilled is not directly affected: the value can be computed on the basis of the assumption that they do not invest

[35]In this case the thresholds differ between skilled and unskilled parents. What matters, of course, in a steady state with upward mobility is the threshold for unskilled parents, and it is this threshold that we refer to here.

[36]Implications for fertility are complex. The wealth effects associated with the transfers will tend to raise fertility among the unskilled and lower them among the skilled. Countering this in the opposite direction are the effects of wage movements induced by the policies, since unskilled wages fall and skilled wages rise.

in education of their children, whence they receive no benefits from the transfers. On the other hand, the continuation value of being skilled rises, since skilled households have more options.[37] This encourages investment in education: the w^* curve shifts down, and the steady state skill ratio rises (hence so does per capita income, while the skill premium declines). The effects are exactly the opposite of an unconditional welfare system.

7.5. **Gender Discrimination and Family Planning Subsidies.** One can extend the model to allow for two parents of differing genders who allocate their time between child-rearing and working outside the home on a labor market. The effects of policies encouraging female labor force participation or reducing gender discrimination on the labor market turn out to be ambiguous, as their effect is similar to a rise in the time cost k of children on parental human capital investment incentives.[38]

The effect of family planning subsidies is similar to a combination of a lump-sum income subsidy, and a rise in f, the incremental goods cost of higher fertility. The latter promotes investment incentives as we have seen above. If the subsidies are funded by lump-sum taxes, there are no wealth effects, and the net effect is the same as an increase in f alone, i.e., a rise in per capita income and skill.

8. Relationship to the Literature

We start by describing related literature on the wage-fertility correlation.

First, as discussed in the Introduction, there is the view that the cross-sectional relationship is indeed positive, and the negative relationship we do in fact see is the result of some omitted variable. Becker (1960) is a proponent of this point of view, emphasizing the possible differences between *desired* and *actual* numbers of children, owing to ignorance

[37] An education subsidy π lowers private education cost to $x - \pi$. Hence budget balance requires $\tau w_1(\lambda) = n_1(\lambda)\pi$, i.e., skilled households pay the taxes to fund the education subsidies they would avail if they chose the same fertility as in the absence of the subsidy. In that case they are as well off as before. But they have the option to have a different number of children, which could make them better off.

[38] The direct effect of an increase in k is similar to that of f: both raise w^*, enhancing skill investment incentives. The indirect effect is difficult to assess. In the positive utility case for instance, it is no longer clear that a rise in k raises $\frac{V_1}{V_0}$, as this effect is proportional to the wage rate which is higher for the skilled.

concerning contraceptive methods. Another important omitted variable (see, e.g., Freedman (1963)) is that family income is correlated with greater female participation in the labor force, and it is the latter that drives the decline. (This view can, of course, be folded into the substitution effect which is driven by time costs of rearing children.) Under this view, then, theory has little to say about the net effect.

Second, there is the view that the cross-sectional relationship is "truly" negative, and must be explained by the theory at hand. Attempts include appropriate calibration of the parameters, so that substitution effects kill off the income effect, introduction of 'quality' of children as an additional choice coupled with suitable assumptions concerning child-rearing costs (Becker and Lewis (1973), Becker and Tomes (1976), Moav (2005))), or the introduction of non-homotheticicities in preferences (see, e.g. Galor and Weil (2000), Greenwood and Seshadri (2002), or Fernández, Guner and Knowles (2005)).

Jones, Schoonbroodt and Tertilt (2008) provide an overview of these explanations, and argue that they all require restrictive assumptions. Instead they suggest an argument that relies on heterogeneity in parental tastes. Parents who want larger families will realize that more time will be spent on bringing up children, and this lowers their optimal choice of human capital, not for their children as in our framework, but for *themselves*. Therefore larger families would tend to have lower incomes.

Steady state models with endogenous fertility, in which the cross-sectional relationship is explicitly addressed, include Alvarez (1999) and Kremer and Chen (1999, 2002). Alvarez introduces Barro-Becker preferences into Loury's (1981) theory of intergenerational inequality. The main objective is to see how the persistence result in inequality (arising from shocks) withstands endogenous fertility, but as a byproduct Alvarez obtains the result that fertility is *positively* correlated with parental wealth.[39] In a modification of the model, Alvarez introduces various non-separabilities in preferences and rearing costs; in particular, increasing marginal costs in child-rearing. If these effects are strong enough, this baseline finding may be reversed. In summary, the model delivers the opposite result on

[39] In fact, in his baseline model, the positive correlation is so strong that per-child parental transfers are independent of parental wealth.

fertility-wealth in its baseline setup and an ambiguous prediction when the modifications are introduced.

