Contractual Structure and Endogenous Matching in Partnerships*

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Abstract

We analyze optimal contracts and optimal matching patterns in a simple model of partnership where there is a double-sided moral hazard problem and potential partners differ in their productivity in two tasks. It is possible for one individual to accomplish both tasks (sole production) and there are no agency costs associated with this option but partnerships are a better option if comparative advantages are significant. We show that the presence of moral hazard can reverse the optimal matching pattern relative to the first best, and that even if partnerships are optimal for an exogenously given pair of types, they may not be observed in equilibrium when matching is endogenous, suggesting that empirical studies on agency costs are likely to underestimate their extent by focusing on the intensive margin and ignoring the extensive margin.

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1 Introduction

Organizations are ways to pool and coordinate the diverse talents of different people. When we look at actual organizations we sometimes observe vertical relationships, e.g., a firm hiring a worker or a landlord hiring an agricultural labourer and sometimes partnerships – sharecropping, professional partnerships, etc. (Eswaran and Kotwal, 1985; Levin and Tadelis, 2005). The choice of organizational structure is influenced by two important factors. On the one hand, it is shaped by incentive considerations since more often than not people's characteristics or actions are unobservable in reality and need to be revealed or implemented through a constrained-optimal organizational design. On the other hand, the organizational structure depends on the relative importance of the inputs that members bring in. By pooling various inputs organizations can exploit specialization and comparative advantage among its members. In general, the gains from members' comparative advantages have to be balanced against the agency costs stemming from incentive problems.

We study the interplay of these two sets of economic forces, namely, incentive provision under double-sided moral hazard and sorting. We are interested in what kind of contracts will be observed in partnerships when they are voluntarily formed, and given this, what kind of partnerships will form in equilibrium. We show that the interplay between these two economic forces makes certain organizational forms more likely to appear than others, and this interaction explain why observed incentive schemes are often more homogeneous relative to what incentive theory would imply.

There is also a large literature on sorting in partnerships or organizations (Becker 1973, 1993; Sattinger, 1975; MacDonald, 1982; Rosen 1982; Kremer, 1993 among many others). The main result is that the equilibrium sorting pattern critically depends on whether the payoff function is sub- or super-modular in agent's characteristics. This in turn is determined to a large extent by the complementarity or substitutability in production of those characteristics and, as we demonstrate, the assumed information structure (first-best vs. moral hazard).

To fix ideas, consider the following example. Suppose production depends on the labor effort and the 'talent' (productivity) supplied by each party in a two-member organization. Agents differ in their talent. Partnerships pool talent but are subject to agency problems with regards to effort provision. To give more incentive to one party means giving less incentives to the other. If talent and effort are complements, the higher the relative ability of one agent, the higher should be his output share. This means that he might be optimally matched with a low-ability agent. However, this undermines the gains from trade due to specialization since a match with a high-ability person would produce more output. Given this trade-off we show that, if the incentive costs outweigh the gains from specialization, voluntarily formed partnerships may not be observed in equilibrium and the high-ability agent may instead simply hire a low-ability agent from the market to work for him or produce on his own . Thus, simply pooling talent is not a sufficient argument for partnerships – strong complementarities among the quality of the supplied inputs are also needed, otherwise with endogenous matching partnerships may not be observed. More interestingly, as we show, the presence of double-sided moral hazard combined with weak complementarity in the inputs can lead to a reversal of the optimal matching pattern relative to the first-best (the case with contractible effort) from positive to negative assortative matching. In contrast, without any incentive problems, even small amount of complementarities among the quality of supplied inputs implies positive assortative matching.

Next, we allow for endogenous matching and show that the reversal in matching patterns can affect the observed contractual forms. We demonstrate that when individuals are heterogeneous in terms of their skills and allowing endogenous sorting, the existing results in the literature can be significantly affected. In particular, sharing the output/revenue/profit leads to the possibility of the abilities of the partners being strategic substitutes. Holding my partner's ability constant, if I am more able, optimally I should have a higher share if ability and my effort are complements in production. But this effect is dampened the higher is my partner's ability. Thus, unless our efforts (or the tasks) are highly complementary, negative sorting might end up being optimal. Therefore, in situations where a standard model of partnerships (e.g., Eswaran and Kotwal, 1985) would predict partnerships for an exogenously given pair of types of players, with endogenous sorting one could instead observe one person doing both tasks (sole production) instead of the first-best partnership arrangement since partnerships always involve some agency costs which need to be traded off relative to the gains from comparative advantage. This has the important implication that empirical studies on agency costs (e.g., Chiappori and Salanie, 2003 or Prendergast, 1999) are likely to underestimate their extent by focusing on the intensive margin and ignoring the extensive margin, i.e., partnerships that are not formed or partnerships in which the sorting pattern is reversed once endogenous matching is allowed.[1] Treating sorting patterns in organizations as exogenously given and/or ignoring the probability of endogenously optimal re-matching can therefore lead to misleading estimates of the welfare gains possible when resolving the problem of asymmetric information.

Our paper is also related to a recent applied literature on contracts and endogenous matching in various settings. The issue of unobserved heterogeneity of principals and agents is now taken quite seriously in the empirical literature on contracts (see Chiappori and Salanie, 2003 for a review). However, it is only recently that authors have systematically tried to explore the role of matching between principals and agents and how this affects contract design. Ackerberg and Botticini (2002) provide an argument as well as empirical evidence that, since risk-neutral tenants are likely to select riskier crops, in equilibrium we could observe the output shares of tenants cultivating risky crops to be higher than those of tenants cultivating less risky crops. This argument has been formalized by Serfes (2005). Dam and Perez-Castrillo (2005) characterize the matching outcome in a principal-agent model with risk-neutral principal and agents, moral hazard in effort choice and limited liability. They show that matching raises efficiency and that wealthier agents are preferred by the principals. Besley and Ghatak (2005) consider a similar setting but allow agents to be intrinsically motivated and principals and agents to differ in terms of their 'mission' preferences which affects the level of agent motivation. They too show that matching raises efficiency, but in their setting, it may reduce incentive pay since agent and principal mission preferences are better aligned. Chakraborty and Citanna (2005) look at a model of occupational choice where individuals can stay self-employed or form matches with another agent where there is a moral hazard problem subject to limited liability. They show that less wealth-constrained individuals choose occupations where incentive problems are more important. A recent paper by Dam (2010) analyzes a two-sided matching problem involving entrepreneurs who vary in terms of wealth and financial intermediaries who vary in terms of monitoring ability and shows that with endogenous matching more efficient monitors lend to borrowers with lower wealth, i.e., negative assortative matching results. Our paper is motivated by a different set of questions compared to this literature. We focus exclusively on the problem of incentive problems within partnerships and two-sided matching. In contrast to the general emphasis of this literature, in our model there are no limited-liability constraints. Comparative advantage and not wealth are the key source of heterogeneity that we focus on.

On the theory side, Legros and Newman (2002) study matching in economies with and without market imperfection and provide sufficient conditions for monotone matching that are weaker than the standard sub- and super-modularity conditions from the earlier literature. Their results, on which we draw upon in this paper, show examples of payoff functions in imperfect markets settings

[1] This fits Bastiat's dictum that an economist must take into account both what is seen and what is not seen (Bastiat, 1873).

where the equilibrium matching pattern is reversed (say from positive assortative to negative assortative) by changing the value of an exogenous parameter. In contrast, we show that the mere fact that a market imperfection exists (here endogenous, due to moral hazard) can lead to a reversal in the matching pattern relative to the first-best due to the optimal endogenous choices by the agents.

The paper proceeds as follows. Section 2 presents a simple two-agent model and characterizes the optimal organizational structures that will be observed in the first best and under double-sided moral hazard. Section 3 extends the model to allow for a varying degree of complementarity in agents' characteristics and more agents and types and derives the main results including the possibility for matching pattern reversal between the first best and moral hazard under endogenous matching. Several empirically testable implications of the model are discussed. Section 4 further extends the analysis of the optimal sorting pattern in partnerships under double-sided moral hazard to general functional forms for production and costs. The last section concludes.

2 A Simple Model

There is one period. A production project generates output which depend on the effort levels supplied in two distinct tasks, $e_1 \geq 0$ and $e_2 \geq 0$. There are two types of economic agents: those who are better at task 1 and those who are better at task 2 (to be referred to as 'task 1 specialists' and 'task 2 specialists'). The quality (ability, talent) of a task 1 specialist is $\theta_1 \geq 1$ and the quality of task 2 specialist is $\theta_2 \geq 1$. Assume that both ability levels take values on the same set $\Theta \subseteq [1, \infty)$. In particular we could look at the case when quality takes two values – high or low, i.e., $\theta_1, \theta_2 \in \{\theta_l, \theta_h\}$ where $\theta_h > \theta_l \geq 1$. The talent parameters, θ_1, θ_2 are assumed to be observable and contractible upon. In contrast, the effort levels e_1 and e_2 may or may not be contractible.

If a task 1 specialist of type θ_1 and a task 2 specialist of type θ_2 form a partnership, output is:

$$q(e_1, e_2) = \theta_1 e_1 + \theta_2 e_2 + \varepsilon$$

where ε is a random term with $E(\varepsilon) = 0$ and finite variance.

