Status, Intertemporal Choice and Risk-Taking

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Abstract

This paper studies endogenous risk-taking by embedding a concern for relative status into an otherwise conventional model of economic growth. We prove that if the intertemporal production function is strictly concave, any Markov perfect equilibrium must converge to a unique steady state in which there is recurrent endogenous risk-taking. (The role played by concavity is clarified by considering a special case in which the the production function is instead convex, in which there is no persistent risk-taking.) The steady state is fully characterized. It displays features that are consistent with the stylized facts that individuals both insure downside risk and gamble over upside risk, and it generates similar patterns of risk-taking and avoidance across environments with quite different overall wealth levels. Endogenous risk-taking here is generally Pareto-inefficient. In summary, a concern for relative status implies that persistent and inefficient risk-taking hinder the attainment of full equality.

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1. Introduction

This paper seeks to explain the coexistence of risk-taking and risk-averse behavior when individuals derive utility from status. Inspired by Veblen (1899) and Duesenberry (1949), we embed a concern for relative payoffs into an otherwise conventional model of economic growth.[1] Individuals care about relative as well as absolute consumption. We presume that all fair gambles are available: there is a competitive industry that can supply such gambles at zero profit. We study the intertemporal equilibrium of such a model. The theory we develop provides an account of endogenous risk-taking, even when there is no intrinsic uncertainty.

The main idea is simple. A deterministic equilibrium of our model induces convergence across dynasties, as in the Solow or Ramsey parables. Such convergence generates large gains in *relative* status from small increases in consumption. Hence the urge to take risks becomes irresistibly strong, destroying the presumption that the equilibrium is deterministic to begin with. We describe the dynamic equilibrium with risk-taking. Our steady state equilibrium generates individual insurance against substantial downside risk and gambling over an intermediate range of outcomes. These turn out to explain the stylized facts that motivated a classic contribution by Friedman and Savage (1948), to which we return below.

Our emphasis on dynamics clarifies that the origin of endogenous risk-taking is the convergence of wealth induced by a strictly concave intertemporal production function. We underline this reasoning by briefly studying the case of a *convex* production function. In this case, when utility depends on status alone, we demonstrate the existence of an equilibrium with no risk-taking. That equilibrium comes about precisely because the convexity of the production function prevents convergence.

The endogenous risk-taking that arises in equilibrium is Pareto-inefficient because status involves a consumption externality. If risk-taking creates a gain in status, for example, this gain must be counterbalanced by a loss to other parties. It is not surprising then that the equilibrium gambling here is Pareto-inefficient— indeed this is true despite the possibility that particular forms of gambling may be Pareto-efficient.

Three strands of the literature inform our approach. Friedman and Savage (1948) reconciled the simultaneous demand for insurance and lotteries by arguing that the former alleviates downside risk and the latter exploits upside risk. They studied a von Neumann-Morgenstern utility function that is first concave, then convex, and finally again concave. This model has been criticized, both for its *ad hoc* specification of the function as well as its dependence on absolute wealth alone. The latter generates marked shifts in behavior with the overall growth of wealth, as individuals display an attenuated appetite for insurance against downside risk.[2] Our main result delivers the Friedman-Savage findings with no assumption at all on the curvature of utility in status. Moreover, the concern with *relative* status creates similar patterns of risk-taking and risk-avoidance across environments with varying wealth levels.

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[1]Frank (1985), Easterlin (2002), Scitovsky (1976), and Sen (1973), among many others, emphasize relative status; for empirical studies., see, e.g., Clark and Oswald (1996), and Dynan and Ravina (2007). There is a small and growing literature dealing with dynamic models that studies how a concern with status influences savings; see, e.g., Corneo and Jeanne (2001), Hopkins and Kornienko (2006), Arrow and Dasgupta (2009) and Xia (2009).

[2]Moreover, as discussed by Friedman (1953), there should be a distinct tendency for all individuals in the convex region and beyond to gamble their way to more extreme final wealth levels.

Second, we build on Robson (1992), who modifies Friedman-Savage utility to depend on both relative as well as absolute wealth.[3] If utility is concave in wealth but convex in status, it is not hard to generate a concave-convex-concave shape for utility as a function of wealth. This construction can be immune to proportional wealth scaling. There is no presumption in favor of Pareto-efficiency — there may be too much gambling, or perhaps too little of it.[4]

The main limitation of such static models is that they require particular assumptions on the curvature of utility in status, as well as on the shape of the wealth distribution, in order to get the requisite concave-convex-concave utility. In the dynamic model we consider, no corresponding assumptions are required. The results are driven by the inevitability of convergence in any deterministic equilibrium, and the resulting need for gambling in order to spread equilibrium status out within any generation.

Our model is also related to a literature that studies a breakdown of convergence induced by "symmetry-breaking": individuals take different actions whenever society-wide distributions are highly equal. One approach in this strand emphasizes the endogenous diversity of occupational choices at identical or near-identical wealth levels, leading to inequality.[5] Another involves the use of income distributions to create endogenous "reference points" with high marginal gains and losses in the departure of wealth from the reference points. Then perfect equality will be destabilized by individuals accumulating capital to different degrees.[6] Risk-taking plays an analogous role here.

Section 2 describes the basic setup. Section 3 studies the special case of a convex production function. Proposition 1 shows that a simple deterministic equilibrium exists, and it is unique in a broad class of strategies. Section 4 studies the central model. Propositions 2 and 3 characterize a unique steady state, in which all dynasties bequeath the same amount, and start each generation with identical wealth. However, endogenous risk-taking induces a nondegenerate distribution of lifetime consumption. Proposition 4 shows that an intertemporal equilibrium must exist, and that any equilibrium path from a positive initial wealth distribution must converge to the unique steady state. Section 5 studies some properties of equilibrium, among them, its inefficiency (Proposition 5). Section 6 sketches two extensions of the central model. First, in line with Veblen, we observe that the equilibrium in this model of consumption-based status can be reinterpreted as a separating equilibrium in which consumption signals unobservable wealth. Secondly, we consider how the introduction of uninsurable productive risk might improve the realism of the model, by creating dispersion in wealth and consumption. Section 7 summarizes and concludes.

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[3]Friedman and Savage actually proposed a rationale of concave-convex-concave utility that involves relative concerns. They sketched a model with two classes where changes in wealth within either class led to decreasing marginal utility, but changes that promoted an individual from the lower class to the upper class led to increasing marginal utility.

[4]In contrast, the Pareto-efficiency of the Friedman-Savage model is the main point in Friedman (1953). In this context, we note that Becker, Murphy and Werning (2005) also consider the incentive to gamble in a static model with rank-dependent status. They extend Robson (1992) in a number of ways, perhaps most significantly to a case in which status is a separate good that can be bought and sold. This assumption restores Pareto-efficiency. On a different note, Hopkins (2010) reexamines the consequences of greater inequality. If there is greater inequality in the exogenous way the status good is distributed, this may lead to more gambling, in contrast to the effect of greater inequality of the initial wealth distribution.

[5]See, e.g., Freeman (1996), Mookherjee and Ray (2003), Matsuyama (2004) and Ray (2006)

[6]Genicot and Ray (2010) develop this idea.

2. The Basic Setup

2.1. **Feasible Set.** There is a continuum of dynasties of measure one. Each dynasty has initial wealth w, distributed according to the cumulative distribution function (cdf) G.

Individuals consume and invest (or bequeath) in every period; each date represents a lifetime. An individual can transform part or all of starting wealth w_t into any gamble with that mean. The realized outcome is then divided between consumption c_t and k_t. Capital produces fresh wealth for the next generation according to the production function

$$w_{t+1} = f(k_t), \tag{1}$$

We assume throughout that f is strictly increasing and continuously differentiable (C^1) in k, with $f(0) \geq 0$. For the special model considered in Section 3, we ask in addition that f be convex. Later, for the more plausible central model of Section 4, we go the other way to require that f be strictly concave, and a little more.

