Aspirations and Inequality

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December 2010

Abstract

The premise of this paper is twofold. First, people’s aspirations for their future well-being (or that of their children) affect their incentives to invest. Second, the experiences of others help shape one’s aspirations. This paper marries a model of aspirations-based choice with a simple theory of aspirations formation to study the relationship between aspirations and the distribution of income. Through its impact on investments, aspirations affect economic mobility and the income distribution, which in turn shape aspirations. Thus aspirations, income, and the distribution of income evolve jointly, and in many situations in a self-reinforcing way. We study the consequences of this model for income distribution as well as for growth rates over different quantiles of the distribution. We show that extreme equality is unstable. Moreover, when the same aspirations are shared in a society, polarization arises. The theory we propose captures both the complacency stemming from low aspirations and the frustration resulting from aspirations that are too high. As a result, for commonly held aspirations, growth rates have an inverted U-shape along the income distribution.

JEL Classification Numbers: J62, D9, O15, 040.

Key Words: Aspirations, Economic Mobility, Growth, Polarization.

Thanks to seminar participants at Georgetown University, the World Bank, Maryland University, the Microfoundations of Development Conference at LSE, the conference on the Economic Theory of Development at ColMex and LAMES 2008. Thanks also to Luis-Felipe Lopez-Calva and the UNDP for funding a background paper [15] on which this paper draws. Ray’s research was funded by the National Science Foundation under grant SES-0962124. Please address all correspondence to gg58@georgetown.edu and debraj.ray@nyu.edu.
1. Introduction

What individuals want for themselves, or what parents want for their children, is conditioned by society in fundamental ways. One such pathway is the creation of individual aspirations (for the individual, or for the future of her children). To some extent, such aspirations are drawn from the past experience of the individual herself, as in the literature on habit formation or reference points, but at the same time they are profoundly affected by one’s social environment. We look at others “around” us, and their experiences and achievements shape our desires and goals.

This is a view of individual preferences that isn’t standard in economic theory. But it should be. Individual goals don’t exist in social isolation as consumer preferences are so often assumed to do. Thus society-wide aggregates of income or investment may depend fundamentally on the ambient distribution of income and wealth. In short, aspirations affect income mobility and income distribution, but in turn, these latter ingredients shape aspirations. Thus aspirations, income (and its distribution), investment and economic mobility evolve jointly, and in many situations in a self-reinforcing way. An examination of this proposition is the subject of our paper.

The relationship between macroeconomic outcomes (such as growth and inequality) and individual aspirations turns on three things. There is the question of how individuals react to the aspirations that they do have. Next, there is the issue of how those aspirations are formed: how they vary with the current economic circumstances of the individual concerned as well with the characteristics of the world around her. Finally, there is the issue of aggregating individual behavior to derive society-wide outcomes. The theory we propose has these corresponding segments. We emphasize the first two, as they are relatively new.

First, individual aspirations determine one’s to incentives to invest, accumulate, and bequeath. We argue below that the existence of realistic, attainable aspirations — targets that are currently beyond one but which are potentially “reachable” — are the most conducive to upward mobility. The “best” sort of aspirations are those that induce a “reasonable distance” between one’s current living standards and where one wants to be, but not a gap that is so large as to induce frustration. For instance, [1], [26] and [27] have argued that individuals with aspirations that are very far away from their current standards of living have little incentive to invest, because the gap would remain very large before and after. There is evidence from cognitive psychology, sports, and lab experiments (see, e.g., [4]) and [16]) that goals that lie ahead — but not too far ahead — provide the best incentives.

This argument captures both encouragement and frustration, and on its own can be used to create an aspirations-based theory of poverty traps. One might add a third effect — complacency — applicable to individuals whose current standards of living comfortably exceed their socially generated aspirations. That may be the case for the very richest sections of society. Thus it is often individuals in the middle — those with a good-sized
aspirations gap as well as the resources to effectively close that gap — who might invest the most.

We develop a simple framework that captures these ideas. We define utilities around a “reference point”, following the lead of [17], [18], [19] and others, and interpret our reference point as an aspiration. We then describe the formation of aspirations from the underlying income distribution, thereby linking social outcomes and individual behavior. We argue that aspirations are likely to depend not only on one’s own historical living standards, as commonly assumed, but also on the experience and lifestyle of others. For instance, individuals may simply use some common function of the income distribution (such as the mean or income at the 75th percentile) to form their aspirations. We use such “common aspirations” as a benchmark but introduce other processes of aspirations formation in which an individual’s aspiration is also defined by her own position in the distribution.

Finally we embed these preferences and model of aspirations formation in a simple growth model. In equilibrium, the overall distribution influences individual aspirations which in turn shape the distribution via individual choices.

Our main results concern stable equilibrium income distributions. We show — even in the absence of any stochastic shocks — that perfect or near-perfect equality is unstable. That is, income distributions cannot converge to a degenerate distribution. Our results are in line with a recent literature on endogenous inequality, in which it is argued that a society must move away from perfect equality for a variety of reasons, including nonconvexities ([14], [21], [22]), occupational choice ([2], [13], [23]) and endogenous risk-taking ([3], [28]).

In the case in which aspirations are common or stratified, we show that in any steady state, incomes must cluster into local poles. Simulations reveal that in the special case of common aspirations, typically two poles emerge, in line with the findings of [24].

We also study the behavior of growth rates along the income distribution. Our propositions attempt to captures the idea that aspirations that are too high can serve to frustrate, while aspirations that are too low might breed complacency. It follows that over a zone of incomes that share the same aspirations, individual growth rates should be inverted U-shaped in income. We also discuss how growth rates along the cross-section of incomes must react to a shift in aspirations brought about, say, through the rise of mass communications media.