Assume that if a task j ($j = 1, 2$) specialist of type θ_j undertakes production entirely on her own she can perform task k ($k = 1, 2$ with $k \neq j$) just as well as a task k specialist of the lowest possible type $\theta_k = 1$. This puts a natural lower bound on θ because otherwise there is no comparative advantage for producing in a partnership. Thus, if a task 1 specialist of type θ_1 undertakes production on her own (she works on both tasks), output is:

$$q(e_1, e_2) = \theta_1 e_1 + e_2 + \varepsilon$$

Basically, she can do both tasks on her own but will do a better job in task 1 where her comparative advantage lies. Similarly, if a task 2 specialist of type θ_2 undertakes production on her own, output is

$$q(e_1, e_2) = e_1 + \theta_2 e_2 + \varepsilon.$$

To summarize there are three possibilities, which we illustrate with the following example. Suppose one party owns a productive asset in fixed supply (e.g., land) and is good at, for instance, supplying managerial inputs (task 1) while the other party is good at another necessary for production task (task 2), e.g., supervision of unskilled labour. The owner of the asset could work on both tasks although she is not very good at task 2. Alternatively, she can rent out the asset to another person who can work on both tasks in exchange of a fixed fee. Finally, the asset owner can form a partnership with a person who is better at the other task. Forming a partnership allows gains from specialization to be realized but will be subject to incentive problems when efforts are unobservable.

All parties are assumed to be risk-neutral and there are no wealth (limited liability) constraints. Each agent has a disutility cost of providing effort in either task which is assumed to be quadratic, $c(e_1) = \frac{1}{2}e_1^2$ and $c(e_2) = \frac{1}{2}e_2^2$ respectively. That is, the utility/profit function of each agent is $q(e_1, e_2) - c(e_1) - c(e_2)$.[2] The reservation payoff of each agent is set to 0.

2.1 The First Best

In the first best (when e_1 and e_2 are contractible), under sole production by a task 1 specialist of type θ_1, we have $e_1 = \theta_1$, $e_2 = 1$ and so expected profits are $\frac{1}{2}\left(\theta_1^2 + 1\right)$. Similarly, under sole production by a task 2 specialist of type θ_2, we have $e_1 = 1$, $e_2 = \theta_2$ and expected profits are $\frac{1}{2}\left(\theta_2^2 + 1\right)$. Finally, in a partnership between a task 1 specialist of type θ_1 and a task 2 specialist of type θ_2, we obtain $e_1 = \theta_1$, $e_2 = \theta_2$ and expected joint profits $\Pi^{FB} = \frac{1}{2}\left(\theta_1^2 + \theta_2^2\right)$.

This immediately implies that partnership would be chosen over sole production by task 1 specialists if and only if

$$\frac{1}{2}\left(\theta_1^2 + \theta_2^2\right) \geq \frac{1}{2}\left(\theta_1^2 + 1\right)$$

which is always true given our assumption that $\theta_2 \geq 1$. Analogously, a partnership will be always chosen over sole production by task 2 specialists. Thus, in the simple setting above, when parties' efforts are contractible, partnerships will be the only organizational form observed in the economy, for any ability levels, $\theta_1, \theta_2 \geq 1$.

2.2 Double-Sided Moral Hazard

Now consider the case in which e_1 and e_2 are non-contractible, i.e., partnerships pool talent but are subject to double-sided moral hazard in contrast to sole production. This implies that the gains from specialization that can be realized in a partnership (the fact that we can have θ_1 and θ_2 larger than one) have to be traded-off with the agency costs occurring because of the moral hazard problem. We restrict attention to linear contracts of the form (s, R) where $s \in [0, 1]$ represents the output share of player 1 and R represents a transfer from player 1 to player 2 (which could be negative or positive).[3] The incentive-compatibility constraints on effort in a partnership imply:

$$\begin{aligned}
e_1 &= \arg\max_{e_1} \left(s\left(\theta_1 e_1 + \theta_2 e_2\right) - R - \frac{1}{2}e_1^2 \right) = s\theta_1 \\
e_2 &= \arg\max_{e_2} \left((1-s)(\theta_1 e_1 + \theta_2 e_2) + R - \frac{1}{2}e_2^2 \right) = (1-s)\theta_2.
\end{aligned}$$

Joint surplus in a partnership is then

$$\begin{aligned}
\Pi^{MH} &= \theta e_1 + \theta_2 e_2 - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 \\
&= \theta_1^2 \left(s - \frac{1}{2}s^2 \right) + \theta_2^2 \left((1-s) - \frac{1}{2}(1-s)^2 \right). \qquad (1)
\end{aligned}$$

[2]We investigate general functional forms for the output and cost functions in Section 4.

[3]See Bhattacharya and Lafontaine (1995) for sufficient conditions under which the optimal sharing rule under double-sided moral hazard can be represented by a linear contract without loss of generality. These conditions are satisfied in our model. Additionally, assume that there is no budget-breaker and so the first-best cannot be achieved as in Holmstrom (1982).

The optimal sharing rule s is chosen to maximize the joint surplus (1), that is:[4]

$$s^* = \frac{\theta_1^2}{\theta_1^2 + \theta_2^2}.$$

Intuitively, the optimal share, s^* of a task 1 specialist in the partnership is higher the greater is her ability relative to her partner's ability. This property follows from the fact that ability and effort are complements in the production function, and so the higher is ability the greater is effort for the same share. The marginal gain from raising the share to elicit greater effort therefore goes up in ability.

Substituting s^* in the expression for joint surplus and simplifying, we obtain

$$\Pi^{MH}(\theta_1, \theta_2) = \theta_1^2 \frac{\theta_1^2}{2\left(\theta_1^2 + \theta_2^2\right)^2}\left(\theta_1^2 + 2\theta_2^2\right) + \theta_2^2 \frac{\theta_2^2}{2\left(\theta_1^2 + \theta_2^2\right)^2}\left(2\theta_1^2 + \theta_2^2\right) \tag{2}$$

$$= \frac{1}{2}\left\{\left(\theta_1^2 + \theta_2^2\right) - \frac{\theta_1^2\theta_2^2}{\left(\theta_1^2 + \theta_2^2\right)}\right\}. \tag{3}$$

It is easily verified that $\Pi^{MH}(\theta_1, \theta_2)$ is strictly sub-modular in θ_1 and θ_2 since $\frac{\partial^2 \Pi^{MH}(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} < 0$. Note that, because of the incentive problem, the modularity of joint surplus cannot be inferred directly from the substitutability or separability of the agents characteristics in the production function as in the first best (e.g., as in Becker, 1973; Sattinger, 1975) but also depends on the endogenous effort choices by the parties and the optimal choice of sharing rule. For any given values of e_1 and e_2, θ_1 and θ_2 are separable in the production function. However, since the optimal share of each partner is equal to her ability relative to the other partner, θ_1 and θ_2 are no longer independent in the joint surplus function once effort levels and the share are determined endogenously. The higher is θ_1, the share of the task 1 specialist goes up while the share of the task 2 specialist goes down, and so e_1 goes up and e_2 goes down. However, if θ_2 is increased, then the positive effect of an increase in θ_1 on the share s, and e_1 (relative to e_2) is dampened, and this is the intuition for the fact that, overall, the abilities of the agents engaged in the partnership under double-sided moral hazard are substitutes.

In contrast, if a type 1 specialist performs both tasks (i.e., output is obtained by setting $\theta_2 = 1$) then $e_1 = \theta_1$ and $e_2 = 1$ as in the first best (there is no incentive problem in sole production) and so

$$\Pi^1(\theta_1) = \frac{1}{2}\left(\theta_1^2 + 1\right). \tag{4}$$

Similarly, if a type 2 specialist performs both tasks $e_1 = 1$, $e_2 = \theta_2$ and

$$\Pi^2(\theta_2) = \frac{1}{2}\left(\theta_2^2 + 1\right). \tag{5}$$

3 Economic Implications

We now characterize the main implications of the model. We show that the matching patterns between parties of different abilities that will be observed under the first best or under double-sided moral hazard, as well as the optimal organization forms observed in these two settings, critically depend on whether the agents' efforts are substitutes or complements in the production function

[4] The same optimal sharing rule obtains if we maximize one party's payoff subject to a participation constraint by the other.

and also on the endogenous effort choices that agents make. We show that it is possible that the optimal matching pattern is reversed relative to the first best when efforts are non-contractible. This has important implications about the organizational forms and contractual structures that will be observed in equilibrium under endogenous matching as well as for estimating the magnitude of the agency costs due to moral hazard present.

We start with analyzing these effects in the context of our simple model from the previous section. Then, in Section 4, we generalize our results for wider classes of functional forms outlining the key differences in the model implications occurring under substitutability and complementarity of skills and efforts in production.

3.1 Matching Pattern Reversal and Agency Costs

Generalize the production function from Section 2 to:

$$q(e_i, e_j) = \alpha\theta_i z_j + \theta_i e_i + z_j e_j + \varepsilon \tag{6}$$

where $z_j = \theta_j$ if production is done in a partnership and $z_j = 1$ if the task i specialist produces alone, for $i, j = 1, 2$. The parameter $\alpha \geq 0$ determines the degree of complementarity in the types (abilities) of the partners.[5] If $a = 0$ agents' abilities are substitutes in production, while their complementarity increases in a for $a > 0$.