Under our intergenerational interpretation, curvature in f is reasonable. The capital stock is an intergenerational bequest. As in Becker and Tomes (1979) and Loury (1981), the "production function" includes human capital accumulation, and — with imperfect capital markets for education — it has household-specific curvature rather than being fully linear as in the case of competitive, complete markets.

2.2. **Utility and Status.** Each individual has a utility function $u(c,s)$, which depends on consumption and status. *Status* is the percentage of the population who are strictly worse off, plus a fraction of those who have exactly the same level of consumption. That is, if F_t is the cdf of consumption in society at date t, then, for any $c \geq 0$, status s is given by

$$s = \bar{F}_t(c) = \mu F_t^-(c) + (1-\mu)F_t(c) \tag{2}$$

where μ is some number strictly between 0 and 1 and $F_t^-(c)$ is the left hand limit of F_t at c.[7]

In the special model considered in Section 3, we assume that u is a function of status alone. We return to the general specification in the central model of Section 4.

2.3. **Risk.** We suppose that all fair gambles are freely available.[8] Each individual can subject part or all of that wealth to randomization, dividing the proceeds between consumption and investment. Such randomization might involve participation in the state lottery, in stock markets, or real-estate speculation. Under our interpretation, "consumption" is *lifetime* consumption, so that within-period risk-taking will include occupational choice or entrepreneurial ventures in addition to the assumption of short-term risk.

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[7]Setting $\mu = 1/2$ implies that total status is always 1/2, across all distributions, with or without atoms. But it is not needed in the formal analysis.

[8]With a finite number of individuals, there is an issue of satisfying the overall budget constraint. This issue disappears as the number of individuals tends to infinity, given the fairness of the gambles. Suppose there are N individuals, each with consumption budget $b > 0$. We wish to allow individuals to take the gamble with cdf F, say. Suppose this cdf has maximum consumption C and minimum 0. It is not hard to show that individuals $n = 1, ..., \tilde{m}$, say, can be given independent draws from F, with individuals $n = \tilde{m} + 1, ..., N$ treated as residual claimants, in such a way that $\tilde{m}/N \to 1$, with probability one, as $N \to \infty$.

For the sake of symmetry, we suppose that actuarially fair insurance against exogenous risk is also available. This only becomes relevant once such exogenous risk is allowed. We will remark further on this assumption in Section 3.

2.4. **Dynastic Objective.** Given initial wealth w, and a sequence of consumption distributions F_t, a typical dynasty maximizes the present discounted value of expected payoffs:

$$\sum_{t=0}^{\infty} \delta^t \mathbb{E} u\left(c_t, \bar{F}_t(c_t)\right) \tag{3}$$

where $\delta \in (0, 1)$ is the discount factor, and expectations are taken with respect to any endogenous randomization. Of course, the constraint (1) must be respected at every date.

2.5. **Equilibrium.** Each dynasty chooses an optimal policy, which involves choices to maximize lifetime payoff as described in (3). At each date, a fair randomization of wealth is chosen, and the resulting budget is split between consumption and investment. The recursive application of these policies, aggregated over all dynasties, yields a sequence of joint distributions for consumption, investment and wealth.

This sequence induces, in particular, a sequence of distributions $\mathbf{F} \equiv \{F_t\}$ for consumption at each date, and a corresponding sequence of distributions $\mathbf{G} \equiv \{G_t\}$ for wealth.

A configuration of policies is a (Markov perfect) *equilibrium* if each policy maximizes expected utility given the sequence of consumption cdfs $\mathbf{F}$ induced by the aggregation of all of these policies. A *steady state* is an equilibrium in which the cdfs of consumption and wealth are time-stationary: there are distributions F^* and G^* such that $F_t = F^*$ and $G_t = G^*$ for all t.

3. A Special Case With No Endogenous Risk-Taking

Often, it helps to understand a phenomenon by studying a situation in which it fails to occur. In this section, we develop the idea that nondecreasing returns to scale in investment, by preventing convergence, permits a remarkably simple equilibrium to exist. In such an equilibrium, individual savings policies depend only on the discount factor, and economic inequality is indefinitely perpetuated.

Consider the following restrictions:

Assumption 1. *u depends on s alone and is C^1, with $u(0) = 0$ and $u'(s) > 0$ for all $s > 0$.*

Assumption 2. *f is strictly increasing, C^1, convex in k, and $f(0) = 0$.*

Assumption 3. *The initial G satisfies $G(0) = 0$, and $u(G(w))$ is concave in w.*[9]

[9] If $u(G(w))$ is not concave, the arguments in the online Appendix can be applied here to show that there will be an equilibrium in which individuals engage in fair bets in initial wealth, but only in the first period. If G' is the post-gambling wealth distribution, then $u(G'(w))$ is concave, with linear ranges of $u(G'(w))$ associated with nontrivial gambling. The subsequent equilibrium is then as described here.

The pure-status restriction on u and the convexity of f are not imposed for reasons of plausibility. We shall discard them soon, but we use them here to illustrate a point. The convexity of f notwithstanding, we find an equilibrium (see Proposition 1(i)) that generates a concave maximization problem for each individual. There is no demand for risk.

This would be less striking if there were many deterministic equilibria, with varying characteristics. However, Proposition 1(ii) shows that this equilibrium is the only "regular" and "smooth" deterministic equilibrium, for general f. A deterministic equilibrium is *regular* if at each date, an individual has a unique deterministic best response at all but possibly a countable many wealths.[10] It is *smooth* if every individual employs a sequence of differentiable consumption policies $\{c_t^i\}$, with $0 < c_t^{i\prime}(w) < 1$ at all wealths w and dates t.

Proposition **1.** (i) *Make Assumptions* 1, 2, *and* 3. *Then there exists an equilibrium in which almost every dynasty makes a deterministic choice of consumption and investment, and has constant status over time. In this equilibrium, the policy*

$$c_t = (1 - \delta)w_t, \tag{4}$$

is employed by almost every dynasty, which depends on neither the utility function nor the initial distribution of wealth.

(ii) *Suppose that Assumption* 1 *holds, and* $f(0) = 0$. *Consider any deterministic equilibrium described by a family* $\{c_t^i\}$ *of consumption functions, that is regular and smooth. Then* $c_t^i(w) = (1 - \delta)w$ *for all* i*, all* t *and all* w.

The proof of this Proposition, and all other omitted proofs, are in the Appendix.

Several remarks apply to part (i) of this proposition. To begin with, as already mentioned, the equilibrium we obtain induces a *concave* optimization problem for each dynasty, despite the presence of convexity in f to an arbitrarily high degree. The convexity of f ensures that wealth and consumption distributions stay suitably dispersed at every date: the marginal gain from status never becomes high enough to create an incentive to deviate.

In particular, despite the strict convexity of the production function, there is no incentive to gamble. While it is true that the gambling has positive net expected value in the sense of output, the endogenous concavity of payoffs from that output more than outweigh the convexity in technology.[11]

Finally, the equilibrium has an extremely simple structure. Equilibrium policy depends neither on the initial distribution of wealth, nor the exact forms of the utility function and the production function. (The equilibrium *distributions* do, however.) In fact, the equilibrium policy that we exhibit is the one that would be followed by an optimizing planner with logarithmic utility defined on absolute consumption given a linear production technology.

[10]Note that each individual faces the same optimization problem conditional on initial wealth.