Finally, we use the percentile distributions of growth rates available for 43 countries in an empirical exercise. We find the return to individual investment that matches in the model the aggregate growth experiences of these countries, and then employ the model to see which aspirations formation structure appears to fit the data best, in that they come closes to the observed growth incidence curves by percentile. We show that a model

\footnote{See [20] for evidence of the importance of social interactions in the formation of aspirations.}

\footnote{This paper develops the ideas laid down in our working paper [15], a Background Paper prepared for the UNDP. Following that paper, [5] also develops a model of socially determined aspirations but focusing on growth.}
of aspirations formation in which individuals use umbrella-shaped weights on incomes in some interval around their own incomes comes closest to replicating the data, and this specification captures 75% of the observed variation in growth.

There is, of course, a large literature which connects social outcomes to individual behavior. In most part, the connection is made by linking aggregate features to an individual’s feasible set (and not her preferences). For instance, macroeconomic outcomes might affect an individual’s access to the credit market, or what she receives as wages. We emphasize, in contrast, the effect on what an individual wants to do. In this sense, the closest literature would be the one which emphasizes the effect of the ambient distribution on status-seeking and therefore behavior (see, e.g., [6], [7], [8], [9], [12], [28], [29], [30], [31], and [32]). This is a direct effect that works through preferences for relative wealth or income. However, the structure we place on aspirations formation as a reference point, and on the “nonlinear” way in which individuals react to the gap between their aspirations and their current standards of living, makes this a distinct exercise, with its own novel distributional and growth implications.

2. The Model

2.1. Preferences and Aspirations. An economy is populated by a large number of families or dynasties. A dynasty is a sequence of individuals, each of whom lives for a single period. A typical member of generation $t$ cares about her own lifetime consumption, $c_t$, and the income $y_{t+1}$ that she leaves for her child, who will grow up to be a member of generation $t + 1$:

$$u(c_t) + \Omega(y_{t+1}, a_t),$$

where $a_t$ is the aspiration that our individual has at time $t$ (much more on this below), and $u$ is a standard utility function satisfying:

[U] $u$ is increasing, smooth and strictly concave, with $u(0) = -\infty$.

Now for aspirations: think of $a$ as a “reference point” as in the work of [17] and [19], but whereas in those models (see especially [19]) $a$ is determined by the own experience of the individuals, we are going to view these aspirations as coming from what individuals see around them.

There are many possible specifications of the utility function $\Omega$; we adopt one that can capture complacency and frustration (as per our informal discussion earlier) in a reasonably tractable way.

First, we write $\Omega$ as follows:

$$\Omega(z, a) = v(z) + w(z, a),$$

where $v$ might be thought of as an intrinsic utility of income tomorrow (denoted by $z$) and $w$ is a term that depends on the comparison of “target income” $z$ to aspirations $a$. We presume that $v$ satisfies all the standard assumptions — it is increasing, smooth and strictly concave, with unbounded steepness at 0. The separation of $\Omega$ into $v$ and $w$ is
formally unnecessary but will help below in understanding the restrictions that we place on the comparison function \( w \).

We assume that \( w \) is increasing in \( z \), and decreasing in \( a \). We also presume that the utility excess (or shortfall) as one moves away from the aspirations target is *increasing at a decreasing rate in either direction*. If I am far ahead of my aspirations, an extra gain is not going to create much additional satisfaction, and likewise if I am way below my aspirations, an increase or decrease is not going to make much of a difference. It is in the region of the aspiration itself that utility gains are most sensitive to an increase in income. See Figure 1. Formally, we maintain the following assumption throughout:

\[ \text{[W]} \ w \text{ is smooth, with } w_1(z,a) > 0 \text{ and } w_2(z,a) < 0. \text{ It is strictly convex (with } w_{11}(z,a) > 0 \text{) for } z < a, \text{ and strictly concave (with } w_{11}(z,a) < 0 \text{) for } z > a. \text{ At the inflection point } (a,a), w_1(a,a) = \infty. \]

2.2. The Determination of Aspirations. We now turn to a discussion of how aspirations might be determined. We follow [1] and [26] in emphasizing the *social construction* of aspirations. There is an alternative view in which aspirations might be determined entirely by one’s own personal history, such as parental income or standard of living. While we do not ignore this aspect of the aspirations formation process, we emphasize social effects. We can capture a fairly broad range of possibilities under the specification

\[ a = \Psi(y,F), \]

where \( a \) stands for the aspiration of an individual (and is applied to the income of the next generation), \( y \) is her current income, and \( F \) is the “relevant” society-wide distribution of income (we elaborate below). It is innocuous to maintain that \( \Psi \) is nondecreasing in \( y \);

\[ \text{[W]} \ w \text{ is smooth, with } w_1(z,a) > 0 \text{ and } w_2(z,a) < 0. \text{ It is strictly convex (with } w_{11}(z,a) > 0 \text{) for } z < a, \text{ and strictly concave (with } w_{11}(z,a) < 0 \text{) for } z > a. \text{ At the inflection point } (a,a), w_1(a,a) = \infty. \]

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3Given that a particular income \( z \) has been achieved, a higher aspiration *per se* can only lower payoffs.
one would not reasonably expect higher incomes to dampen aspirations. If \( \Psi \) is strictly increasing in \( y \), then personal histories have an active role to play in the determination of aspirations, something that we find reasonable but do not necessarily insist upon.

We maintain the following assumption on \( \Psi \):

\[ \text{[A]} \, \Psi \text{ is continuous in } y \text{ and } F, \text{ and takes values within the range of } F. \]

The substantive restriction in Condition A is that aspirations do not wander outside the society-wide range of incomes.

Consider some particular processes of aspirations formation:

**Common Aspirations:** This is the simplest case. All individuals have exactly the same aspirations, which are given by some common function of the income distribution, and do not depend on the specific value of individual income:

\[
\Psi(y, F) = \psi(F).
\]

The terminology “aspirations” is a bit strained when aspirations are “common” in the sense just described. There will typically be individuals with starting incomes that exceed those aspirations. “Reference point” might be a better phrase. A leading example of common aspirations is one in which aspirations equal the mean value of income.