Proposition 1 (First-best matching)

Suppose agents efforts are contractible. Then:

(a) partnership dominates any form of sole production.

(b) only partnerships with positive assortative matching are optimal. Moreover, optimal matches are of the 'segregation' type, i.e., all partnerships consist of agents of the same ability.

Proof:

(a) Since $\alpha \geq 0$ and $\theta_i \geq 1$, the result follows directly from our discussion in Section 2.1 where we showed that for $\alpha = 0$ partnerships always (weakly) dominate sole production in the first best.

(b) The first-best expected joint payoff in a partnership is $\Pi^{FB}(\theta_1, \theta_2) = \alpha\theta_1\theta_2 + \theta_1^2 + \theta_2^2$. To prove the proposition statement we use the techniques of Legros and Newman (2002) which allow us to provide a sharper characterization of the matching pattern than simply showing sub- or super-modularity by looking at the sign of the cross-partial of joint surplus. More precisely, for any $\theta_1 = a, \theta_2 = b$ with $a, b \in \Theta$, define the 'surplus function',

$$\begin{aligned}
\sigma^{FB}(a,b) &\equiv \max\{0, \Pi^{FB}(a,b) - \frac{1}{2}(\Pi^{FB}(a,a) + \Pi^{FB}(b,b))\} = \\
&= \max\{0, \alpha ab + a^2 + b^2 - \frac{1}{2}\alpha a^2 - a^2 - \frac{1}{2}\alpha b^2 - b^2\} = \\
&= \max\{0, -\frac{\alpha}{2}(a-b)^2\} = \\
&= 0 \ \text{ for all } a, b \text{ and } \alpha \geq 0
\end{aligned}$$

The proposition statement then follows directly from Proposition 4 in Legros and Newman.■

[5] We rule out the case $\alpha < 0$ in which higher partner's ability *reduces* the joint payoff. In this case, if α is sufficiently negative, sole production can dominate partnership and negative assortative matching occurs if partnership is optimal. The proof is similar to the proof of Proposition 1 below.

The intuition for Proposition 1 follows from Becker's (1973) famous model of the marriage market – when agents' types are complements positive assortative matching results (e.g., Becker, 1993, ch. 4). Consider now the case in which the parties' efforts are non-contractible. From our results in Section 2.2 we obtain:

Proposition 2 (Matching under double-sided moral hazard)

Suppose the parties' efforts are non-contractible. Conditional on partnership being the optimal organizational form,

(a) for $\alpha \geq 0$ but not too large (i.e., $\alpha < M_1$ for some $M_1 \in (0, 1/2)$) negative assortative matching results.

(b) for α positive and sufficiently large, ($\alpha \geq M_1$) positive assortative matching (possibly heterogeneous) results. More precisely, for any $\alpha \geq 1/2$ the optimal matching pattern is 'segregation' (homogeneous partnerships).

Proof:

(a) In Section 2.2 we showed that for $\alpha = 0$ joint profits are sub-modular under double-sided moral hazard and the cross-partial $\frac{\partial^2 \Pi^{MH}(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2}$ is strictly negative (it equals $-\frac{4\theta_1^3\theta_2^3}{(\theta_1^2+\theta_2^2)^3}$). Since we are simply adding α to the cross-partial here, the existence of $M_1 > 0$ such that for $\alpha \in (0, M_1)$ the cross-partial remains negative and negative matching results follows by the continuity of Π^{MH} and its derivatives. To show that $M_1 \in (0, 1/2)$ note that $\frac{4\theta_1^3\theta_2^3}{(\theta_1^2+\theta_2^2)^3} \leq \frac{1}{2}$ is equivalent to $(\theta_1^3 - \theta_2^3)^2 + 3\theta_1^2\theta_2^2(\theta_1 - \theta_2)^2 \geq 0$ which is always true. That is, for the cross-partial of Π^{MH}, $\alpha - \frac{4\theta_1^3\theta_2^3}{(\theta_1^2+\theta_2^2)^3}$ to be negative, it must be that $\alpha < 1/2$ hence the threshold M_1 cannot exceed 1/2.

(b) By part (a) there exists an $M_1 > 0$ for which $\frac{\partial^2 \Pi^{MH}(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} \geq 0$ for $\alpha \geq M_1$. Thus, joint surplus in a partnership is super-modular and positive assortative matching results (possibly heterogeneous). For a sharper characterization, as in the proof of Proposition 1, we follow Legros and Newman (2002) and compute the surplus function under moral hazard (omitting the algebra):

$$\sigma^{MH}(a,b) = \max\{0, (a-b)^2[-\frac{\alpha}{2} + \frac{(a+b)^2}{8(a^2+b^2)}]\}$$

To show the rest, note first that $\frac{(a+b)^2}{8(a^2+b^2)} \leq 1/4$ is equivalent to $(a-b)^2 \geq 0$ which is always true. Thus, for $\alpha \geq 1/2$ we have $\frac{(a+b)^2}{8(a^2+b^2)} \leq 1/4 \leq \frac{\alpha}{2}$ i.e. $\sigma^{MH}(a,b) = 0$ for any $a, b \in \Theta$. By Proposition 4 in Legros and Newman (2002) the unique equilibrium matching pattern is therefore segregation.■

Proposition 2 implies that the presence of agency costs can lead to a *reversal* in the equilibrium matching pattern in partnerships. Specifically, when $\alpha > 0$ so that $\Pi^{FB}(\theta_1, \theta_2)$ is super-modular but α is small enough so that $\Pi^{MH}(\theta_1, \theta_2)$ is sub-modular, the presence of moral hazard not only implies the usual loss of surplus compared to the first-best due to the agency costs, but also an opposite pattern of sorting.

Due to the reverse endogenous sorting the true extent of agency costs is higher compared to when matches are exogenously given. To see this, suppose we observe negative assortative matching in a partnership under double-sided moral hazard and would like to assess the welfare loss due to agency costs relative to the first-best outcome. If we took this partnership as exogenously given, resolving the moral hazard problem would indicate a smaller increase in welfare compared to the true magnitude of welfare gains that will be obtained when parties are allowed to endogenously re-match in a positive assortative pattern (as they optimally would in the first best). Treating sorting patterns in organizations as exogenously given and/or ignoring the probability of endogenously optimal re-matching can therefore lead to misleading estimates of the welfare gains possible when resolving asymmetric information or other causes of market imperfections.

3.2 Optimal Contractual Forms with Endogenous Matching

3.2.1 Sub-Modular Joint Surplus

We now concentrate on the case of non-contractible efforts since otherwise (as shown in Section 3.1) partnership are the only organizational form that will be observed. Suppose the production function is such that negative assortative matching is optimal in a partnership. In particular, take our setting from Section 3.1 with $\alpha = 0$ (the results below are easily generalizable for α positive and sufficiently small, the case $\alpha \in (0, M_1)$ from Proposition 2). We characterize the case of more general production functions in Section 4. Also, assume initially that each agent's abilities can take only two values, θ_l and θ_h.

For the sake of concreteness, think of the setting studied by Eswaran and Kotwal (1985) where Agent 1 (the productive asset owner) is a landowner who has comparative advantage in managerial inputs (e.g., making production decisions based on market information and technical know-how) and Agent 2, who has no land, is a tenant who has comparative advantage in labour supervision. Both parties' efforts are not easily verifiable which leads to a double-sided moral hazard problem. Given that, both parties need to be given incentives to provide their inputs which is precisely what a share contract does. There are three possible forms of production: (i) the agents form a partnership and produce together subject to the double-sided moral hazard problem ('sharecropping'); (ii) Agent 1 is the sole producer paying unskilled workers a fixed wage; and (iii) Agent 2 is the sole producer paying Agent 1 a fixed rent. Since output is increasing in θ, in the latter two cases it is the higher productivity agent who is the sole producer, i.e., a necessary condition for the 'fixed wage' scenario to occur is $\theta_1 > \theta_2$, while the 'fixed rent' scenario occurs if $\theta_2 > \theta_1$.

While retaining some key elements of the Eswaran-Kotwal model, we relax two of their assumptions. First, we generalize their analysis for more general production functions and show how this affects the optimal choice of organizational form. Second, more importantly, we allow for many players with different degrees of absolute advantage in the two tasks (including many of the same type) who are free to choose any partner. In contrast, Eswaran and Kotwal consider only two, exogenously given agents, each with an absolute advantage in performing one of the tasks.

A straightforward comparison of (4) and (5) implies the following result: $\Pi^1 \gtreqless \Pi^2$ according as $\theta_1 \gtreqless \theta_2$. Furthermore, a partnership would be chosen if and only if

$$\frac{1}{2}\left\{\left(\theta_1^2+\theta_2^2\right)-\frac{\theta^2\theta_2^2}{\left(\theta_1^2+\theta_2^2\right)}\right\} \geq \max\left\{\frac{1}{2}\left(\theta_1^2+1\right), \frac{1}{2}\left(\theta_2^2+1\right)\right\}$$

Suppose we have a same-skill exogenously given match, i.e., $\theta_1 = \theta_2 = \theta$ (where θ is θ_l or θ_h). From the above result, a partnership would be chosen if and only if

$$\frac{1}{2}\left\{2\theta^2-\frac{\theta^2\theta^2}{2\theta^2}\right\} \geq \frac{1}{2}\left(\theta^2+1\right)$$

or,

$$\theta^2 \geq 2 \tag{7}$$

Note that this condition is stronger than the corresponding condition for the first best ($\theta \geq 1$) derived in Section 2.1.