[11]It is worth noting, however, that one can design simultaneous gambles on investment and insurance contracts based on the output from such endogenous gambles that do better than no randomization at all, when the production function is strictly convex. Such an outcome requires the individual to commit to investing the realizations from endogenous gambling, and we disallow such insurance against endogenous risk. Indeed, the attendant moral hazard would be extremely high. No one with a high realization of the gamble would want to take up such insurance *ex post*.

What accounts for this structure is the delicate balance achieved across time periods: status matters today, which increases the need for current consumption, but it matters tomorrow as well, which increases the need for consumption tomorrow. In equilibrium — with a convex production function — the two effects nicely cancel in a way that induces a particularly simple equilibrium structure.[12]

Proposition 1(ii) states that our simple equilibrium is the only deterministic equilibrium in a broad class of policies. It is worth emphasizing that this result is independent of the curvature of f. This observation acts as an entry point into the concave case that we shall discuss in the next section. To see how, consider the following minor result:

Remark **1.** *Suppose f satisfies Assumption* 4*, as given in the next section, and that Assumption* 1 *holds. Then there is no regular, smooth deterministic equilibrium.*

The proof of this assertion is simple. If such a policy sequence were to be an equilibrium, then it must be of the linear form described in Proposition 1(ii). So convergence of all wealth and consumption levels would occur. A deviator could then adopt a strategy that led to large gains in status for an arbitrarily large number of periods, yielding the desired contradiction. While this remark leaves open the possibility of rather pathological deterministic equilibria, the obvious resolution is to allow randomization.[13]

4. The Central Model With Endogenous Risk-Taking

We develop our central model in the light of the preceding discussion. If an equilibrium deterministic policy keeps dynasties sufficiently "apart," that policy is a full equilibrium. But if the policy induces convergence, "small" deviations from the policy will generate "large" gains in status. Such convergence must be therefore be prevented by the endogenous creation of risky opportunities.

We describe the model. First, the production function is taken to be strictly concave, and a little more:

Assumption 4. *f is C^1, strictly increasing and strictly concave in k, with $f(0) \geq 0$, $\delta f'(0) > 1$, and $f(k) < k$ for all k large enough.*

Next, utility is taken to be a function both of consumption and status:

[12]This observation may be viewed as a counterpart for rank-dependent status of the result established by Arrow and Dasgupta (2009) where status derives from the average consumption level.

[13]Allowing randomization in consumption restores existence in this case, not surprisingly, since it is a limiting case of the central model for which existence is available. For simplicity, suppose that all individuals have the same initial wealth, w_0, say. Now, if the assumptions of Remark 1 hold, there is an equilibrium that can be derived simply from the standard one-agent problem of maximizing $\sum_{t=0}^{\infty} \delta^t \ln b_t$ subject to $w_0 = k_0 + b_0$ and $f(k_t) = k_{t+1} + b_t$ for $t = 0, ...$ That is, if b_t^* and k_t^* solve this standard problem, then they also are the basis of an equilibrium here. In this equilibrium everyone invests k_t^* and randomizes the residual b_t^* with the cdf of consumption determined by the requirement that $u(F_t(c_t)) = c_t/2b_t^*$, for $c_t \in [0, 2b_t^*]$, for $t = 0, 1, \ldots,$ under the harmless normalization that $u(1) = 1$.

Assumption 5. *$u(c,s)$ is C^1, with $u(0,0) = 0$. It is increasing in s, with $u_s(c,s) > 0$ for all (c,s). It is strictly increasing and strictly concave in c, so that $u_c(c,s) > 0$ and is strictly declining in c. For every s, $u_c(c,s) \to 0$ as $c \to \infty$, and $u_c(c,s) \to \infty$ as $c \to 0$.*

Finally, we assume that wealths are strictly positive and bounded to begin with.

Assumption 6. *The initial distribution G has compact support, with $G(0) = 0$.*

We make two remarks. First, under Assumption 4, an individual will never randomize on k, whether or not continuation values are convex in investment. Any such randomization can be dominated by investing the expected value of the investment, and then taking a fair bet using the produced output. This domination is independent of the curvature of utility or continuation values. Without loss of any generality, then, we can work with the equation:

$$f(k_t) = b_{t+1} + k_{t+1},$$

where b_t is the *consumption budget* of an individual at date t, and k_{t+1} is deterministic.

Second, Assumption 5 does not impose any restriction on the curvature of $u(c,s)$ in s. However, we assume that utility is strictly increasing in consumption, which rules out the pure status model. While our argument for the existence of equilibrium uses this restriction, the main results on steady states and convergence do not; see Ray and Robson (2010).

We first describe the equilibrium outcome in every period, and then embed this solution in reduced form into the fully dynamic model.

4.1. **Within-Period Equilibrium and Reduced Form Utility.** Consider an equilibrium. Suppose that at some date, the distribution of consumption budgets is given by H. With risk-taking, there will be a new distribution of consumption *realizations*, F. To understand the conditions that follow, suppose that F is continuous (so that $\bar{F} = F$); we will prove this later.[14] Because F is obtained from H by fair randomizations (some possibly degenerate),

[R1] F is a mean preserving spread of H.

The "reduced form utility" to any agent will be given by $\mu(c) \equiv u(c, \bar{F}(c))$:

[R2] $\mu(c) = u(c, F(c))$ is concave and continuous.

For if μ were not concave, deviations with gambling in some "nonconcave segment" of μ must exist.

Finally, all individuals who engage in randomization must do so willingly. That requires utility to be convex over the range of any randomization. [R2] calls for concavity throughout. These two restrictions imply linearity over the range of any randomization:

[R3] μ is affine over the range of any randomization used in converting H to F. Specifically, suppose that

$$\int_0^c F(x)dx > \int_0^c H(x)dx \text{ for all } c \in (\underline{c}, \bar{c}),$$

[14] See the end of the proof of Proposition (pr:RFU) in the online appendix.

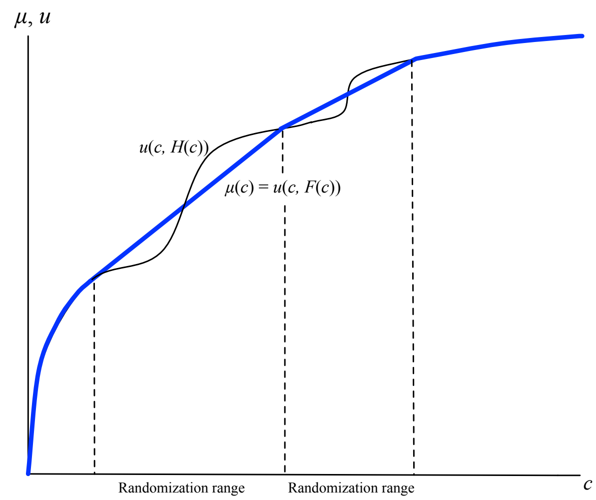

Figure 1. The Reduced Form Utility.

Then $\mu(c) = u(c, F(c))$ must be affine on $(\underline{c}, \bar{c})$. Figure 1 illustrates a μ satisfying [R1]–[R3].

Proposition **2.** *Under Assumption* 5*, if H has compact support, there is a unique F associated with H satisfying* [R1]–[R3].

Under Assumptions 5 *and* 6*, if* $\{H_t, F_t\}$ *is an equilibrium sequence of consumption budgets and realizations, then at each t,* F_t *satisfies* [R1]–[R3] *relative to* H_t.

The proof of this proposition is technical and quite lengthy; see appendix for outline and online appendix for details. The second part formally connects [R1]–[R3] to equilibrium, and it is here that we justify the interpretation $\bar{F}_t = F_t$ for all t.