**Stratified Aspirations:** Aspirations are likely to depend on one’s position in the income distribution. Divide up the income distribution into \( n \) quantiles. Define

\[
\Psi(y, F) = a_i
\]

for individuals with income \( y \) in quantile \( i \), where \( a_i \) is a scalar representing some summary statistic of the distribution in that quantile: e.g., average income conditional on being in quantile \( i \), or the income of the 75th conditional percentile in that quantile.

**Upward-Looking Aspirations:** Aspirations might vary more finely with personal incomes. For instance, say that individuals look “upwards” at all families who are richer than them, and that aspirations are the conditional mean of all such incomes:

\[
\Psi(y, F) = \frac{\int_{y}^{\infty} x dF(x)}{1 - F(y)}.
\]

**Local Aspirations With Population Neighborhoods:** [26] discusses aspirations “windows”, in which people draw upon the experiences of those in some cognitive window around them. For instance, suppose that weight is placed only on the surrounding

\[ ^4 \text{Continuity in } F \text{ is with respect to the topology of weak convergence on distributions.} \]
\( d \) (income) percentiles of the population. That is, we consider an income \( y' \) only if 
\[ |F(y') - F(y)| \leq d, \]
so that
\[ \Psi(y, F) = \frac{1}{d} \int_{L(y)}^{H(y)} x \, dF(x), \]
where \( L(y) \) and \( H(y) \) are the appropriately defined edges of the cognitive window for a person situated at \( y \).\(^5\)

**Local Aspirations With Income Neighborhoods:** Now suppose that weight is placed instead only on incomes within an interval \( N(y) \) of the individual’s income. Then
\[ \Psi(y, F) = \frac{1}{F(N(y))} \int_{N(y)} x \, dF(x), \]
where \( F(N(y)) \) has the obvious meaning.

There is an interesting distinction to be drawn between these last two examples. With population neighborhoods, an individual’s cognitive window includes a certain percentage of the population, which may contain individuals far richer than her. In particular, if she occupies a sparsely populated income segment, then this sort of cognitive window may lead to unduly high aspirations and consequently frustration, as discussed earlier. In contrast, with income neighborhoods, the possibly sparse population around an individual’s income is of no consequence to her: she anchors her aspirations on the basis of what is attainable regardless of the number of individuals actually earning those incomes. For obvious reasons, these aspirations are less sensitive to the ambient distribution of income.

There is an important aspect of aspirations formation which we have not emphasized so far. Recall that \( a \) is to be interpreted as the aspiration that an individual holds for her progeny’s income in the next generation. In an environment of ongoing intergenerational growth (or decay), it is entirely plausible that the anticipated future distribution of income should enter the aspirations formation process. This requires a reinterpretation of \( F \) in (1): it is not, then, the current ambient distribution but the anticipated distribution of income in the next generation. In a rational expectations equilibrium, we will take this to be a correct point forecast of the true income distribution that will actually prevail. When future distributions are used, we will also need to scale up \( y \) to the corresponding value anticipated in the future, in all specifications where an individual’s own income makes an appearance. The correct interpretation of this scaling is that the individual is not scaling up his own income, but the income of others who have the same income as him.\(^6\)

\(^5\)That is, \( L(y) \) is the lowest income in the support of \( F \) with \( F(y) - F(L(y)) \leq d \), and \( H(y) \) is the highest income in the support of \( F \) with \( F(H(y)) - F(y) \leq d \).

\(^6\)That is, \( y \) does conceptual double-duty. It is, first, the individual’s own income, as in, say, the example of upward-looking aspirations. But (continuing with that example) when we write the integral \( \int_y x \, dF(x) \) and interpret \( F \) as the future income distribution, we will need to integrate starting from \( z(y) \), where \( z(y) \) is the anticipated income of all individuals (other than himself, but including himself, in “equilibrium”) who has income \( y \) today.
To refer to these two alternatives, we will use the terminology “current aspirations” to describe the case in which current income distributions are used to form aspirations, and “future aspirations” for the case in which aspirations are formed by using the anticipated income distribution for the next generation.

2.3. **Equilibrium.** To describe equilibrium, we embed our model of aspirations formation into a standard growth model. For each individual in generation $t$, lifetime income $y_t$ is divided between consumption $c_t$ and an investment or bequest for the future, $k_t$.

$$y_t = c_t + k_t,$$

That bequest gives rise to fresh income for the next generation.

$$y_{t+1} = f(k_t, \theta_t),$$

where $\{\theta_t\}$ is some idiosyncratic shock, independent and identical across individuals and over generations, and $f$ is a smooth increasing function. For much of the paper, we will consider the case in which there is no uncertainty, but in any case it is easy enough to include it in our description of equilibrium.

A policy maps current lifetime income $y$ to bequests $k$.

An *equilibrium with current aspirations* (ECA) from some initial distribution $F_0$ is a sequence of policies $\{\phi_t\}$ and income distributions $\{F_t\}$ such that

(i) For every $t$ and $y$ in the support of $F_t$, $k = \phi_t(y)$ maximizes

$$u(y - k) + \mathbb{E}[v(f(k, \theta)) + w(f(k, \theta), \Psi(y, F_t))],$$

over $[0, y]$, and at every date $t$,

(ii) $F_{t+1}$ is generated in the obvious way, given $F_t$ and the policy $\phi_t$.

An *equilibrium with future aspirations* (EFA) is defined exactly in the same way, except that the term $F_t$ in (2) is replaced by $F_{t+1}$.

**Proposition 1.** *An equilibrium exists.*

With current aspirations, establishing existence is a simple recursive exercise of no great interest. Existence in the case of future aspirations, in which agents need to be predictive, is much less trivial, but still an exercise of largely technical interest. In what follows, we take Proposition 1 as given, and move on.