If condition (7) is satisfied by both θ_l and θ_h and (this is crucial) if the above homogeneous matches were given *exogenously*, we would conclude that partnerships would be the only organizational form observed in this economy. For example, if there were only two agents in the economy (either both type θ_l or both type θ_h) or if we think of these homogeneous pairs of agents as living in two separate villages, we would expect to observe only partnerships (and no sole production) in each location. By taking pairs of agents in isolation (as, for instance, Eswaran and Kotwal do) and inferring which contract form they would choose we, however, ignore the possibility that endogenous sorting may cause such a pair of agents to never actually match in equilibrium.

Consider the following example which makes the latter point clearer. Suppose there are two agents of each type θ_l and θ_h, i.e., four individuals in total. If each agent is allowed to match with any partner they want to, out of the three available, we can show that partnerships may actually not be observed in equilibrium although they would be if the pairs (θ_l, θ_l) or (θ_h, θ_h) with $\theta_l^2, \theta_h^2 \geq 2$ were studied in isolation. This is due to the fact that joint surplus in a partnership under double-sided moral hazard is sub-modular in the quality of the agents engaged in the two tasks under the assumed parameters in this sub-section (see Proposition 2, part (a)).

The sub-modularity of joint surplus implies that if partnerships are optimal they must take the form (θ_h, θ_l). Therefore, with endogenous matching it is no longer the case that partnership dominates the alternative organization forms for any θ_l and θ_h satisfying (7). Indeed, with negative assortative matching, sole production by the high type would dominate partnership if

$$\frac{1}{2}\left\{\left(\theta_h^2+\theta_l^2\right)-\frac{\theta_h^2\theta_l^2}{\left(\theta_h^2+\theta_l^2\right)}\right\}<\frac{1}{2}\left(\theta_h^2+1\right)$$

or, equivalently, if

$$\theta_h^2 > \theta_l^4 - \theta_l^2 \qquad (8)$$

For instance, condition (8) always holds for $\theta_l^2 = 2$ and hence with endogenous matching we would not observe partnerships in an economy with double-sided moral hazard and $\alpha \in [0, M_1)$.

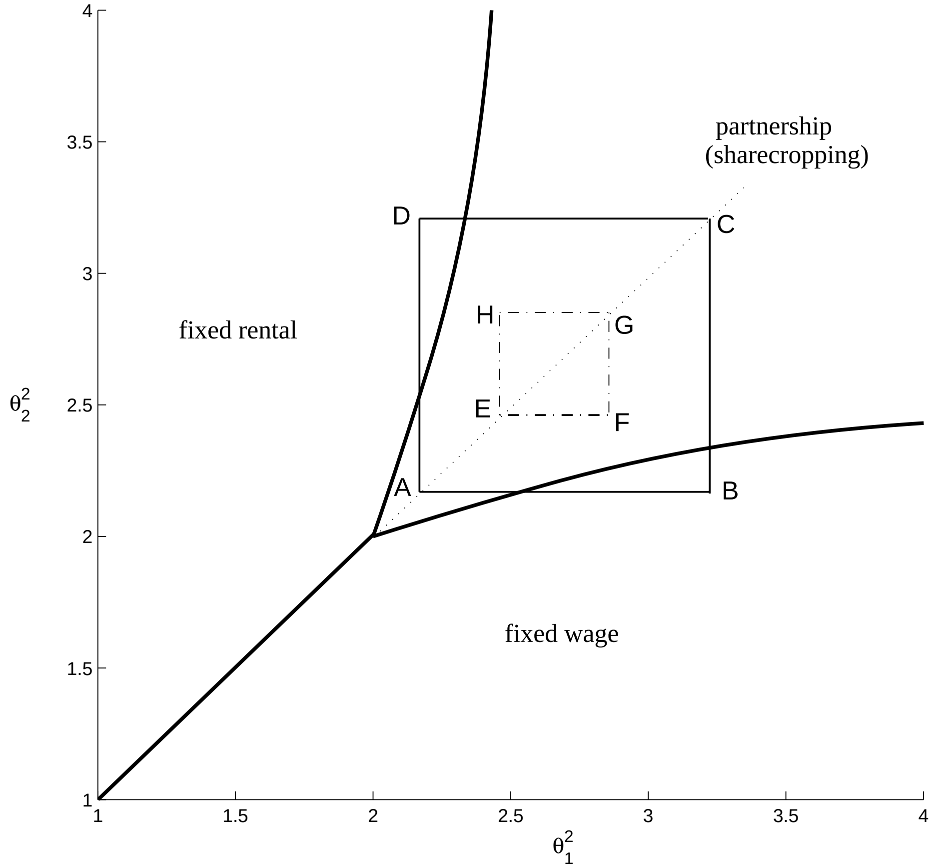
More generally, the (θ_1, θ_2) parameter space can be divided into three areas depicted on Fig. 1 below.

A1. 'fixed wage' area corresponding to the set $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_2 \text{ and } \theta_1^2 > \theta_2^4 - \theta_2^2\}$

A2. 'fixed rental' area corresponding to the set $\{(\theta_1, \theta_2) : \theta_2 \geq \theta_1 \text{ and } \theta_2^2 > \theta_1^4 - \theta_1^2\}$

A3. 'partnership' area corresponding to the set $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_2 \text{ and } \theta_1^2 \leq \theta_2^4 - \theta_2^2\} \cup \{(\theta_1, \theta_2) : \theta_2 \geq \theta_1 \text{ and } \theta_2^2 \leq \theta_1^4 - \theta_1^2\}$

Fig. 1 − Contract Forms



Note that partnerships would occur only when both agents have relatively high and not too dissimilar productivities (similar to Eswaran and Kotwal, 1985). However, unlike in Eswaran and Kotwal, our analysis implies that models of partnerships that ignore endogenous matching could predict partnerships to be optimal for parameter values for which they may not be optimal in many realistic situations where free partner choice is possible. For example, look at Fig. 1 and suppose we take two agents in isolation at either point A (both with high θ) or point C (both with low θ). Since $\theta^2 > 2$ at both points A and C we would conclude that partnerships between them will be observed. If we consider, however, an economy with the four agents together and they are allowed to match endogenously, the total surplus in two heterogeneous partnerships (which are optimal when Π^{MH} is sub-modular) is higher than that of the two homogeneous partnerships. However, for the thetas corresponding to points A and C on the figure, heterogeneous partnerships corresponding to the points B and D on the graph are dominated by contractual forms in which the high-skilled agent is a sole producer. This implies that no partnerships would be observed in this economy with endogenous matching.

Contrast the above scenario with another in which we instead start with two isolated agents at either points E and G (i.e., when the diversity in skills between the high and low type is not so large). Under matching in isolation (or, exogenously given pairs) we would observe partnerships. Unlike in the previous example, with all agents together and free to match with anyone they like we now observe partnerships corresponding to points H and F. Notice, however, that the matching pattern is reversed under endogenous sorting.

Clearly, the same intuition generalizes for the case of different skill levels. For example, the initial points A or C may be off the 45-degree line, i.e., we can have skill level combinations $((\theta_1^A)^2, (\theta_2^A)^2)$ and $((\theta_1^C)^2, (\theta_2^C)^2)$ with $\theta_i^A > \theta_i^C$. As long as A and C are inside the triangular shaped area labeled "partnership" on Fig. 1 while the corresponding points B and D obtained by permuting their coordinates (i.e., $((\theta_1^A)^2, (\theta_2^C)^2)$ and $((\theta_1^C)^2, (\theta_2^A)^2)$ are outside this area, we obtain the same results. Introducing more players also does not change the main intuition.

The above results imply that under double-sided moral hazard and endogenous matching partnerships will be only observed if the agents' skills are relatively close to each other. Moreover, the smaller the values that θ_h^2 and θ_l^2 take, the smaller the difference between the two must be to observe partnerships in equilibrium. Remember that for the production function (6) the optimal share in a partnership is $\frac{\theta_h^2}{\theta_h^2+\theta_l^2}$. Thus, (as in Eswaran and Kotwal, 1985) the optimal share when a partnership is actually observed would be close to 1/2.

Note that the endogenous matching makes the dispersion in ability values for which partnerships would be observed narrower than if the pairings were exogenously given (see Fig. 1, points H and F) and so the corresponding optimal share is closer to 1/2. Another prediction of our model is that the lower the levels of θ_h and θ_l are, the lower the dispersion of observed shares around 1/2 would be. For example, it is clear that if $\theta_l^2 = 2$, partnerships can only be observed if $\theta_h = \theta_l$ and the share will be exactly 1/2. More generally, if $\theta_l^2 > 2$ but close to 2 partnerships will be observed only if the ability difference between the two partners, $\theta_h - \theta_l$ is sufficiently small and the optimal sharing rule will be close to 1/2.