Because it will matter for steady states, we dwell on the special case in which everyone has a *common* positive consumption budget, so that H is concentrated on a single point:

Corollary **1** (to Proposition 2)**.** *Make Assumption* 5*. For each* $b > 0$*, there is a unique cdf F satisfying* [R1]–R3] *with the following properties:*

(i) *The mean of F equals b:*

$$\int c dF(c) = b. \tag{5}$$

(ii) *The support of F is an interval* $[a, d]$*, and there exists* $\alpha > 0$ *such that*

$$u(c, F(c)) = u(a, 0) + \alpha(c - a) \text{ for all } c \in [a, d]. \tag{6}$$

(iii) $a > 0$*, so* $\mu(c) = u(c, F(c))$ *everywhere, and* $\alpha = u_c(a, 0)$.

Moreover, the slope of the affine segment α is a nonincreasing function of b.

This corollary describes how a common consumption budget must be spread out by gambling. All such gambles are fair, so we have (5). Moreover, [R3] tells us that μ must be linear over the overall domain of the gambles, which yields (6).

4.2. **Steady State.** A *steady state* is an equilibrium outcome in which the same distribution of wealth recurs period after period. We characterize a unique steady state with positive wealth:

Proposition **3.** *Make Assumptions* 4 *and* 5*. Then there is a steady state such that:*

(i) *Every individual (in a set of full measure) makes an identical investment k^*, given by the unique solution to $\delta f'(k^*) = 1$, and has equal starting wealth $w^* = f(k^*)$ at every date.*

(ii) *The distribution of realized consumption is as in Corollary* 1 *with $b = b^* \equiv f(k^*) - k^*$.*

Moreover:

(iii) *There is no other steady state with positive wealth for almost every individual.*

Observe that any steady state can be augmented by the addition of any mass of dynasties with zero initial wealth. Such dynasties will have zero investment and consumption. However, Proposition 3 asserts that zero wealth is the *only* impediment to uniqueness.

4.3. **Existence and Convergence.** Two crucial results are needed to justify our focus on the steady state. That steady state would be rather meaningless if equilibrium convergence to this steady state were not guaranteed from an arbitrary initial distribution of wealth. Second, we need existence of equilibrium. The following proposition resolves both these issues.

Proposition **4.** *Make Assumptions* 4, 5 *and* 6*, and suppose, moreover that the infimum of initial wealth is strictly positive. Then*

(i) Under any equilibrium, the sequence of consumption distributions must converge over time to the steady state distribution identified in Proposition 3*.*

(ii) An equilibrium exists.

While existence is largely of technical interest, it may be useful to provide an outline of the long argument leading to part (i) of Proposition 4; see Appendix for details. Begin with any intertemporal equilibrium. By [R2], the the reduced-form utility functions $\mu_t(c) = u(c, F_t(c))$ are concave at every t. This generates an optimal growth problem with time-varying one-period utilities. By a "turnpike theorem" due to Mitra and Zilcha (1981),[15] starting from any two (positive) initial wealths, the resulting path of capital stocks must converge to *each other* over time. Therefore capital stock sequences bunch up very closely. When they do, the preservation of concavity in μ_t requires that consumption be suitably spread out using endogenous risk-taking. Further, the supports of all the gambles involved must overlap. Hence all consumption budgets ultimately fall into a range over which utility is linear (see [R3]). Thus, the marginal utility of consumption of all agents is fully equalized after some date, so not only are capital stocks are close together, they *coincide* after some finite date. The

[15]In the formal proof, we use an extension of the Mitra-Zilcha theorem due to Mitra (2009).

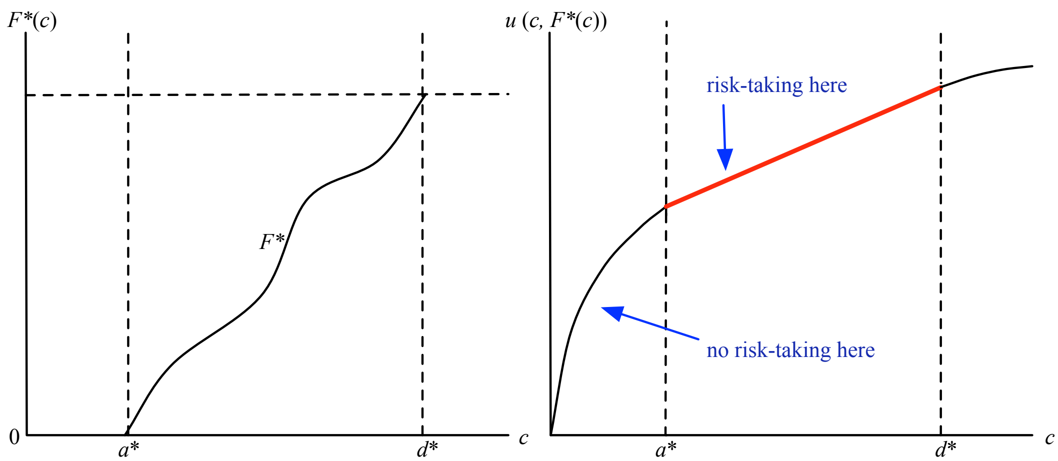

FIGURE 2. THE FRIEDMAN-SAVAGE PROPERTY

remainder of the argument consists in showing that this (common) capital stock sequence must converge.

5. SOME IMPLICATIONS OF THE CENTRAL MODEL

5.1. **Risk-Averse and Risk-Preferring Choices.** Figure 2 provides a diagrammatic representation of the steady state. The first panel depicts the steady state cdf F^*. This panel is deliberately drawn to suggest that F^* has no particular shape, only that it "cancels" all curvature in u to create the affine segment (between a^* and d^*) in the second panel. In addition, a zone $[0, a^*]$ must be present, over which no bets are taken and the utility function is strictly concave.[16]

The two regions taken together generate the phenomena that Friedman and Savage (1948) sought to explain by their postulate of an (exogenous) utility function which is alternately concave and convex. In the steady state, there is aversion to downside risk; no individual would ever take bets that would lead them into the consumption region $[0, a^*]$. Yet there *must* be risk-taking in the region $[a^*, d^*]$, as emphasized throughout the paper.

In the stark specification we study, the zone $[0, a^*]$ is actually not inhabited in steady state. This outcome is an artificial consequence of our assumption that there is no exogenous risk. But this is easy to incorporate; see the discussion in Section 6.1 below. If this exogenous risk would have realizations in the zone $[0, a^*]$, insurance has the effect of avoiding such outcomes. The central model then generates both a demand for insurance and a demand for gambling. (If some of the exogenous risk is uninsurable, both the zones will generally be actively inhabited in steady state.)

It is of interest is that this phenomenon — risk-aversion at the lowest end of the distribution coupled with risk-taking elsewhere — arises "naturally" in an environment where utility

[16]This follows because u has unbounded steepness in consumption at the origin.

depends on status. There is no need to depend on an *ad hoc* description of preferences with varying curvature for an explanation.[17]

5.2. **Scale-Neutrality.** The use of relative consumption guarantees that the model is, in a certain sense, scale-neutral. Two insulated societies with, say, two different production technologies, will generally settle into two different steady states. *Both* the steady states will generally exhibit the requisite patterns of risk-taking and risk-avoidance, even though they may be located at different ranges in the wealth and consumption distribution. Unless the Friedman-Savage utility function magically moves around to accommodate different wealths in exactly the right way, it is not possible in their approach to generate the same phenomenon at diverse aggregate wealth levels.