3. **The Instability of Equality**

A recent literature argues that high levels of equality may be “unstable”, in the sense that there are large incentives to move away from such configurations. The purpose of this section is to show that a natural theory of aspirations, such as the one we have outlined here, shares similar properties. The intuition is very simple: when income is excessively bunched around a common value, there is a large gain to be had in accumulating a bit more relative to others. (In this model, this gain comes through the relative ease of
meeting and exceeding one’s aspirations.) That leads to a race to the bottom in which all agents accumulate too much. Eventually, the pressure must ease as symmetry is broken by some agents taking present consumption instead and falling behind in accumulation. The resulting outcome separates near-identical agents, thus destroying equality.

3.1. **Nonconvergence.** The following proposition formalizes this idea.

**Proposition 2.** If \( \{F_t\} \) is an equilibrium sequence of income distributions, then it cannot converge to a degenerate distribution.

*Proof.* We only need to consider the case in which there is no uncertainty, so we write \( f(x, \theta) \) as \( f(x) \). Suppose, contrary to our assertion, that \( F_t \) converges weakly to \( F \), where \( F \) is degenerate. Then by an obvious continuity argument, the stationary sequence \( \{F_t\} \) centered on \( y^* \) is an equilibrium. Because \( v \) has unbounded steepness at 0, it must be that \( y^* > 0 \). By \([A]\), every individual must have a common aspiration, given by \( a = y^* \). Thus each individual solves the maximization problem

\[
\max_k u(y^* - k) + v(f(k)) + w(f(k), y^*)
\]

and does so by choosing \( k \) so that \( f(k) = y^* \). Using \([W]\), in which \( w_1(a, a) = \infty \) and an interior first-order condition for a maximum, which must necessarily hold, we must conclude that \( y^* - k = 0 \). But then, by \([U]\), payoffs are negative infinity, a contradiction, for any individual can guarantee finite utility by, say, dividing his resources equally across the two periods. \(\blacksquare\)

Several remarks on this proposition are warranted. First, to prove this result, we make use of the assumption that \( w_1(a, a) = \infty \), so that there is an irresistible urge to break the symmetry of perfect equality. It is easy enough to see that \( w_1 \) does not literally have to be infinite; a large enough slope will suffice. On the other hand, if the slope is relatively flat, it may well be that an equal limit distribution is stable; see the numerical example in Section 3.4.

Second, note that convergence of the equilibrium to some stationary distribution is not needed for this result, provided we restrict ourselves to the case of EFA. When aspirations are formed using future distributions of income, the sequence of distributions cannot approach perfect equality, or anywhere close to it, at any date. For if this were to be false, then \( z = a \) and the same argument used in the proof above goes through.

The same is not entirely true of ECA. It is possible that perfect equality could be maintained provided that all incomes grow fast enough at the same common rate. For in this way, we will have \( a_t = y_t \) and \( y_{t+1} = (1 + g_t)y_t \) at every date \( t \), where \( y_t \) is the common level of income and \( g_t \) the common rate of growth at date \( t \). It is easy enough to provide examples of such a phenomenon, but equality will necessarily break down if the common rate of growth is nonnegative but small; for instance, if it is 0 as in the statement of Proposition 2.
3.2. The Case of Common Aspirations. We can examine the nature of the limit distribution more closely in special cases. Take first the case of common aspirations. We begin by assuming that there are no stochastic shocks, and write the production function as \( f(k) \).

Now suppose that income distributions converge to some limit distribution \( F^* \), with attendant aspirations \( a^* \) (common to all) within the support of \( F^* \). Then for each income \( y \) in the support of the limit distribution, an individual solves the maximization problem

\[
\max_k u(y - k) + v(f(k)) + w(f(k), a^*).
\]

A standard single-crossing argument informs us that the optimal choice of \( k \) must be nondecreasing in \( y \).\(^7\) It follows that if \( F^* \) is a stationary distribution, then \( y \) must map into \( y \) again. Let \( k(y) \) be the capital stock that permits this to happen; then \( f(k(y)) = y \), so that \( k(y) \) is just the inverse of \( f(y) \). Because of the unbounded steepness of both \( u \) and \( v \), the solution must be interior for every \( y > 0 \), and so the (necessary) first-order condition informs us that

\[
(3) \quad u'(y - k(y)) = f'(k(y)) \left[ v'(y) + w_1(y, a^*) \right]
\]

for every positive \( y \) in the support of the limit distribution. Now observe that \( W \) is concave to the right of \( a^* \), so that there can be just one solution for steady state income that lies above the common aspiration. In other words, there is convergence among “rich” individuals: those with incomes exceeding the common aspiration.

But as we’ve just argued in the preceding proposition, these cannot be the only individuals in society. If they were, aspirations cannot be located where they are. There must be individuals with limit incomes lower than \( a^* \), and they, too, must satisfy the first-order condition (3). For \( y < a^* \), \( W \) is convex, and in principle there may be a number of solutions to (3). Generically, there must be finitely many such solutions, so that the steady state distribution develops multiple poles. This clustering of incomes is a robust feature of the common aspirations model.

It goes without saying that the clustering into degenerate poles described above is not to be taken literally. When there are stochastic shocks, the distribution will always be dispersed, but there will be a tendency for it to exhibit local modes: one above the common aspirations level, and one or more modes below it. See the numerical example in Section 3.4 for an illustration.

These observations can be usefully related to different aspects to the literature on evolving income distributions. The closest relationship is to endogenous inequality, in which high levels of equality are destabilized by forces that tend to move the system away from clustering. In [13] and [23], this happens because of imperfect substitutes among factors of productions, so that a variety of occupations with different training costs and returns must be populated in equilibrium. Together with imperfect capital markets, this implies

\(^7\)The assumption that \( u \) is strictly concave in consumption is sufficient to deliver this result, using a familiar revealed preference argument.
that in steady state, there must be persistent inequality, even in the absence of any stochastic shocks. In related work, [3] and [28] argue that endogenous risk-taking can also serve to disrupt equality, as relative status-seeking effectively “convexifies” the utility function at high levels of clustering.