These results also generalize for multiple skill levels. Suppose, for example, the squares of the two task qualities are drawn from the continuous symmetric distribution[6] $\phi(\theta^2)$ with support on the interval $(\theta_m^2 - \Delta, \theta_m^2 + \Delta)$ where $\theta_m^2 - \Delta \geq 2$. The above discussion then implies that, under endogenous matching, the locus of all partnerships which would be observed in equilibrium must lie on the segment AB of the line through point C with coordinates (θ_m^2, θ_m^2) depicted on Fig. 2 below. Clearly, the lower θ_m is, the 'harder' will be to observe partnerships. Holding θ_m fixed, partnerships will be observed only between agents of skills relatively close to θ_m (which would be the mean skill level if ϕ is uniform). In particular, the most heterogeneous partnership (corresponding to points A and B on Fig. 2) can be between agents with skills $\sqrt{\theta_m^2 - \delta}$ and $\sqrt{\theta_m^2 + \delta}$ where δ solves[7]:

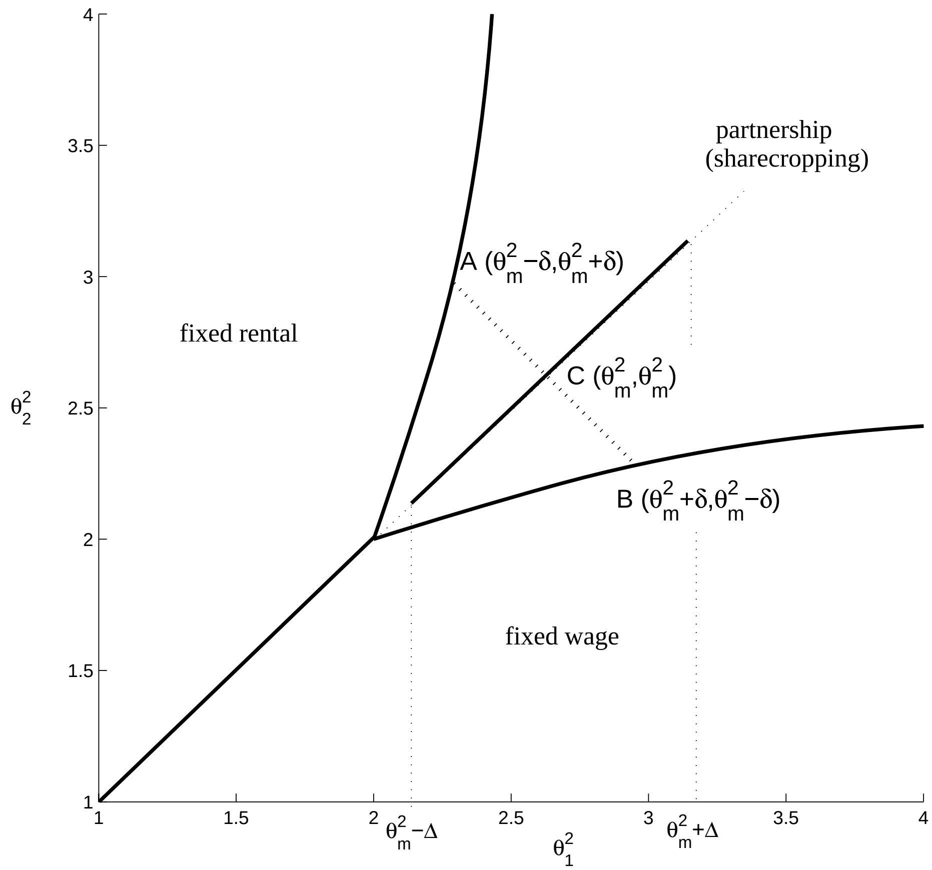
$$\theta_m^2 + \delta = (\theta_m^2 - \delta)^2 - (\theta_m^2 - \delta)$$

i.e., for $\delta = \theta_m^2 - \sqrt{2}\theta_m$.

For example, if ϕ is uniform and $\theta_m^2 = 3$ (mean skill of 1.73), the most heterogeneous partnership that will be observed in equilibrium is between agents with skills 1.57 and 1.88 and it features an optimal share of 0.59. That is, there would be no partnerships observed in this economy with shares outside the interval (0.41,0.59).

[6]Everything said here applies also to the asymmetric abilities case which however is harder to depict graphically.
[7]This is simply the equation of the curved line through point B.

Fig. 2 − Contract Forms (multiple skill levels)



3.2.2 Super-modular Joint Surplus

The results in Section 3.2.1 use crucially the fact that when Π^{MH} is sub-modular the optimal matching pattern under double-sided moral hazard is negative. Suppose instead that Π^{MH} is super-modular (e.g., $\alpha > 0$ and high enough – Proposition 2, part (b)), so that positive assortative matching would optimally occur. Now the argument from the sub-modular case goes the other way. Namely, if we start with heterogeneous agent pairs corresponding to the points B or D on Fig. 1 in isolation, we would conclude that no partnerships will be observed in the economy. However, with endogenous matching allowed, the homogeneous partnerships corresponding to points A and C will arise in equilibrium.

More generally, compared to the sub-modular case, positive assortative matching will lead to partnerships between agents with close skill levels, i.e., the observed optimal shares will be even closer to 1/2 on average, holding other things equal. For example, in the continuous skill distribution setting described in Section 3.2.1 with endogenous matching we would observe only partnerships between agents of equal skill ('segregation') and hence shares of 1/2 in each of them. No other sharing rules would be observed in equilibrium. In addition, as long as $\theta^2_m - \Delta \geq 2$ all agents in the economy would form partnerships, no matter what the values of the mean skill θ^2_m or the support range Δ are, unlike in the sub-modular case.

3.3 Discussion and Testable Implications

We now derive and discuss some economic applications and testable implications which follow from our theoretical analysis.

1. Within vs. between group heterogeneity

Suppose that negative sorting is observed in equilibrium under double-sided moral hazard (e.g., the case $\alpha = 0$). This means there is more heterogeneity *within* groups (partnerships), but less heterogeneity *across* groups. In our simple two-type (θ_l, θ_h) example, the joint surplus of both groups is the same. While we do not determine the exact individual payoffs, we showed that the optimal output shares within a heterogeneous partnership are unequal. If instead we observed positive sorting, there is little intra-group inequality (zero in the simple two-type setting where both tasks are equally important), but considerable inter-group inequality. The endogenous sorting pattern can thus have significant implications about inequality in the economy as a whole.

2. Unobserved ability

Negative sorting implies that partnerships will be heterogeneous. If ability is not directly measured but instead proxied (e.g., by wealth) such heterogeneity is often interpreted as suggestive for risk-sharing – the relatively less risk-averse (e.g., richer) partner insures the more risk-averse (e.g., poorer) partner. We provide an alternative explanation for heterogeneous partnerships which does not rely on risk aversion and insurance motives.

3. Variance of observed optimal shares

A testable prediction of our theory is that, holding other things equal, the observed distribution of shares in the population would have greater variance under negative sorting than under positive sorting. However, one needs to be mindful of the fact that different sets of partnerships would be observed depending on the optimal sorting pattern.

4. Variance of output

If we measure the output of partnerships under double-sided moral hazard (ignoring the cost of effort) it is easy to verify that it equals:

$$\alpha\theta_1\theta_2 + \left(\theta_1^2 + \theta_2^2\right) - \frac{2\theta_1^2\theta_2^2}{\left(\theta_1^2 + \theta_2^2\right)}$$

Hence, if $\alpha = 0$ or positive but not too high output is sub-modular in θ_1 and θ_2 as well. This has the implication that the variance of output across partnerships will be less with negative sorting compared to the case with positive sorting.

5. Skewness of the income distribution

With positive sorting Kremer (1993) shows that the income distribution is skewed to the right. His model has no agency problems, just a production function depending on the product of skills, $y = A\theta_1\theta_2$ which leads to completely homogeneous groups ($\theta_1 = \theta_2$) being formed. Kremer shows that if the skill gap between two agents is $\theta_h - \theta_l > 0$ this translates into an income gap proportional to $\theta_h^2 - \theta_l^2$ since the competitive market wage is proportional to the square of one's skill given the homogeneous matching. Since x^n is a convex function for $n > 1$, if $x_3 - x_2 = x_1 - x_0$ where $x_3 > x_1$ then by convexity $(x_3)^n - (x_2)^n > (x_1)^n - (x_0)^n$. Hence, with positive sorting the income distribution in Kremer's model economy is more skewed than the skill distribution.

Similar logic applies in our setting. Suppose both parties' skills are drawn from the same distribution. Under positive sorting there will be homogeneous matching and total profits are: $\Pi_{pos}^{MH} \equiv \frac{3}{4}\theta^2$. Since the optimal share is 1/2, each partner earns income of $\frac{3}{8}\theta^2$ which, as in Kremer's model, is proportional to the square of her skill level.