5.3. **Pareto Inefficiency.** Gambling in the Friedman-Savage world is *ex-ante* efficient: there is an assumed convexity in the utility function, and this convexity is well-served by risk-taking. In our model, however, there is a pervasive consumption externality. I consider the status consequences for me of my choice to gamble, but there must be consequences for others as well, and these I ignore. Equilibrium risk-taking will then generally be Pareto-inefficient.

Consider first the special case in which u is jointly strictly concave in (c, s). Assign a status rank of 1/2 to every individual that lives in a society of perfect equality. It then follows that the steady state identified in Proposition 3 must be Pareto-inefficient. To prove this, simply ban all gambling at the steady state. All individuals continue to invest k^*, and utility in each period is $u(b^*, 1/2)$. In contrast, in the steady state with gambling, utility is given by

$$\int u(c, F^*(c))dF^*(c) < u\left(\int cdF^*(c), \int F^*(c)dF^*(c)\right) = u(b^*, 1/2),$$

where we use the strict concavity of u, and Jensen's inequality.

But inefficiency is more pervasive, and it does not require the concavity of u in status. In such cases, *some* gambling may well be Pareto efficient, in line with the static model of Robson (1992) and one of the models of Becker, Murphy and Werning (2005). Nevertheless, such efficient gambling cannot be an equilibrium outcome in the steady state derived here:

Proposition **5.** *Under Assumptions* 4 *and* 5*, the steady state identified in Proposition* 3 *must be Pareto-inefficient.*

5.4. **Consumption Distribution in Steady State.** Our model predicts the equilibrium distribution of lifetime consumption. If u is jointly strictly concave in (c, s), then Jensen's inequality implies that $u(b^*, F^*(b^*)) < u(b^*, 1/2)$, so that $b^* < (F^*)^{-1}(1/2)$. (This conclusion holds whether or not $\mu = 1/2$.) That is, the mean of the distribution is less than the median, which conflicts with the stylized fact that the mean exceeds the median. Utility functions which are concave in consumption, but convex in status, on the other hand, are capable of generating the realistic prediction that the mean exceed the median.[18] Concavity in consumption and

[17]One might object that (unlike Friedman and Savage) our individuals do not *strictly* prefer to bear risk. In the aggregate, however, risk-taking arises as a robust phenomenon. From a revealed preference perspective, we have accounted for the same observations as did Friedman and Savage.

[18]It is not hard to produce an example of such a utility function where the mean exceeds the median.

convexity in status was the central formulation of Robson (1992), although the motivation there was independent of the argument here.

6. Two Extensions

6.1. **Exogenous Risk.** Suppose that there are production or ability shocks, so that can write the production function as $f(k, \theta)$ where $k \geq 0$ is the bequest as before, and $\theta \in [0, 1]$ is the realization of a random variable. We can replace Assumption 4 by

Assumption 7. *f is increasing in (k, θ), C^1 and strictly concave in k, with $f(0, \theta) \geq 0$ for all $\theta \in [0, 1]$. Moreover, for all $\theta \in [0, 1]$, $\delta f_k(0, \theta) > 1$ and there exists $K \geq 0$ such that $f(k, \theta) < k$ for all $k > K$.*

Suppose that this risk can be *fully* insured in actuarially fair fashion. Then, without any loss of generality, the production function can then be taken as $\mathbb{E}_\theta f(k, \theta)$ which is deterministic and satisfies Assumption 4. So insurable risk makes no difference at all to the analysis.

However, the assumption that all risk is insurable is strong. There may be ability shocks (f includes all income sources, including wage income), or part of k may be in the form of human capital bequests that are subject to moral hazard. In this case, Assumption 7 genuinely replaces 4. An equilibrium can still be shown to exist (see the online Appendix). Now all three regions of the utility function — see Figure 2 — will generally be populated even in the long run. As long as the effect of the random variable θ is small enough, however, there will be a generalized stochastic steady state (or invariant distribution) that is close to the deterministic steady state found here, with endogenous and persistent risk-taking. Individuals who find themselves in a concave region of utility above or below the region where it is linear would then consume and invest in a way that tends to restore them to the linear region. The stochastic steady state would then essentially involve balancing the noise introduced by uninsurable risk and this restorative behavior.

6.2. **Status from Wealth.** Status in the current model derives explicitly from consumption rather than wealth. Veblen (1899) coined the phrase "conspicuous consumption" as a reflection of the capacity of observable consumption goods to signal underlying wealth and thereby generate status. Although the current model cannot do justice to Veblen, the equilibrium here can be reinterpreted as a fully separating equilibrium in which observable consumption signals underlying unobservable wealth.[19]

To see this, reconsider the steady state of our model, in which consumption generates status. At the start of any date, almost all individuals have wealth $w^* = f(k^*)$; but the after-gambling wealth distribution has continuous cdf G^*, say, which is the cdf of after-gambling wealth $k^* + c$, where the cdf of c is F^*. That is, $G^*(k^* + c) = F^*(c)$, and status is $F^*(c)$ for all $c \geq 0$. For

[19]There is yet another formulation in which wealth is *observable*, just as consumption is in our model. This is a variant which is not accommodated. It would be of interest to extend our analysis to this case. It is not obviously more difficult, but it is the task of another paper. (It is fairly obvious that endogenous risk-taking would also arise if there is convergence, but convergence would have to be proven.

individual optimality, an individual who has any after-gambling wealth $w \geq 0$ must solve

$$\max_{c,c'} \Big[u(c, F^*(c)) + \delta u(c', F^*(c')) \Big] \text{ subject to } f(w-c) = c' + k^*, \tag{7}$$

where everyone else behaves in accordance with the equilibrium. That is, the individual would choose to invest k^* over the range of after-gambling wealth levels generated by the consumption gamble F^* and would find it optimal to take the gamble G^* in the first place.

It is easy to interpret this as a separating equilibrium in which observable consumption signals unobservable after-gambling wealth. Suppose everyone else behaves as before, and consider the two-period problem faced by any individual. This is

$$\max_{c} \Big[u(c, G^*(w(c))) + \delta u(c', G^*(w(c'))) \Big] \text{ subject to } f(w-c) = c' + k^*, \tag{8}$$

where $w(x)$ is the after-gambling wealth level inferred from observing consumption level x. However, since $w(x) \equiv k^* + x$ for all $x \geq 0$, it follows that $G^*(w(x)) = F^*(x)$, our individual solves (8) exactly as she solves (7). Moreover, by the one-shot deviation principle, no profitable deviation across multiple dates can exist. It follows that the steady state equilibrium where consumption generates status directly can be reinterpreted as a separating equilibrium where consumption signals after-gambling wealth.[20]

7. Conclusion

In this paper, we embed a concern for relative consumption into an otherwise conventional model of economic growth, and examine its consequences. In our main result, obtained with conventional concavity restrictions on the utility and production functions, there must be persistent, endogenous and inefficient risk-taking in equilibrium.

More generally, there must be persistent consumption inequality. When that inequality is generated "naturally", as it is with a constant- or increasing-returns technology, behavior is simple and deterministic. On the other hand, when inequality tends to diminish, as it does under concavity, it is recreated by endogenously generated recurrent risk-taking.

What might be the real-world manifestations of such risk-taking? We take as wide a view as possible. We do not emphasize state lotteries,[21] as did Friedman and Savage, but also view the choice of career in this light, such as entrepreneurship or the decision to become a professional sportsperson, situations in which the expected rate of return may be quite low.[22] Consider, for example, a restauranteur who invests heavily in a new eatery, despite a

[20]Out of steady state, initial wealth levels may differ across individuals. However, realized observed consumption is still strictly increasing in total after-gambling wealth, by the weak concavity of current utility and the strict concavity of the relevant continuation value, so the argument can be generalized.