The clustering of incomes into local poles also speaks to the work of [10], [24] and [25]. These authors make a strong case for local clustering in the world income distribution and argued that convergence was a local phenomenon “within the cluster” but not globally. Durlauf and Quah [11] summarize by writing that there is an “increase in overall spread together with [a] reduction in intra-distributional inequalities by an emergence of distinct peaks in the distribution”. This is exactly the relationship predicted by a common aspirations model.

3.3. Stratified Aspirations. While the failure to converge to equality is endemic in all the models of aspirations formation (at least under the assumptions that we’ve imposed), the emergence of clustering — and the number of such clusters or poles — deserves further investigation. One extension to the case of stratified aspirations, in which different segments of the economy each harbor common aspirations, but those aspirations vary across segments. For instance, we might think that the economy is divided among the “poor”, the “middle class”, and the “rich”, and each inhabitant of this coarse classification has common aspirations drawn from the going (or anticipated) income distribution.

Recall that under stratified aspirations, the income distribution is segmented into $n$ quantiles with aspirations

$$\Psi(y, F) = a_i$$

for individuals with income $y$ in quantile $i$, where $a_i$ is a scalar representing some summary statistic of the distribution in that quantile.

**Proposition 3.** Under stratified aspirations, if $\{F_t\}$ is an equilibrium sequence of income distributions converging to some limit distribution $F^*$, then $F^*$ must generically be concentrated on a finite set of points, at least two in number.

The proof of this proposition is a direct extension of the argument made for common aspirations, and we omit it.

The only way to get away from clustering is to have aspirations that are fine-tuned to one’s own personal circumstances. Such is the case with models of aspirations formation in which *individual* income enters in a highly sensitive way. One example is upward-looking aspirations, in which the aspirations of an individual are given by the conditional expected value of all higher incomes in the distribution. In some circumstances, this specification is compatible with a non-clustered steady state distribution of income. (We omit the proof of this observation, which is available on request.)

3.4. A Numerical Illustration. We illustrate the observations above with some numerical simulations. In the examples that follow, we suppose that the production function exhibits the constant elasticity form $f(k, \theta) = \frac{1}{\alpha} \theta k^\alpha$, where $\alpha = 0.8$ and $\theta$ is a stochastic
shock with mean 1. Preferences are as follows: \( u(c) = \ln(c) \) and \( v(z) = 0.8 \ln(z) \), while \( w \) is taken to have the logistic structure

\[
w(a, z) = \frac{1.6}{1 + \exp\left[ -\kappa (\frac{z}{a} - 1)\right]}.
\]

Note that the parameter \( \kappa \) controls the steepness of \( w \) at the point \( z = a \), where aspirations are met, while leaving the level of utility \( w \) at \( z = a \) unaffected.

Our first exercise illustrates our results on polarization and common aspirations, which we take to be at median income (but any other specification would do just as well). We begin with an initial distribution of income that is uniform over a population of 800 individuals, and iterate the distribution over time. The simulated distributions converge to a steady state (where the only mobility is due to the noise in the production function). When \( \kappa \) is large so that \( w \) is suitably steep at \( z = a \), equality is impossible and the distribution converges to a bimodal limit, in line with the discussion above. Over time, the distribution of income clusters around two poles. The first panel of Figure 2 illustrates this outcome for \( \kappa = 5 \).

If the value of \( \kappa \) is lower, then \( w \) is relatively flat at \( z = a \) and aspirations are less focal as a goal. Now full equality is possible. In the second panel of Figure 2, constructed for \( \kappa = 0.5 \), we see convergence to a perfectly equal income distribution.

In our second exercise, we contrast stratified and unstratified aspirations. We illustrate our point in the “steep” case — \( \kappa = 5 \) — but without any noise. The thought experiment goes as follows. Consider a society in which individuals are “cognitively stratified” into two income classes, perhaps as a result of social or spatial segregation by income. The

\[\text{Figure 2. Polarization and Common Aspirations.}\]

...
poorer half of the population draws his aspiration from the median income among the poor while, in similar fashion, the richer half use the conditional median among their group. In this case, the distribution develops multiple poles and exhibits some mobility (despite the absence of noise). In the numerical example illustrated in Figure ??, a group of poor individuals cluster around an income of just below 5, while a group of rich individuals earn around 20. Both experience hardly any mobility. In the middle, groups of individuals earn between 6 and 9 and experience some mobility with their dynasties switching regularly from one class to the other (around 40% of individuals in the last period switched class).

It is possible to contrast this outcome with that in a similar society with common aspirations. Say that owing to less segregation or higher media exposure, individuals learn more about the incomes of the entire population and aspirations are commonly tagged at the median income. In this society, the distribution becomes much more polarized and converges to one with twin peaks at 2 and 27 with zero mobility (there is no production noise, in contrast with the previous example). With stratified aspirations, aspirations windows are smaller and aspirations consequently more attainable. This permits the emergence of a middle class and generates an income distribution with less inequality.

4. Aspirations and Growth on Income Cross-Sections

Moving away from steady state analysis, we want to allow for growth and look more closely at how growth rates vary along the cross-section of incomes. To do so, it will be useful to provide some more structure on the utility functions $u$ and $v$, as well as the production function $f$. Let us suppose that $u$ and $v$ have the same constant-elasticity functional form; i.e., $u(c) = c^{1-\sigma}/(1 - \sigma)$ and $v(z) = \rho z^{1-\sigma}/(1 - \sigma)$ for some $\sigma > 0$ and $\rho > 0$. Suppose, moreover, that the production function is linear: $f(k) = (1+r)k$, where $r$ is the rate of return on investment.
Then individuals maximize

\[ u(c) + v(z) + w(z, a) = \frac{1}{1 - \sigma}c^{1-\sigma} + \rho \frac{1}{1 - \sigma}z^{1-\sigma} + w(z, a), \]

subject to the constraint

\[ z = (1 + r)(y - c). \]

for a given starting income level \( y \) and aspiration level \( a \).