Constraining ourselves to partnerships, we can show that the income distribution is even more skewed than the skill distribution under negative sorting compared to under positive sorting. For simplicity consider the case with only two skill levels for each task, $\theta_h > \theta_l > 2$. From above we know that heterogeneous partnerships will arise in equilibrium and each will make joint profits of $\Pi^{MH}_{neg} \equiv \frac{1}{2}\left\{\theta_h^2 + \theta_l^2 - \frac{\theta_h^2\theta_l^2}{\theta_h^2+\theta_l^2}\right\}$. The optimal share is $s^* = \frac{\theta_h^2}{\theta_h^2+\theta_l^2}$, so the high-skill partner earns expected income of $s^*\Pi^{MH}_{neg}$ while the low skilled one earns an income of $(1-s^*)\Pi^{MH}_{neg}$. Therefore, the difference in the incomes of the high- and low-skill partners is:

$$(2s^* - 1)\Pi^{MH}_{neg} = \frac{1}{2}(\theta_h^2 - \theta_l^2)[1 - \frac{\theta_h^2\theta_l^2}{(\theta_h^2+\theta_l^2)^2}] > \frac{3}{8}(\theta_h^2 - \theta_l^2)$$

since $\frac{\theta_h^2\theta_l^2}{(\theta_h^2+\theta_l^2)^2} < \frac{1}{4}$. Thus, the income distribution among agents who form partnerships under both positive and negative sorting will be more skewed under negative sorting. The intuition is that the optimal share now also biases income in favor of the high-skill agent – incentive reasons dictate that he earns a higher share of total profits, which enhances the rewards to skill in addition to what is coming solely from technology. The reversal in the matching pattern from positive to negative due to the presence of moral hazard shown in the previous section thus exacerbates income inequality in our model economy above and beyond the technological complementarity reasons identified by Kremer. Depending on the social welfare function one may wish to adopt this may add to the agency costs due to asymmetric information or effort non-contractibility.

4 General Functional Forms

We now investigate in more detail the conditions on the production and effort cost functions that lead to sub- or super- modularity of the joint surplus function under double-sided moral hazard and endogenous matching. Since the payoff function is implicitly defined by the endogenous effort and share choices, we first need to characterize the optimal efforts and share that arise in a partnership. In particular, with general functional forms we cannot use directly the conditions of Legros and Newman (2002) to verify whether positive or negative assortative matching will occur since it is hard to verify them for the endogenously defined surplus function. Instead, we use the implicit function theorem and rely on the cross-derivative sign to characterize the conditions for sub- or super-modularity of joint payoffs directly.

The optimal sorting literature (e.g. Becker, 1993, Sattinger, 1975) has argued that the sub- or super-modularity of the payoff function is closely related to whether the agents' characteristics over which matching is done are substitutes or complements. With substitutes negative assortative matching occurs while with complements there is positive assortative matching. As pointed out in Legros and Newman (2002) the link between the degree of substitutability of agents' characteristics and the modularity of the payoff function may break down in more general environments featuring credit market imperfections (Legros and Newman, 1996; Sadoulet, 1998), restrictions on how output is shared (Farrell and Scotchmer, 1998), or specific output technologies (Kremer and Maskin, 1996). Given our double-sided moral hazard environment, it is thus necessary to derive the precise conditions under which joint profits are sub- or super-modular. The connection between substitutability in skills (over which agents match) and the modularity of joint profits is not immediate due to the endogenous choice of effort levels and the optimal sharing rule for output. We show that modularity is still related to the degree of substitutability in agents' skills but the degree of substitutability in efforts plays an additional important role.

We first look at two general classes of production functions – one where agents' efforts and skills are substitutes in production and one in which they are complements. We then study several particular widely-used functional forms.

4.1 Substitutes

Assume that the joint payoff function takes the following form in which the two partners' inputs and skill levels are substitutes in production:

$$\Pi(e_1, e_2, \theta_1, \theta_2) = F(e_1, \theta_1) + F(e_2, \theta_2) - c(e_1) - c(e_2) + \varepsilon,$$

and where the functions F and c satisfy:

Assumption S: $F_1 > 0$, $F_{11} < 0$, *c is increasing and convex and* $c' F_{111} - c''' F_1 \leq 0$.

Among standard requirements on the production and cost functions, Assumption S requires that c' is convex enough relative to F (which can be concave as shown in the Cobb-Douglas example later in the paper). Let e_1^*, e_2^* denote the first-best effort levels of the two partners and $\hat{e}_1$, $\hat{e}_2$ their effort levels under double-sided moral hazard.

Proposition 3

(a) In the first best (contractible efforts): $\dfrac{\partial^2 \Pi(e_1^*, e_2^*, \theta_1, \theta_2)}{\partial\theta_1 \partial\theta_2} = 0$, *i.e. joint surplus is both sub- and super-modular in* θ_1, θ_2.

(b) Under double-sided moral hazard and endogenous matching $\dfrac{\partial^2 \Pi(\hat{e}_1, \hat{e}_2, \theta_1, \theta_2)}{\partial\theta_1 \partial\theta_2} < 0$, *i.e., joint surplus is sub-modular in* θ_1, θ_2.

Proof: (see Appendix)

Proposition 3 is basically a generalization of the case $\alpha = 0$ from our simple model in Section 2. The first-best joint surplus in a partnership is weakly super-modular while the joint surplus under double-sided moral hazard is sub-modular. The corresponding discussion in Section 3 applies.

The following corollary extends Proposition 3 to allow for direct complementarity between agents' skills (but not between their efforts). The proof follows directly from the proof of Proposition 3 and is hence omitted.

Corollary 1 to Proposition 3

Suppose the joint payoff function is given by

$$\Pi(e_1, e_2, \theta_1, \theta_2) = f(\theta_1, \theta_2) + F(e_1, \theta_1) + F(e_2, \theta_2) - c(e_1) - c(e_2)$$

where $\frac{\partial^2 f(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} > 0$. *Then:*

(a) In the first best $\frac{\partial^2 \Pi(e_1^*, e_2^*, \theta_1, \theta_2)}{\partial\theta_1\partial\theta_2} > 0$, *i.e. joint surplus is super-modular in the skill levels* θ_1, θ_2.

(b) Under double-sided moral hazard and endogenous matching the sign of $\frac{\partial^2 \Pi(\hat{e}_1, \hat{e}_2, \theta_1, \theta_2)}{\partial\theta_1\partial\theta_2}$ *depends on the magnitude of* $\frac{\partial^2 f(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2}$, *and if the latter is small enough joint surplus is sub-modular in* θ_1, θ_2.

The Corollary results are a generalization of the case $\alpha > 0$ in Section 3.1 and Proposition 2. Again, the corresponding discussion from Section 3 applies.

4.2 Complements

Next, we analyze the case in which the agents' skills and efforts are complements. Specifically, let the joint payoff function be

$$\Pi(e_1, e_2, \theta_1, \theta_2) = F(\theta_1, \theta_2)G(e_1, e_2) - c(e_1) - c(e_2)$$

and suppose the following assumption holds:

> **Assumption C:** *F is symmetric and increasing in both arguments, G is symmetric and increasing in both arguments, $G_{11}, G_{22} \leq 0$, $G_{12} \geq 0$ and $G_{11}G_{22} - G_{12}^2 \geq 0$*

Note that this case assumes complementarity between agents' skills, between agents' efforts, and between skills and efforts. Also note that the conditions on the partial derivatives of G are satisfied for any Cobb-Douglas function with exponents less than one.

Proposition 4

(a) In the first best (contractible efforts): $\dfrac{\partial^2 \Pi(e_1^*, e_2^*, \theta_1, \theta_2)}{\partial\theta_1 \partial\theta_2} > 0$, *i.e. joint surplus is super-modular in θ_1, θ_2.*

(b) Under double-sided moral hazard and endogenous matching: $\dfrac{\partial^2 \Pi(\hat{e}_1, \hat{e}_2, \theta_1, \theta_2)}{\partial\theta_1 \partial\theta_2} > 0$, *i.e. joint surplus is also super-modular in θ_1, θ_2.*

Proof: (see Appendix)

Proposition 4 extends our previous analysis to allow for complementarity in agents' efforts. The result is that homogeneous partnerships result independently of whether effort is contractible or not. Intuitively, the complementarities in both efforts and skills work in the same direction to make high-skill agents optimally match with other high-skilled agents, as in Kremer's (1993) model.

4.3 Special Cases of Production Functions

We now look at some specific, widely-used functional forms as we continue to investigate the modularity of joint surplus with respect to the agents' skill levels and hence the optimal matching pattern that will be observed in partnerships. Depending on whether skills are sub- or super-modular in the indirect joint surplus function $\Pi^{MH}(\theta_1, \theta_2)$, the respective discussion and conclusions from Section 3 apply.

4.3.1 Perfect Complements

Suppose output is given by a Leontief / perfect-complements production function in the ability-augmented efforts of the two partners:[8]

$$q(e_1, e_2) = \min\{\theta_1 e_1, \theta_2 e_2\} + \varepsilon.$$

We first show that at the optimum it must be the case that $\theta_1 e_1 = \theta_2 e_2$. Suppose not, e.g., for example, $0 < \theta_1 e_1 < \theta_2 e_2$. Then the expected joint surplus maximization objective would be

[8] Note that we analyzed the corresponding 'perfect substitutes' in $\theta_i e_i$ case in Section 2.