[21]The salient feature of state lotteries is that they offer a very small probability of a very large gain, and a probability near one of a small loss. It is difficult to explain why these would be the *only* unfair gambles taken in an expected utility framework. See Chew and Tan (2005) for an explanation using weighted utility.

[22]The risks emphasized here are idiosyncratic, in contrast to the stock market, for example, which has a large aggregate component. What would the current approach predict for the attitude of individuals to pure aggregate risk? In the simplest case, each realization of an aggregate shock would leave each individual's status unaffected. The only effect then would be that on absolute consumption, c. The current assumption that $u(\cdot, s)$ is concave then implies that individuals would decline fair aggregate shocks, in sharp contrast to the gambling that is

half-life of six months for such establishments. Alternatively, consider a low-level member of a drug gang who earns only about the minimum wage, faces the possibility of arrest and imprisonment and of being murdered, and can only on average have only a modest chance of promotion within the gang. Such phenomena admit of alternative explanations — most obviously that the subjective probabilities of success in these two cases are exaggerated (perhaps — as in the case of professional sports — by the media). But the current explanation is attractive in that it does not rely on such misperception.

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robustly generated here for fair idiosyncratic shocks. Individuals are then more risk-averse with respect to aggregate risks, such as the stock market, than they are to idiosyncratic risks, such as entrepreneurial activities.

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Appendix

This appendix outlines the proofs; detailed arguments are in an online appendix. We begin with a central lemma (proof omitted):

Lemma **1.** *At any date with equilibrium cdf of consumption F, $\mu(c) \equiv u(c, \bar{F}(c))$ must be concave.*

Proof of Proposition 1. Part (i). Suppose that all individuals in a set of unit measure use the policy function (4). Let $\mathbf{G} = \{G_t\}$ be the resulting sequence of wealth distributions. Clearly, for every date t and for every w in the support of G_{t+1}, $G_{t+1}(w) = G_t\left(f^{-1}(w)/\delta)\right)$. A recursive argument (omitted) using Assumption 3 now establishes

LEMMA **2.** *$u(G_t(w))$ is concave for all dates t on the support of G_t.*

Fix t. Suppose that an individual employs the policy (4) for all $s \geq t+1$, and that everyone else employs (4) at all dates. Define $V_{t+1}(w')$ to be the discounted value to our individual under these conditions, starting from wealth w' and date $t+1$. Then $V_{t+1}(w') = (1-\delta)^{-1}u\left(G_{t+1}(w')\right)$. Consequently, if $k \in [0, w]$ is chosen from starting wealth w,

$$\begin{aligned} u\left(\bar{F}_t(w-k)\right) + \delta V_{t+1}\left(f(k)\right) &= u\left(\bar{F}_t(w-k)\right) + \delta(1-\delta)^{-1}u\left(G_{t+1}\left(f(k)\right)\right) \\ &= u\left(G_t\left([w-k]/(1-\delta)\right)\right) + \delta(1-\delta)^{-1}u\left(G_t\left(k/\delta\right)\right). \end{aligned}$$

By Lemma 2, this expression is concave in both w and k so no randomization is necessary. Moreover, given the concavity of $u(G_t(w))$ and the fact that u is C^1, G_t must have left-hand and right-hand derivatives everywhere ($G_t^-(w)$ and $G_t^+(w)$ respectively), with

$$G_t^-(w) \geq G_t^+(w) \tag{9}$$

for all w. So a solution to the first-order condition

$$\begin{aligned} & -u'(r_t)G_t^+\left([w-k]/(1-\delta)\right)(1-\delta)^{-1} + \delta(1-\delta)^{-1}u'(r_{t+1})G_t^-\left(k/\delta\right)\delta^{-1} \geq 0 \\ \geq\; & -u'(r_t)G_t^-\left([w-k]/(1-\delta)\right)(1-\delta)^{-1} + \delta(1-\delta)^{-1}u'(r_{t+1})G_t^+\left(k/\delta\right)\delta^{-1} \end{aligned} \tag{10}$$

(where r_s is the resulting status in date s, for $s = t, t+1$) is an optimum. Using (9), we see that $k = \delta w$ is indeed a solution to (10), so that by the one-shot deviation principle and the fact that t and w are arbitrary, (4) is an equilibrium policy.

Part (ii).[23] Notice that each individual is atomless and therefore has the same intertemporal utility criterion as any other. Because the equilibrium is regular, we see that at any date, the solution to the optimization problem is unique except at countably many wealth levels. But it is easy to see that such a solution cannot admit more than one differentiable selection. Therefore all individuals must use the same savings policy, which we denote by $\{c_t\}$. Given this environment, let $V_t(w)$ be the (lifetime) value to a person with wealth w at date t. By using exactly the same steps as in Part (i), we see that for every w, $c = c_t(w)$ must maximize

$$u\left(G_t\left(c_t^{-1}(c)\right)\right) + \delta(1-\delta)^{-1}u\left(G_t\left(s_t^{-1}(w-c)\right)\right), \tag{11}$$

where $s_t(w) \equiv w - c_t(w)$ is also strictly increasing and differentiable, by regularity. By Lemma 1, $u(F_t(c))$ is concave in c, and so F_t is differentiable almost everywhere. Consequently, because $G_t(w)) = F_t(c_t(w))$ and c_t is differentiable and strictly increasing, G_t is also differentiable at a.e. w. Using the fact that optimal c and $w - c$ are both strictly increasing in w, we may therefore differentiate the expression (11) with respect to c at almost every w, set the resulting expression equal to zero (it is the first-order condition) and cancel common terms all evaluated at the same rank or same wealth to obtain

$$\frac{1}{c_t'(w)} = \frac{\delta}{1-\delta}\frac{1}{s_t'(w)} = \frac{\delta}{1-\delta}\frac{1}{1-c_t'(w)},$$

or $c_t'(w) = (1-\delta)$ for every t and for a.e. w. This completes the proof of the proposition. □

[23]We are indebted to a referee for suggesting this line of proof, which is simpler than the one we had.

In what follows, we maintain Assumptions 4–6 throughout.

Proof of Proposition 2. First suppose that H has finitely many mass points. For any "initial point" a such that $H(a) < 1$, and for any "terminal point" $d > a$, let $[aHd]$ be the affine segment that connects $u(a, H(a))$ to $u(d, H(d))$. Associated with $[aHd]$ is a positive slope α, given by

$$\alpha \equiv \frac{u(d, H(d)) - u(a, H(a))}{d - a},$$

Say that $[aHd]$ is *allowable* if $\alpha \geq u_c(a, H(a))$.

LEMMA **3.** *If* $[aHd]$ *is allowable, then the following distribution function* F *is well-defined and strictly increasing:* $F(c) = H(c)$ *for all* $c \notin (a, d)$*, and*

$$u(c, F(c)) = u(a, H(a)) + \alpha(c - a) \tag{12}$$

for all $c \in (a, d)$.

Proof. See the online appendix.

For allowable $[aHd]$ with associated distribution function F as described in Lemma 3, define

$$I_{[aHd]}(x) \equiv \int_a^x [F(z) - H(z)]dz$$

for $x \geq a$. Say that the allowable segment $[aHd]$ is *feasible* if

$$I_{[aHd]}(x) \geq 0 \tag{13}$$

for all $x \in [a, d)$, with equality holding at $x = d$:

$$I_{[aHd]}(d) = 0 \tag{14}$$

Because H has finitely many jumps and is flat otherwise, and because u is concave in c, it is easy to see that from any a, there are at best finitely many feasible segments (there may not be any). Construct a function $d(a)$ in the following way. If, from a, there is no feasible segment with $d > a$, set $d(a) = a$. Otherwise, set $d(a)$ to be the largest value of d among all d's that attain the highest value of α. The proofs of the following four lemmas are omitted.