The expositional advantage of constant-elasticity utility with linear production is that, in the absence of an aspirations effect, bequests are proportional to incomes and therefore growth rates are constant across the cross-section of current incomes. We can therefore be sure that any cross-sectional variation in the presence of aspirations stems entirely from aspirations alone. Without restricting individuals to be at steady state, we would like to describe the \textit{growth incidence curve}, a relationship that links baseline income to subsequent rates of growth.

Rewriting (4) in terms of the growth rate \( g = (z/y) - 1 \), we get

\[ \frac{1}{1 - \sigma} \left( y \left[ \frac{r - g}{1 + r} \right] \right)^{1-\sigma} + \rho \frac{1}{1 - \sigma} \left( [1 + g]y \right)^{1-\sigma} + w((1 + g)y, a). \]

The problem is nonconvex and may exhibit more than one solution. However, any solution to problem (6) is obviously interior in the choice of \( g \) and is therefore described by the first-order condition

\[ (r - g)^{-\sigma}(1 + r)^{\sigma-1} - \rho (1 + g)^{-\sigma} = w_1((1 + g)y, a) y^{\sigma}. \]

The chosen growth rate will lie between a minimum of \(-1\) and a maximum of \( r \).

4.1. \textbf{Baseline Income and Growth.} The condition (7) permits us to study the effect of income on the (chosen) rate of growth \( g \). To gain intuition, Figure 4 describes how the rate of growth \( g \) is determined by this first-order condition. The upward-sloping bold line is the left hand side of the first-order condition, it is obviously increasing in \( g \).\(^{10}\) The right-hand side (which is the other bold line) is \textit{also} increasing in \( g \), at least as long as aspirations are \textit{unattained}, in the sense of having \((1 + g)y < a\). Once aspirations are \textit{exceeded}, so that \((1 + g)y > a\), the right-hand side will decline in \( g \).\(^ {11}\)

There could, in principle, be several intersections between the two lines. The second-order condition, however, assures us that we only need to consider those intersections in which the right-hand side cuts the left-hand side “from above”. (Even that isn’t enough to fully pin the solution down, but it is certainly necessary.) For ease of exposition, the diagram only has the two lines intersecting once, at \( g_1 \).

\(^{10}\)Indeed, the left-hand side tends to minus (or plus) infinity when \( g \) tends to \(-1\) (or \( r \)), with curvature switching from concavity to convexity in between.

\(^{11}\)It is easy to check that under our assumptions, it will not be the case that \((1 + g)y = a\), so these definitions are exhaustive.
Suppose aspirations are high enough so that they remain unattained. Figure 4 carries out the exercise of raising \( y \) in such a context. The right-hand side of (7) is unambiguously shifted upwards, and we see that the new growth rate is higher, at \( g_2 \). While this intuitive argument in quite far from a formal proof,\(^{12}\) it motivates

**Proposition 4.** Suppose that aspirations are commonly held. Consider any income level \( y \) at which aspirations are unattained. Then growth rates decline as incomes decline below \( y \).

For a proof, see the Appendix.

What of income levels for which aspirations are exceeded? As before, the answer hangs on what happens to the right-hand side of the first-order condition (7) as income rises:

**Proposition 5.** Suppose that aspirations are commonly held, and assume that

\[ W^\prime (z,a)z^\sigma \text{ is declining in } z \text{ when } z > a. \]

Then growth rates decline as incomes increase, once aspirations are attained.

\(^{12}\)In particular, local second-order conditions are not sufficient for optimality, and a change in \( y \) could move the optimal choice to an entirely different location instead of simply precipitating a local change. The proofs of Propositions 4 and 5 therefore employ revealed-preference arguments that are not based on local conditions.
See the Appendix for a proof.

Proposition 4 captures the idea of frustration, and Proposition 5 the notion of complacency. In the former case, as incomes fall further and further below a commonly held aspiration, the incentivizing effects of aspirations weakens: this comes naturally from the S-shaped specification that we’ve borrowed from the behavioral economics literature. There is, therefore, less of an attempt to save when baseline incomes are lower. This observation is in line with the arguments in [1] and [26].

A different effect comes into play as outcomes exceed aspirations. Now individuals turn complacent. Under the additional condition \([W']\) in the statement of Proposition 5, an increase in \(y\) lowers growth.

How reasonable is \([W']\)? If \(w\) has unbounded steepness at \(z = a\), the condition certainly holds in some region above \(z = a\). Whether it holds more globally will depend on the value of \(\sigma\), as well as the specific form of \(w\), and in particular on the degree of concavity exhibited by it when \(z > a\). Specifically, it can be shown that the curvature elasticity \(\sigma\) of the conventional one-period utility indicators \(u\) and \(v\) should not be too high, and in particular should not exceed that of \(w\).13

It is easy to see that \([W']\) is equivalent to the requirement that the curvature elasticity of \(w\), given by \(-w_{11}(z, a)z/w_1(z)\) exceed those of the conventional utility indicators \((\sigma)\) when \(z > a\).

The two propositions together represent a formal statement of our informal assertion that “attainable” aspirations, which can be closed by a round of sustained growth, are the most conducive to investment. It is tempting to conclude that growth should be maximized at “intermediate” levels of income, but that will depend on more restrictions.

Certainly, when aspirations are commonly held and condition \([W']\) is applicable, the equilibrium rates of growth rise and then fall on the cross-section of incomes, though the relative sizes of the two segments will depend on just where the common aspirations are placed. More generally, the theory makes no particular prediction regarding the shape of growth rates in income, once aspirations are also suitably endogenized. For instance, in the case of upward-looking aspirations, it is possible that every income grows at exactly the same rate, once the initial distribution is suitably chosen (details available on request).

At the same time, it is intriguing to observe — as we do in Section 4.2 below — that in actual contexts, much of the variation in observed cross-sectional growth rates can be accommodated with simple models of aspiration formation; more on this below.