$\theta_1 e_1 - \frac{e_1^2}{2} - \frac{e_2^2}{2}$ and it is clearly optimal to set $e_2 = 0$, that is $\theta_2 e_2$ would be smaller than $\theta_1 e_1$ – contradiction. Therefore, we must have,

$$\theta_1 e_1^* = \theta_2 e_2^* \tag{9}$$

From the FOCs we obtain:

$$e_1^* = s\theta_1 \text{ and } e_2^* = (1-s)\theta_2 \tag{10}$$

Equations (9) and (10) imply that the optimal share is equal to:

$$s^* = \frac{\theta_2^2}{\theta_1^2 + \theta_2^2} \tag{11}$$

Note that the above expression for s^* is the 'opposite' of that in the perfect substitutes case derived in Section 2.2. Given the expressions for s^*, e_1^* and e_2^*, joint surplus at the optimum equals:

$$\Pi^{MH} = \frac{1}{2} \frac{\theta_1^2 \theta_2^2}{\theta_1^2 + \theta_2^2}$$

It is clear upon inspection that Π^{MH} is super-modular in θ_1 and θ_2. Moreover, it is easy to verify that the 'surplus function' (Legros and Newman, 2002) is identically equal to zero for any $\theta_1, \theta_2 \geq 1$ and thus *only homogeneous partnerships will be observed* in equilibrium under double-sided moral hazard and endogenous matching.

4.3.2 Cobb-Douglas Substitutes

Suppose joint payoffs are given by

$$\Pi = \theta_1 \frac{e_1^\alpha}{\alpha} + \theta_2 \frac{e_2^\alpha}{\alpha} - \frac{e_1^\beta}{\beta} - \frac{e_2^\beta}{\beta} + \varepsilon$$

where $\alpha \in (0,1)$ and $\beta > 1$. This surplus function belongs to the general class studied in Section 4.1 but here we do not necessarily impose the sufficient conditions in Assumption S. From the general analysis above it is clear what happens in the first best, so let us focus on the second best. The FOCs are:

$$\begin{aligned} s\theta_1 \alpha e_1^{\alpha-1} &= e_1^{\beta-1} \\ (1-s)\theta_2 \alpha e_2^{\alpha-1} &= e_2^{\beta-1} \end{aligned}$$

i.e.,

$$\hat{e}_1 = (s\theta_1)^\gamma \text{ and } \hat{e}_2 = ((1-s)\theta_2)^\gamma$$

where $\gamma \equiv \frac{1}{\beta - \alpha}$. Plugging in and differentiating we obtain:

$$\frac{\partial \hat{\Pi}}{\partial s} = \gamma \theta_1^{\alpha\gamma} s^{\alpha\gamma-1}[1-s] - \gamma \theta_2^{\alpha\gamma}(1-s)^{\alpha\gamma-1} s$$

Setting the above derivative to zero we have:

$$\frac{s^*}{1-s^*} = (\frac{\theta_2}{\theta_1})^{\frac{\alpha\gamma}{\alpha\gamma-2}}$$

Clearly then, $\frac{\partial s^*}{\partial \theta_1}$ and $\frac{\partial s^*}{\partial \theta_2}$ must have opposite signs. Assume that $\alpha < \frac{\beta}{2}$ (for example, $\beta > 2$ is sufficient) which implies

$$\frac{\alpha}{\beta - \alpha} = \alpha\gamma < 1 \tag{12}$$

Now look at the function $G(a, b)$ from the Proof of Proposition 3 in Section 4.1. In the Cobb-Douglas case we have:

$$G_{11}(a, b) = \gamma b^{\alpha\gamma}[(\alpha\gamma - 1)a^{\alpha\gamma-2}(1 - a) - a^{\alpha\gamma-1}] < 0$$

given (12). This inequality, together with the opposite signs of $\frac{\partial s^*}{\partial \theta_1}$ and $\frac{\partial s^*}{\partial \theta_2}$ is enough to guarantee the negativity of the cross partial of Π^{MH} as shown in Proposition 3. Thus, if $\beta > 2\alpha$ the optimal sorting pattern is negative and heterogeneous partnerships are optimal.

To clarify the restrictions under which Proposition 3 holds, we analyze what Assumption S implies for the parameters α and β in the Cobb-Douglas case. Using the notation from Section 4.1, we have $c' = \hat{e}^{\beta-1}$, $F''' = (\alpha - 1)(\alpha - 2)\hat{e}^{\alpha-3}$, $c''' = (\beta - 1)(\beta - 2)\hat{e}^{\beta-3}$, $F' = \hat{e}^{\alpha-1}$. Thus, the inequality involving the derivatives of c and F in Assumption S is equivalent to:

$$3(\beta - \alpha) \leq (\beta - \alpha)(\beta + \alpha)$$

Since $\beta > \alpha$, the above inequality holds whenever $\alpha + \beta > 3$, i.e., when c is convex enough (relative to the concavity of F). Note that this is stronger than the condition $\beta > 2\alpha$ in (12) but not excessively restrictive.

4.3.3 CES Production Function

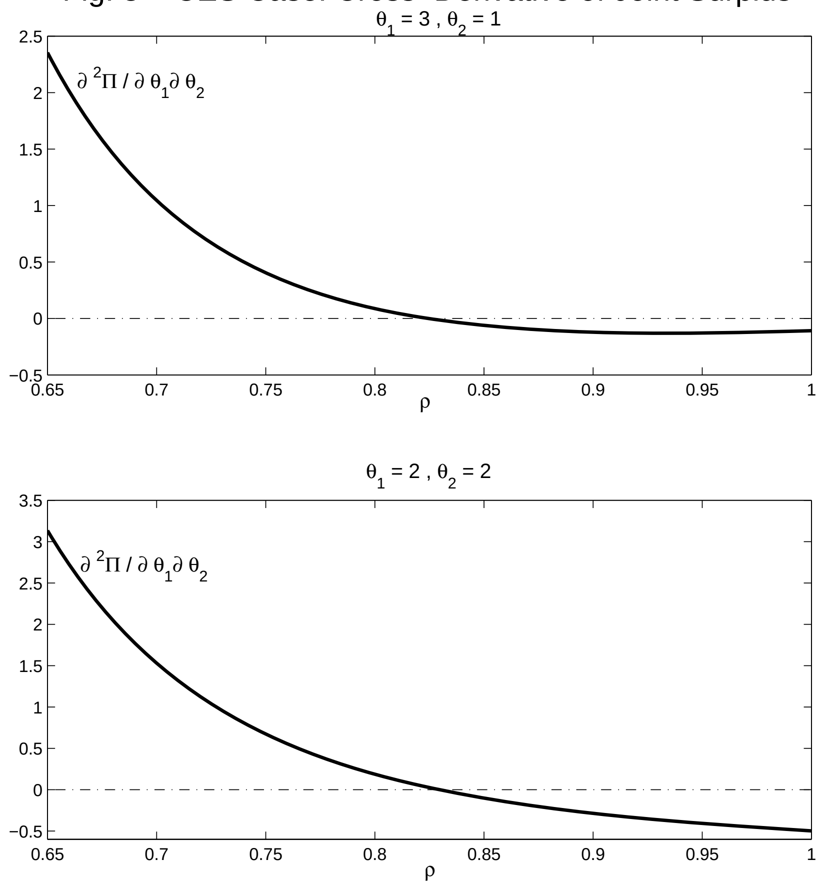
In our analysis above we saw that, depending on whether the efforts and/or abilities of the two agents are substitutes vs. complements, joint surplus can be sub- or super-modular in the agents' abilities, θ_1 and θ_2. A natural way to combine and extend the results from the perfect substitutes and perfect complements is to look at a production function of the CES type:

$$q(e_1, e_2) = (\theta_1 e_1^{\rho} + \theta_2 e_2^{\rho})^{\frac{1}{\rho}}$$

When $\rho \to 1$ expected output becomes $\theta_1 e_1 + \theta_2 e_2$, i.e., perfect substitutes in ability-augmented efforts (our benchmark case from Section 2) while when $\rho \to -\infty$ expected output equals $\min\{\theta_1 e_1, \theta_2 e_2\}$ – the perfect complements case from Section 4.3.1.

By a standard continuity argument our previous results imply that for high values of ρ, the CES function leads to super-modularity of joint surplus under moral hazard, Π^{MH}, while for lower ρ joint surplus will be sub-modular. It is hard to analyze analytically the intermediate case $\rho \neq 1, -\infty$, thus we compute numerically the cross-partial derivative of joint surplus $\frac{\partial^2 \Pi^{MH}(\theta_1,\theta_2)}{\partial \theta_1 \partial \theta_2}$ as a function of ρ. Fig. 3 demonstrates the existence of a cutoff level for ρ so that the cross-partial is strictly positive for ρ lower than the threshold and strictly negative for ρ higher than the threshold. This result is shown for both equal or different ability values for the two agents. The main implication is that for sufficiently high degree of complementarity between the inputs in production positive assortative matching is always optimal, whereas for production functions characterized by relatively low input complementarity negative assortative matching provides higher joint surplus under double-sided moral hazard and endogenous matching.

Fig. 3 − CES Case. Cross−Derivative of Joint Surplus



5 Conclusions

We analyzed partnerships between risk-neutral parties that are subject to double-sided moral hazard. We contribute to the emerging literature in theory of contracts and organizations that looks at endogenous matching of the contracting parties by looking at the case of partnerships. We allow agents to be able to freely to choose any partner. We present two main results. First, the presence of moral hazard can reverse the optimal matching pattern in organizations relative to the first-best. Second, even if partnerships are optimal for an exogenously given pair of types we show that if matching is endogenous they may not be observed in equilibrium and so any measure of agency costs based on partnerships that exist are likely to underestimate the true extent of agency costs due to double-sided moral hazard, because of partnerships that will not form relative to the first-best.