LEMMA **4.** *Let* $[aHd]$ *and* $[aHd']$ *be two feasible segments. If* $\alpha' > \alpha$*, then* $d' > d$.

LEMMA **5.** *For every* a *with* $H(a) < 1$*,* $\alpha(a)$ *and* $d(a)$ *are well-defined.*

LEMMA **6.** *Suppose that* $a^n \downarrow a$ *with* $d(a^n) > a^n$ *for all* n*. Then* $d(a) > a$.[24]

LEMMA **7.** *Suppose that* $[aHd]$ *with slope* α *is allowable, but* (13) *fails at* $x = d$*. Then the maximum slope* $\alpha(a)$ *from* a *strictly exceeds* α.

Now construct a utility function μ^* on consumption alone. In the sequel this will be the unique reduced-form utility satisfying [R1]–[R3] for the distribution H. The construction is always in one of two phases: "on the curve" or "off the curve", referring informally to whether we are "currently" following the original function $u(x, H(x))$ or are changing it in some way. Start at $a = 0$, follow the original function $u(a, H(a))$ as long as $d(a) = a$ (stay "on

[24] This assertion is false for arbitrary sequences a^n; consider a distribution H with a unique mass point at a. It is clear that $d(a) = 0$, while $d(a') > 0$ for all $a' < a$.

the curve"); at the first point at which $d(a) > a$ — and Lemma 6 guarantees that if any $d(a) > a$ exists, there is a *first* such a — move along the line segment $[aHd(a)]$ (go "off the curve"). Repeat the same process once back again "on the curve" at $d(a)$.[25] The reduced-form function — call it μ^* — will be made up of affine segments in the regions in which $d(a) > a$, and when $d(a) = a$, of stretches that locally coincide with $u(c, H(c))$. It is easy to see that there are at most finitely many affine segments involved in the construction of μ^*.[26]

When H has finite support, this generates a reduced-form utility [RFU] (proof omitted):

Lemma 8. *μ^*, as given by the construction, satisfies* [R1]–[R3].

Now we to prove that a larger class of consumption budget distributions all admit reduced-form utilities satisfying [R1]–[R3]. We begin by proving the uniqueness of such functions.

Lemma 9. *For every distribution of consumption budgets H, there is at most one RFU.*

Proof. Suppose on the contrary that μ and μ' are two distinct RFUs for H, both satisfying the needed properties. Let a^* be the infimum value of c such that $\mu^*(c) \neq \mu(c)$. Without loss of generality, $\mu(a^*) \geq \mu'(a^*)$ and $\mu(c) > \mu'(c)$ for all $c \in (a^*, b)$, for some $b > a$.[27] Let F and F' be the associated distribution of consumption realizations. Then $F(c) = F'(c)$ for all $c < a^*$, $F(a^*) \geq F'(a^*)$, and $F(c) > F'(c)$ for all $c \in (a^*, b)$, so that

$$\int_0^c F(x)dx > \int_0^c F'(x)dx \geq \int_0^c H(x)dx.$$

for all $c \in (a^*, b)$, where the second inequality follows from second-order stochastic dominance (Condition R1 applied to F'). Using [R3] applied to F, we must conclude that μ is affine over (a^*, b). Let $[a^*, d]$ be the maximal interval over which μ is affine. Then

$$\int_0^d \Big[F(t) - H(t)\Big]dt = 0.$$

At the same time, because μ is linear and μ' is concave on $(a^*, d]$ (Condition R2), it must be that $\mu(c) > \mu'(c)$ for all $c \in (a^*, d]$, so that $F(c) \geq F'(c)$ for all $c \in [0, d]$ with strict inequality on $(a^*, d]$. It follows that

$$\int_0^d \Big[F'(t) - H(t)\Big]dt < 0,$$

which contradicts the fact that F' must satisfy [R1]. □

To complete the proof of the proposition, we use an extension argument starting from finite H. Consider the collection $\mathcal{H}$ of all cdfs H on $[0, M]$, where $M < \infty$. We seek the existence of a mapping ϕ that assigns to each $H \in \mathcal{H}$ its unique RFU μ. Let $\mathcal{H}^{\text{fin}}$ be the subspace of $\mathcal{H}$ containing all H with finite support. Then $\mathcal{H}^{\text{fin}}$ is dense in $\mathcal{H}$ in the weak topology. Lemma 8 tells us that the mapping ϕ is already well-defined on $\mathcal{H}^{\text{fin}}$. To extend it, we use the following three lemmas (proofs omitted):

[25] It could be that $d(d(a)) > d(a)$ so that we immediately leave the curve again at $d(a)$.

[26] Indeed, the number of affine segments cannot exceed the number of atoms in H.

[27] That such an interval must exist follows from the concavity of both μ and μ'.

Lemma **10.** *Let G^n converge weakly to G, and (a^n, b^n) to (a, b). Then*

$$\int_{a^n}^{b^n} G^n(x)dx \to \int_a^b G(x)dx \text{ as } n \to \infty.$$

Lemma **11.** *Consider any sequence $H^n \in \mathcal{H}$ converging weakly to $H \in \mathcal{H}$, and suppose that there exist associated RFUs μ^n, along with distributions of realized consumption F^n. If F^n converges weakly to F, then μ given by $\mu(c) \equiv u(c, F(c))$ for all c is the RFU for H.*

Lemma **12.** *Every sequence in $\phi(\mathcal{H}^{\text{fin}})$, the space of all RFUs for distributions in $\mathcal{H}^{\text{fin}}$, admits a weakly convergent subsequence.*

Now proceed as follows. Pick any distribution $H \in \mathcal{H}$. We know that there is a sequence $H^n \in \mathcal{H}^{\text{fin}}$ that converges weakly to H. Each H^n has its (unique) RFU μ^n, with associated distribution of realized consumptions F^n. By Lemma 12, $\{F^n\}$ admits a convergent subsequence that weakly converges to some distribution F. By Lemma 11, this is an RFU for H. By Lemma 9, it is the only one.

To establish the second part of the proposition, we show (see online appendix) that $F_t = \bar{F}_t$ for all t (Assumption 6 is used here). This allows us to interpret $F_t(c)$ as equilibrium status at c, and then deduce [R1]–[R3] (Lemma 1 plays a crucial role). □

Proof of Corollary 1. See the online appendix.

Proof of Proposition 3. Let F^* be any steady state distribution of consumption. Then we know that the RFU $\mu^*(c) = u(c, F^*(c))$ is concave. By Assumption 5 and the fact that $\mu^*(c) \geq u(c, 0)$ for all c, μ^* has unbounded steepness at 0. Consider the problem of choosing $\{b_t(i), k_t(i)\}$ to maximize $\sum_{t=0}^{\infty} \delta^t \mu^*(b_t(i))$, subject to $w_t(i) = b_t(i) + k_t(i)$ and $w_{t+1}(i) = f(k_t(i))$ for all t, with $w_0(i)$ given. Because μ^* is concave and f is strictly concave, there is a unique optimal investment strategy, assigning an investment k and consumption budget b for every starting wealth w.

One can check (see, e.g., Mitra and Ray (1984)) that for each individual, k_t must converge to a steady state. Because μ^* has unbounded steepness at 0, this steady state value k^* is defined by $\delta f'(k^*) = 1$, provided $w_0(i) > 0$. Finally, F^* must be the distribution associated with the degenerate consumption budget $b^* = f(k^*) - k^*$. That verifies that if there is any steady state with positive wealth for all individuals, it must be the one described in the Proposition.