The theory is also particularly well-suited to assess what might happen to cross-sectional growth rates when some mechanism that predictably shifts aspirations at each level of income comes into play. For instance, it is possible to consider the rise of mass media in

\[13\]For instance, suppose that \(w\) has constant curvature elasticity \(\mu\) to the right of \(a\); i.e., \(-w_{11}(x + a, a)x/w_1(x + a) = \mu\). Then it is easy to check that \(\sigma \leq \mu\), along with the already-assumed strict concavity of \(w\) to the right of \(a\), is sufficient for \([W']\).
developing countries, in which advertisements and television programs showcase the lives of the rich, thus raising aspirations across the board. Figure 5 illustrates the outcome for the case of common aspirations.

Up to income level \( Y \), aspirations are not met at the (common) aspiration \( a \). When \( a \) increases to \( a' \), income \( Y \) can now be identified with a lower income in the earlier situation, so the growth rate associated with \( Y \) — and indeed, with all income levels below \( Y \) — fall. We can do the same argument in reverse by defining \( Y'' \) as the income level after which aspirations are exceeded, in the situation with common aspirations \( a' \). Assume that growth rates are declining after that point; this will automatically be satisfied when condition \([W']\) is met. When \( a' \) is brought back to \( a \), all income levels to the right of \( Y'' \) can be identified with still higher incomes, so growth rates are lower relative to \( a \) than they are relative to \( a' \). These “swivels” in the growth incidence curve translate into predictions about both the aggregate growth rate as well as the evolution of income inequality over time.

4.2. An Illustrative Empirical Exercise. This section uses percentile distributions available for 43 countries over at least two distinct years to illustrate the growth incidence curves predicted by different models of aspirations formation.\(^{14}\) Throughout, we take \( w(z,a) \) to have a CARA shape in \( z/a \) above 1 and be symmetric around 1,\(^{15}\) and set \( \rho = 0.8 \).

\(^{14}\)Special thanks to Claudio Montenegro at the Development Research Group, Poverty Unit, The World Bank.

\(^{15}\)

\[
\begin{align*}
    w(z,a) &= 10 - e^{-20(z/a-1)} \text{ for all } z \geq a \\
    &= 8 + e^{-20(1-z/a)} \text{ for all } z < a;
\end{align*}
\]
In our first exercise, we assume common aspirations tagged to societal mean income. For any distribution, a given return to capital $r$ generates specific individual growth rates as a function of income. These incomes and growth rates imply a specific aggregate growth rate for the country. In this exercise, we find the return to capital $r$ that generate in our model the actual aggregate annual rate growth observed. Figure 6 shows the resulting growth incidence curves for nine Latin American countries in the nineties.\footnote{We would be happy to provide anyone interested with the graphs for the remaining countries.} As seen in Propositions 4 and 5, our model predicts that with common aspirations these growth rates follow an inverted U-shape.

Clearly, the actual growth pattern by percentiles observed in these countries has, in most case, a different shape. This is not surprising as our simple model does not even come close to capture the many factors that drive percentile growth, but on top of that there is no reason to believe that aspirations are commonly held. That suggests the following thought experiment: using these data and our model, we can study the characteristics of the process of aspirations formation that best fit the actual growth incidence curves. Our second exercise does just this.

Our data consists in 55 percentile distribution and growth incidence curves. For each of these, we search among a class of aspiration formation processes where 1) the weight put by percentile $i$ on another percentile income $j$ only depends on $(i-j)$ (the percentile
distance and whether i is richer or poorer than j; 2) these weights have a quadratic shape in $|i - j|$ on either side of $i$ (and are not necessarily symmetric around $i$). This class includes among others common and upward aspirations processes. As before, the return to capital $r$ in any country is selected to match actual aggregate annual rate growth observed.

The growth incidence curve predicted by our “best fit” aspirations are illustrated in Figure 6. Although they are not perfect match, we see that they come much closer to the actual percentile growth. This specification captures 75% of the observed variation in growth (as opposed to 3% for the common aspirations model).

What we find is umbrella-shaped aspirations that a. are centered: for 85% of the countries (47 out of 55) individuals put the most weight in forming their aspirations on the income in their own percentile; and b. have narrow aspirations windows: in more than half the countries individuals put no weight in forming their aspirations on the incomes that are more than two percentiles away from themselves. This is shown in Figure 7 for the same nine countries studied earlier. On the left-hand-side of the picture, we see the weights that the median percentile puts on the neighboring percentiles when forming its aspirations. On the right-hand-side of the pictures we see the resulting level of aspirations (in log) for the various percentiles.
Although the limitations of this exercise are obvious, it suggests a fair amount of stratification in the aspiration formation process. This raises the growth rates of the poorest percentiles but reduces the growth rates in the upper middle range of the distribution.

5. Conclusion

This paper builds a theory of aspirations formation that emphasizes the social foundations of individual aspirations, and relates those aspirations in turn to investment and growth. Following a familiar lead from behavioral economics (see, e.g., [17], [18], and [19]), we define utilities around a “reference point”, and interpret that reference point as an aspiration. Our main departure from this literature is in the determination of aspirations: rather than emphasizing the past experiences of the individual herself in shaping aspirations, we stress the social basis of aspirations formation. We argue that aspirations are likely to depend not only on one’s own historical living standards, as commonly assumed, but also on the experience and lifestyle of others.

The theory we propose has three segments. First, individual aspirations determine one’s to incentives to invest, accumulate, and bequeath. Second, aspirations are determined by the going distribution of income. Finally, individual behavior is aggregated to derive the social distribution of income, thus closing the model.

A central ingredient of our setup is that aspirations can serve both to incentivize and to frustrate. We argue that aspirations that are above — but not too far — from current incomes can encourage high investment, while aspirations that are too high may discourage it. Formally, this is well-captured by the reference-based utility function, in which departures of income from the reference point in either direction lead to diminishing returns in utility (or disutility).

Our main results concern stable equilibrium income distributions. We show that perfect or near-perfect equality is unstable: income distributions cannot converge to a degenerate distribution. Indeed, if aspirations are common or stratified, we show that in any steady state, incomes must polarize over time. We also study the behavior of growth rates along the income distribution.