6 Appendix

Proof of Proposition 3

(a) The first-best efforts solve:

$$\begin{aligned} F_1(e_1,\theta_1) &= c'(e_1) \\ F_1(e_2,\theta_2) &= c'(e_2) \end{aligned}$$

Let the solutions to the above system be denoted by $e_1^* = e^*(\theta_1)$ and $e_2^* = e^*(\theta_2)$. Plugging back into the expression for Π we have:

$$\frac{\partial^2 \Pi^*}{\partial\theta_1\partial\theta_2} = 0 \tag{13}$$

due to the assumed separability of the function Π.

(b) Now assume that agents' efforts are unobservable, i.e., there is a double-sided moral hazard problem. The FOCs are:

$$sF_1(e_1,\theta_1) = c'(e_1) \tag{14}$$
$$(1-s)F_1(e_2,\theta_2) = c'(e_2) \tag{15}$$

Denote the solutions to the above equations by $\hat{e}_i = \hat{e}(a,b)$ where a is s or $1-s$ and b is θ_1 or θ_2 respectively for $i = 1,2$. We start by deriving some useful properties of $\hat{e}$. Differentiating (14) and (15) with respect to a we have:

$$F_1 + aF_{11}\frac{\partial\hat{e}}{\partial a} = c''\frac{\partial\hat{e}}{\partial a} \tag{16}$$

i.e.,

$$\frac{\partial\hat{e}}{\partial a} = \frac{F_1}{c'' - aF_{11}} > 0$$

by Assumption S. Differentiating (16) with respect to a we obtain:

$$F_{11}\frac{\partial\hat{e}}{\partial a} + F_{11}\frac{\partial\hat{e}}{\partial a} + a[F_{111}(\frac{\partial\hat{e}}{\partial a})^2 + F_{11}\frac{\partial^2\hat{e}}{\partial a^2}] = c'''(\frac{\partial\hat{e}}{\partial a})^2 + c''\frac{\partial^2\hat{e}}{\partial a^2}$$

i.e.,

$$\frac{\partial^2\hat{e}}{\partial a^2} = \frac{[2F_{11} + (aF_{111} - c''')\frac{\partial\hat{e}}{\partial a}]\frac{\partial\hat{e}}{\partial a}}{c'' - aF_{11}} < 0$$

by Assumption S and by the fact that $c' = aF'$ from the FOCs.

Denote by $\hat{\Pi}(s)$ the joint surplus under double-sided moral hazard, i.e.,

$$\begin{aligned} \hat{\Pi}(s) &= F(\hat{e}(s,\theta_1),\theta_1) + F(\hat{e}(1-s,\theta_2)) - c(\hat{e}(s,\theta_1)) - c(\hat{e}(1-s,\theta_2)) = \\ &\equiv G^1(s,\theta_1) + G^2(1-s,\theta_2) \end{aligned}$$

where $G(a,b) \equiv F(\hat{e}(a,b),b) - c(\hat{e}(a,b))$. We proceed to derive some properties of the function G. Note first that,

$$G_1 = [F_1 - c']\frac{\partial\hat{e}}{\partial a} > 0 \tag{17}$$

using the FOCs and the properties of $\hat{e}$ derived above. Similarly, differentiating once more in a we have,

$$G_{11} = F_{11}(\frac{\partial\hat{e}}{\partial a})^2 + [F_1 - c']\frac{\partial^2\hat{e}}{\partial a^2} - c''(\frac{\partial\hat{e}}{\partial a})^2 < 0. \tag{18}$$

Let $s^*(\theta_1,\theta_2)$ be the share which maximizes $\hat{\Pi}(s)$. It must solve:

$$G_1(s^*,\theta_1) - G_1(1-s^*,\theta_2) = 0 \tag{19}$$

Differentiating with respect to θ_1 we get:

$$G_{11}(s^*,\theta_1)\frac{\partial s^*}{\partial \theta_1} + G_{12}(s^*,\theta_2) + G_{11}(1-s^*,\theta_2)\frac{\partial s^*}{\partial \theta_1} = 0 \quad (20)$$

By the envelope theorem,

$$\frac{\partial \hat{\Pi}(s^*)}{\partial \theta_1} = G_2(s^*,\theta_1)$$

and therefore

$$\frac{\partial^2 \hat{\Pi}(s^*)}{\partial \theta_1 \partial \theta_2} = G_{21}(s^*,\theta_1)\frac{\partial s^*}{\partial \theta_2} = -[G_{11}(s^*,\theta_1) + G_{11}(1-s^*,\theta_2)]\frac{\partial s^*}{\partial \theta_1}\frac{\partial s^*}{\partial \theta_2} < 0$$

where we have used the expression for $G_{21}(s^*,\theta_1)$ from (20). The sign of the above cross-partial derivative is negative by (18) and by the fact that $\frac{\partial s^*}{\partial \theta_1}$ and $\frac{\partial s^*}{\partial \theta_2}$ must have different signs at an interior optimum.■

Proof of Proposition 4

(a) The first-best efforts solve:

$$\begin{aligned} FG_1(e_1,e_2) &= c'(e_1) \\ FG_2(e_1,e_2) &= c'(e_2) \end{aligned}$$

where $F \equiv F(\theta_1,\theta_2)$ to save on notation. Denote the optimal effort levels by $e_1^*(F)$ and $e_2^*(F)$. Differentiating the above FOCs with respect to F yields:

$$\begin{aligned} G_1(e_1^*,e_2^*) + FG_{11}\frac{\partial e_1^*}{\partial F} + FG_{12}\frac{\partial e_2^*}{\partial F} &= c''(e_1^*)\frac{\partial e_1^*}{\partial F} \\ G_2(e_1^*,e_2^*) + FG_{21}\frac{\partial e_1^*}{\partial F} + FG_{22}\frac{\partial e_2^*}{\partial F} &= c''(e_2^*)\frac{\partial e_2^*}{\partial F} \end{aligned}$$

This is a linear system in $\frac{\partial e_1^*}{\partial F}$ and $\frac{\partial e_2^*}{\partial F}$. Its solutions can be obtained by Cramer's rule:

$$\frac{\partial e_1^*}{\partial F} = \frac{\begin{vmatrix} G_1 & -FG_{12} \\ G_2 & c''(e_1^*) - FG_{22} \end{vmatrix}}{\begin{vmatrix} c''(e_1^*) - FG_{11} & -FG_{12} \\ -FG_{12} & c''(e_2^*) - FG_{22} \end{vmatrix}}$$

and similarly for $\frac{\partial e_2^*}{\partial F}$, where $|A|$ denotes the determinant of the matrix A. Notice that both the numerator and denominator are non-negative by our assumptions. Thus $\frac{\partial e_1^*}{\partial F}$ and $\frac{\partial e_2^*}{\partial F}$ are non-negative. Plugging back into Π and using the envelope theorem we have:

$$\frac{\partial \Pi^*}{\partial \theta_1} = F_1 G$$

and thus

$$\frac{\partial^2 \Pi^*}{\partial \theta_1 \partial \theta_2} = F_{12}G + F_1[G_1\frac{\partial e_1^*}{\partial F}F_2 + G_2\frac{\partial e_2^*}{\partial F}F_2] > 0$$

(b) Now suppose agents' efforts are unobservable. The FOCs become:

$$sFG_1(e_1, e_2) = c'(e_1) \tag{21}$$
$$(1-s)FG_2(e_1, e_2) = c'(e_2) \tag{22}$$

Denote the solutions to the above system by $\hat{e}_1 = \hat{e}(s, F)$ and $\hat{e}_2 = \hat{e}(1-s, F)$. Proceeding as in part (a), we have,

$$\frac{\partial \hat{e}_1}{\partial F} = \frac{\begin{vmatrix} sG_1 & -sFG_{12} \\ (1-s)G_2 & c''(\hat{e}_1) - (1-s)FG_{22} \end{vmatrix}}{\begin{vmatrix} c''(\hat{e}_1) - sFG_{11} & -sFG_{12} \\ -(1-s)FG_{12} & c''(\hat{e}_2) - (1-s)FG_{22} \end{vmatrix}}$$

which is once again positive by our assumptions on G. The same holds for $\frac{\partial \hat{e}_2}{\partial F}$.

It is crucial to note that, because of the assumed symmetry, the optimal share $\hat{s}$ cannot depend on F. Suppose this is not true, e.g., $\hat{s}$ is increasing in F. But then, since the production function is completely symmetric across the two agents, it must be that $1-s$ is also increasing in F which is a contradiction. This implies that F affects the optimal effort choices only directly and not through s, i.e., $\frac{\partial s^*}{\partial F} = 0$. Given the latter, we have:

$$\frac{\partial^2 \hat{\Pi}}{\partial \theta_1 \partial \theta_2} = F_{12}G + F_1[G_1 \frac{\partial \hat{e}_1}{\partial F} F_2 + G_2 \frac{\partial \hat{e}_2}{\partial F} F_2] > 0$$

thus joint profits remain super-modular under double-sided moral hazard.■

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