Kremer and Chen use a special specification of parental preferences, in which utility is given by $c + \log n$. Parents must spend time in child-rearing, but do not care about the quality of children or the payoffs enjoyed by them. This assumed absence of income effects implies that rising parental wages exert only substitution effects on the demand for fertility. Hence fertility is decreasing in parental wages. In this sense, the Kremer-Chen exercise is a special case of the parametric calibration that forces substitution effects to dominate income effects.

The same is true of Doepke (2004), who assumes that utility functions exhibit lower curvature than the log function, and that there are no goods costs involved in child-rearing. In this case, as we have seen in Proposition 5, wealth effects are weaker than substitution effects, ensuring a negative wage-fertility correlation.

Our Proposition 9 is clearly very different from all of the preceding explanations of the wage-fertility correlation, as it does not rest on assumed weakness of wealth effects or unobserved preference heterogeneity, but instead on the endogeneity of occupational wages and related steady state restrictions.

Some of our partial equilibrium findings, which serve as the preliminaries leading up to Proposition 9, are related to results in Doepke (2004). While Doepke considers the positive utility case alone and does not address the general equilibrium finding of Proposition 9, his Proposition 3 implies the jump-down as occupational boundaries are crossed (derived for a more restrictive class of preferences and rearing/schooling costs). Doepke's Proposition 4 makes the point that in equilibrium, there could be upward, downward or zero mobility outcomes, and also observes that in the first two cases the parents in the "out-migrating" category will need to be indifferent between educating and not educating their children. Doepke subsequently expresses the view that the first of these three outcomes is the most relevant one, though without providing any theoretical justification, and then moves on to calibration and empirical investigation. Our contribution is to provide a theoretical justification for the focus on this class of steady states. An added contribution are the theoretical results concerning local determinacy and comparative static properties of steady states.

Finally, we discuss the connections to the occupational choice literature. The generic model of occupational choice with fixed fertility[40] takes as given a set of occupations (typically two, as in the exercise here). The steady-state conditions are simple: no dynasty currently in one occupation must wish to move their progeny to another with a different setup cost. This occupational persistence follows from single-crossing, which rules out cycles in occupational choice. In particular, there is no mobility in steady state. Such a model can generate intergenerational mobility by adding to it stochastic shocks in ability or income (as in Banerjee and Newman (1993) or Mookherjee and Napel (2007)). Our theory ascribes, however, a predictable *direction* to mobility; it occurs (even without any stochastic shocks) to compensate for the endogenous fertility differentials between skilled and unskilled households. The local isolation of steady states in our theory permits tractable comparative statics analysis, with the absence of stochastic shocks greatly simplifying the analysis.

9. Concluding Remarks

We have developed a theory on the implications of interactions between fertility and human capital in a setting with imperfect financial markets. This framework provides new insights into the wage-fertility relationship, the determinants of intergenerational mobility, and the extent of macroeconomic history dependence. It provides a tractable framework permitting comparative static analysis of effects of changes in a variety of fiscal and human capital policies. It also illustrates a number of possible factors underlying observed features of the demographic transition in developing countries, whereby economic and social factors associated with urbanization and modernization induce large declines in fertility

[40]Versions of this model appear in Banerjee and Newman (1993), Ljunqvist (1993), Galor and Zeira (1993), Ghatak and Jiang (2002), and Mookherjee and Ray (2003). Ray (1990, 2006) contains a model which exactly aligns with this generic description in the two-occupation case.

(e.g., falling costs of education, rising costs of childcare, regulations on child labor). Underlying all of these is an induced shift towards occupations requiring higher investments in education.

A number of gaps in our results still remain, which we hope can be filled by future research. One concerns the generality of the finding that steady states with downward or zero mobility do not exist. Others include the existence of interior steady states in the case of zero fixed costs of childcare, and local determinacy of steady states without mobility in the case of negative utility.

An obvious next step would be to subject the findings of this paper to empirical testing. The central finding of this paper is that the occupational shift effect accounts for a robust negative correlation between parental wages and fertility, which may work against a positive wealth effect. Hence *within* occupations or human capital categories, or in contexts where there is not much scope for occupational or human capital variations, the wage-fertility correlation may be positive. Similarly over short periods of time when occupations of the adult population are unchanging, fertility may move in the same direction as income. But the correlation *across* occupational categories will typically be negative. These are consistent with empirical findings reported in some earlier literature such as Freedman (1963) and Simon (1969), and it would be interesting to see if they continue to be confirmed more broadly using more powerful econometric techniques and other datasets (e.g. pertaining to developing countries).

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