The theoretical results are complemented by an empirical exercise, which uses percentile distributions of growth rates available for 43 countries. We use aggregate growth experiences to estimate a rate of return to individual investment, and then employ the model to see which aspirations formation structure appears to fit the data best, in that they come closest to the observed growth incidence curves by percentile. We show that a model of aspirations formation in which individuals use umbrella-shaped weights on incomes in some interval around their own incomes comes closest to replicating the data, and that a large fraction of the observed variation is indeed “explained” by our specification.

The goal of this paper has been to take a modest step towards thinking about the social determinants of aspirations or reference points. As in the case of any model with social effects on individual behavior, which are then aggregated to yield those social outcomes,
there are difficulties in undertaking a full-blown dynamic analysis, and this paper is no exception. It would be of great interest to fully describe income-distribution dynamics for different models of aspirations formation. In the same spirit, one might ask for a more comprehensive structural exercise which would allow us to exploit the model to uncover more fully the process of aspirations formation. We believe that this approach will shed new and complementary light on the endogenous emergence of inequality.

References

Appendix: Proofs Not in the Main Text

Define a function \( L(g) \) by

\[
L(g) = \frac{1}{1-\sigma} \left( \frac{r-g}{1+r} \right)^{1-\sigma} + \rho \frac{1}{1-\sigma} (1+g)^{1-\sigma}.
\]

Then it should be clear from (6) that for each \( y \), the optimal choice of \( g \) maximizes

\[
L(g) + g^{\sigma-1} w ((1 + g)y, a).
\]

Let \( g_1 \) and \( g_2 \) be optimal choices at \( y_1 \) and \( y_2 \) respectively. For additional simplicity of notation, let \( G_1 \equiv (1 + g_1) \) and \( G_2 \equiv (1 + g_2) \), and let \( h(z) \equiv w(z, a) \). A standard
revealed preference argument tells us that
\[ L(g_1) + y_1^{\sigma-1}h(G_1y_1) \geq L(g_2) + y_2^{\sigma-1}h(G_2y_2) \]
and
\[ L(g_2) + y_2^{\sigma-1}h(G_2y_2) \geq L(g_1) + y_1^{\sigma-1}h(G_1y_1) \].

Combining these two inequalities, we must conclude that
\[ \Psi(y_1) \geq L(g_2) - L(g_1) \geq \Psi(y_2), \]
where the function \( \Psi \) is defined by
\[ \Psi(y) \equiv y^{\sigma-1} [h(G_1y) - h(G_2y)]. \]

**Proof of Proposition 4.** Suppose that \( y_1 < y \), and that aspirations are unattained at \( y \): \((1 + g)y < a\). Suppose, contrary to our assertion, that \( g_1 > g \). Then, with a little work, we can find \( y_2 > y_1 \) such that \( g_1 > g_2 \) and \((1 + g_1)y_2 < a\).\(^{17}\)

Now, simple differentiation of \( \Psi \) tells us that
\[
\Psi'(y) = y^{\sigma-2} \left[ \{ h'(G_1y) G_1y - h'(G_2y) G_2y \} + (\sigma - 1) \{ h(G_1y) - h(G_2y) \} \right]
\]
\[
\geq y^{\sigma-2} \left[ \{ h'(G_1y) G_1y - h(G_1y) \} - \{ h'(G_2y) G_2y - h(G_2y) \} \right].
\]

At the same time, the function \( h'(z)z - h(z) \) is strictly increasing in \( z \) for \( z \in [G_2y_1, G_1y_2] \), because \( z < a \) over this entire range.\(^{18}\) Using this information in (9), and recalling that \( G_1 > G_2 \) by assumption, we must conclude that \( \Psi'(y) > 0 \) for all \( y \in [y_1, y_2] \), which contradicts (8). \( \Box \)

**Proof of Proposition 5.** Suppose that \( y_1 < y_2 \), and that aspirations are exceeded at \( y_1 \): \((1 + g_1)y_1 > a\). Suppose, contrary to our assertion, that \( g_2 > g_1 \). Just as in the proof of Proposition 4, we know that
\[
\Psi'(y) = y^{\sigma-2} \left[ \{ h'(G_1y) G_1y + (\sigma - 1)h(G_1y) \} - \{ h'(G_2y) G_2y + (\sigma - 1)h(G_2y) \} \right].
\]

But condition \([W']\) informs us that the function \( h'(z)z + (\sigma - 1)h(z) \) is strictly decreasing in \( z \) for \( z \in [G_1y_1, G_2y_2] \), because \( z > a \) over this entire range.\(^{19}\) We must therefore conclude that \( \Psi'(y) > 0 \) for all \( y \in [y_1, y_2] \), which contradicts (8).

---

\(^{17}\)Suppose that \( y_1 < y \) and \( g < g_1 \). A standard argument establishes the monotonicity of aggregate investment in initial income, so that aspirations are unattained for all \( y' < y \). Define \( y' \equiv \inf \{ y' < y \} \) for \( y' < g_1 \). If \( y' = y_1 \), we are done by choosing \( y_2 \) slightly above \( y_1 \): because \((1 + g_1)y_1 < a\), we will have \((1 + g_1)y_2 < a\) and \( g_1 > g_2 \), as desired. If \( y' > y_1 \), then \( g' > g_1 \) for all \( y' \in [y_1, y') \), so that once again, \( y \) choosing \( y_2 \) slightly above \( y' \), we have \((1 + g_1)y_2 < a\) and \( g_1 > g_2 \).

\(^{18}\)To see this, differentiate to see that \( \frac{d}{dz} [f'(z)z - f(z)] = zf''(z) > 0 \).

\(^{19}\)To see this, differentiate to see that \( \frac{d}{dz} [h'(z)z + (\sigma - 1)h(z)] = zh''(z) + \sigma h'(z) < 0 \), by \([W']\).