We need to complete the formalities of showing that this outcome is indeed a steady state. All we need to do is exhibit an optimal consumption policy. If the consumption budget b at any date equals $b^* \equiv f(k^*) - k^*$, take a fair bet with cdf F^*, consuming the proceeds entirely.

We already know that the investment policy is optimal. So is the consumption policy, because utilities are linear in realized consumption over the support of F^*. □

Proof of Proposition 4. The steps to follow assume the conditions in the proposition.

Part (i): Convergence. We review the main argument. The first step is Lemma 13, based on a turnpike theorem due to Mitra and Zilcha (1981) and Mitra (2009). It states that in any equilibrium, the paths followed by all agents converge *to one another*. Lemmas 14 and

15 ensure that convergence occurs to some common sequence which has a strictly positive limit point (in time). The second step is Lemma 17, which states that when all stocks cluster sufficiently close to this common limit point, a bout of endogenous risk-taking must force all consumption budgets to lie in the same affine segment of the "reduced-form" utility function μ at that date. Lemma 18 states that all individual capital stocks must fully coincide thereafter. The remainder of the proof shows that this common path must, in turn, converge over time to k^*, with consumption distributions converging to F^*, the unique cdf associated (as in Corollary 1) with $d^* = f(k^*) - k^*$. Lemmas 13–17 are stated without proof.

LEMMA **13.** *In any equilibrium,* $\sup_{i,j} |k_t(i) - k_t(j)| \to 0$ *and* $\sup_{i,j} |b_t(i) - b_t(j)| \to 0$ *as* $t \to \infty$*.*

LEMMA **14.** *In any equilibrium, for any* i *with initial wealth strictly positive,* $b_t(i) > 0$ *for every* t *and* $\limsup_t b_t(i) > 0$*.*

LEMMA **15.** *There exists* $\sigma > 0$ *so that for every* $\epsilon > 0$*, there is a date* T

$$b_T(i) \in [\sigma - \epsilon, \sigma + \epsilon] \tag{15}$$

for all i*.*

LEMMA **16.** *For any* $\sigma > 0$*, there exists* $\psi > 0$ *such that for all* $\epsilon < \sigma/2$*,*

$$F_t(\sigma + \epsilon) - F_t(\sigma - \epsilon) \le \psi\epsilon \tag{16}$$

independently of t*.*

We now combine Lemmas 15 and 16 to prove

LEMMA **17.** *There exists a date* T *such that for every* i*,* $b_T(i)$ *belongs to the interior of the same affine segment of* μ_T*; in particular,* $\mu'_T(b_T(i))$ *is a constant independent of* i*.*

LEMMA **18.** *For every date* $t \ge T + 1$*, where* T *is given by Lemma 17, the wealths, investments and consumption budgets of all agents must fully coincide.*

Proof. By Lemma 17, we see that $\mu'_T(b_T(i)) = \alpha_T$ for all i, where $\alpha_T > 0$ is independent of i. Let $V_{T+1}(w)$ be the value function at date $T + 1$; it is concave because μ_t is concave for all t. Therefore $V_{T+1}(f(k))$ is *strictly* concave.

Note that $b_T(i) > 0$ for all i by Lemma 14. So using the Bellman equation between dates T and $T + 1$, and writing the first order condition, we see that

$$\alpha_T \ge \delta\beta_i(k_T(i)), \text{ with equality if } k_T(i) = 0$$

for every agent i, where $\beta_i(k)$ denotes some supporting hyperplane to $V_{T+1} \circ f$ at k. It follows that the wealths of all agents fully coincide at date $T+1$. It is easy to see that optimal programs are unique starting from any initial wealth at any date,[28] so all wealths, investments, and consumption budgets must fully coincide from date $T + 1$ onward. □

In what follows, we consider only dates $t > T$. By Lemma 18, the equilibrium program has common values at all dates thereafter: (w_t, b_t), where all these values are strictly positive. By Proposition 2 and Corollary 1, the distribution F_t is also fully pinned down at all these

[28]The optimization problem facing each individual is strictly concave.

dates. Denote by α_t the corresponding slopes of the affine segments of μ_t, given by (6); these too are all strictly positive.

Lemma **19.** *Suppose that for some $t \geq T+1$, $k_t \leq k_{t+1}$ and $\alpha_t \leq \alpha_{t+1}$. Then $k_s \leq k_{s+1}$ for all $s \geq t$.*

Proof. Suppose that for some t, we have $k_t \leq k_{t+1}$ and $\alpha_t \leq \alpha_{t+1}$. If $k_{t+1} = 0$, then so is k_t, and then $b_{t+1} = f(0) \geq b_{t+2}$, so that by then very last part of Corollary 1, $\alpha_{t+1} \leq \alpha_{t+2}$. Otherwise, $k_{t+1} > 0$, and using the Euler equations for utility maximization (with appropriate inequality at date t, and with equality at date $t+1$) and combining them with the concavity of f,

$$\frac{\alpha_t}{\alpha_{t+1}} \geq \delta f'(k_t) \geq \delta f'(k_{t+1}) = \frac{\alpha_{t+1}}{\alpha_{t+2}},$$

which permits us to conclude that $\alpha_{t+1} \leq \alpha_{t+2}$ once more. Using again the very last part of Corollary 1, we must also conclude that $b_{t+1} \geq b_{t+2}$. Therefore

$$k_{t+1} = f(k_t) - b_{t+1} \leq f(k_{t+1}) - b_{t+2} = k_{t+2}.$$

We have therefore shown unambiguously that $\alpha_{t+1} \leq \alpha_{t+2}$ and $k_{t+1} \leq k_{t+2}$. We can continue the recursive argument indefinitely to show that $k_s \leq k_{s+1}$ for all $s \geq t$. □

Lemma **20.** *The common sequence of investments $\{k_t\}$, defined for $t \geq T+1$, must converge to k^*, which solves $\delta f'(k^*) = 1$.*

Proof. First we establish convergence. If the sequence $\{k_t\}$ is either eventually nondecreasing or eventually nonincreasing, it must converge. Otherwise, there is some date $t \geq T+1$ with $k_t \geq k_{t+1} < k_{t+2}$. Then

$$b_{t+1} = f(k_t) - k_{t+1} > f(k_{t+1}) - k_{t+2} = b_{t+2},$$

which implies (by Corollary 1) that $\alpha_{t+1} \leq \alpha_{t+2}$. But now all the conditions of Lemma 19 are satisfied, so that in this case $\{k_t\}$ must eventually be nondecreasing. By Assumption 4, k_t is bounded and so must converge. It follows that b_t and therefore α_t also converge. Passing to the limit using the Euler equations, and noting from Assumption 4 that $\delta f'(0) > 1$, we must conclude that $\lim k_t = k^*$. □

Proof of Proposition 4. Part (i): Lemma 17 assures us that there exists a date T at which consumption budgets $b_T(i)$ belong to the same affine segment of μ_T for every i. Lemma 18 states that for every date $t \geq T+1$, the wealths, investments and consumption budgets of all agents must fully coincide. Lemma 20 states that the common sequence of investments $\{k_t\}$, defined for $t \geq T+1$, must converge to k^*, which solves $\delta f'(k^*) = 1$.

At the same time, Corollary 1 asserts that for all $t \geq T+1$, the equilibrium distribution of consumptions must be the unique cdf associated with the common consumption budget b_t, where "association" is defined (and uniqueness established) in Proposition 2. Therefore the sequence of consumption distributions must converge to the unique cdf associated with $b^* = f(k^*) - k^*$. This is the unique steady state of Proposition 3, so the proof is complete.

Part (ii): Existence will follow as a corollary of Proposition 5 in the online appendix. This proves existence for a more general model than the one in the paper, in which the production function can have convex segments and there are possibly stochastic shocks to production. □