# Dynamic Moral Hazard, Learning and Belief Manipulation*

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**Abstract**

We study dynamic moral hazard, with symmetric ex ante uncertainty and learning. Unlike Holmstrom's career concerns model, uncertainty pertains to the difficulty of the job rather than the general talent of the agent, so that contracts are required to provide incentives. Since effort is privately chosen, the agent can always cause a misalignment of beliefs between the principal and himself, by shirking. We show that such a misalignment is always profitable for the agent, and must be dissuaded by providing more high powered incentives. However, high powered incentives in the future only aggravate the incentive problem today, so that the problem is compounded as the interaction becomes longer. We also study the benefits of long term contracts with full commitment, and the role of random effort choice.

Keywords: moral hazard, learning. JEL codes: D83, D86.

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# 1 Introduction

We study a multi-period principal agent model, with moral hazard, and ex ante symmetric uncertainty. Our underlying setting is reminiscent of Holmstrom's career concerns model (1999). The main difference is that uncertainty pertains to the difficulty of the job (or the job-specific ability of the agent), rather than the general ability of the agent, as in Holmstrom. This implies that explicit incentives must be provided, in order to induce the agent to put in effort. Consequently, we allow complete contracting so that the principal can commit to output-contingent wages, at least within each period. In contrast with Holmstrom and most of the following literature, we analyze a rather general information structure.

The main conceptual insight of our analysis arises from the possible difference in beliefs, between the principal and agent, were the agent to deviate and choose low effort.[1] We show that this *potential* private information, even if unrealized, is an important factor that increases the dynamic agency cost associated with inducing high effort, since the principal must provide high powered incentives. However, high powered incentives in the future only aggravate the incentive problem today, i.e. the dynamic agency cost compounds with the length of the relationship. Thus, in long term relationships, the principal may find it optimal not to incentivise effort, at least till uncertainty is reduced. Organizational solutions may take the form of reducing the agent's tenure, e.g. managers in a multi-plant firm may be transferred to other locations or plants, even when this is costly in terms of loss of location specific capital.

To introduce our main intuitions, consider the standard principal agent model with moral hazard and one period, where there is uncertainty regarding job difficulty, where the probability distribution over output signals depends both on effort and upon job difficulty. Specifically, suppose that the agent believes that the job is good (i.e. easy) with probability $\lambda$ and bad with complementary probability. That is, the agent's *first order belief* equals $\lambda$. In the standard model with common priors and no higher order uncertainty, the principal's belief regarding the agent's belief are degenerate. That is, her *second order belief* assigns probability one to the event that the agent's first order belief is $\lambda$. Notice that the principal designs her optimal contract based on her second order belief. In particular, assuming that the principal is risk neutral, the

[1] Note that in Holmstrom and in the career concerns literature, this difference in beliefs plays no substantive role, since this does not affect the agent's continuation strategy.

optimal contract minimizes expected wage payments, given the individual rationality and incentive constraints, where $\lambda$ defines the probability distributions underlying these constraints. Indeed, both these constraints bind at the optimum.

This observation regarding beliefs is the key to our analysis in the dynamic context. Suppose that effort must be chosen from the set $\{0, 1\}$, and that the principal designs a contract to induce high effort in both periods. Now suppose that the agent chooses high effort in period one, and suppose that some signal $y^k$ is realized. The agent's first order belief is now given by Bayesian updating given the signal realization and high effort. Denote the agent's belief by $\mu_1^k$ (the superscript indexing the signal realization, and the subscript effort choice). The principal does not observe effort; however, since the agent chooses high effort with probability one in equilibrium, the principal's second order belief is degenerate and assigns probability one to $\mu_1^k$. Assuming that the principal can only commit for one period, the second period contract minimizes expected wage payments subject the incentive and individual rationality constraints defined by $\mu_1^k$.

Now suppose that the agent deviates in the first period to $e = 0$. Given signal realization $y^k$, the agent updates to a belief $\mu_0^k$. However, the principal's second order beliefs will be incorrect since they assign probability one to the agent having belief $\mu_1^k$. That is, the principal continues to believe that the agent has chosen high effort, and therefore her second order belief is both certain and wrong. In consequence, the contract that she chooses for the second period will be subject to the (incorrect) incentive and individual rationality constraints defined by $\mu_1^k$.

Our focus is on the second period continuation value of the agent when he deviates to low effort in the first period. We shall show that under general conditions, the agent's continuation value *strictly increases* if he deviates to low effort in the first period. The intuition comes from the fact that the individual rationality constraint always binds given belief $\mu_1^k$. This implies that if the agent is more pessimistic about the job (i.e. $\mu_0^k < \mu_1^k$), then the constraint is violated, while if the agent is more optimistic (i.e. $\mu_0^k > \mu_1^k$), then the individual rationality constraint holds strictly, and the agent makes a surplus above his reservation utility. Now, when the IR constraint is violated, the agent will simply refuse the contract and earn his reservation utility, and therefore suffers no loss. Since the agent accepts the payoff gains but can refuse the payoff losses, he will benefit as long as there is some signal $y^k$ such that $\mu_0^k > \mu_1^k$, i.e. where he is more optimistic regarding the job than the principal thinks that he is.

We show that there always exists some signal $y^k$ such that $\mu_0^k > \mu_1^k$. This follows from the martingale property of beliefs. The expectation of the agent's posterior, over all signal realizations, must equal his prior, $\lambda$, regardless of whether the agent performs the experiment $e = 1$ or the experiment $e = 0$. Since good signal have higher probability under $e = 1$ than under $e = 0$, this equality of expectations can only be satisfied if there are some signals such that $\mu_0^k > \mu_1^k$.

Since the agent's second period continuation value is higher when he deviates to low effort in period one, as compared to the case where he does not deviate, this implies that the incentive constraint in the first period must be modified. That is, the principal must provide greater incentives for high effort than she would need to do in a static context, where there was no second period. This argument is quite general, and holds as long as the signal structure satisfies the property that a signal that is informative of high effort is also informative of the job being easy.

The incentive problem only worsens when the period of interaction lengthens, or if the agent is more patient. If we augment the interaction by one more period, from $T$ to $T + 1$, then the incentive scheme in period $T$ must become more high powered. But high powered incentives mean greater informational rents, thereby increasing the deviation gain in previous periods. Thus the solution to the incentive problem in period $T$ only aggravates it in period $T - 1$. This in turn requires more high powered incentives in period $T - 1$, compounding the incentive problem in previous period. Thus, in contrast with the repeated game literature (e.g. Radner, 1985), long term relationships accentuate the incentive problem.

Our analysis, in the main, assumes that long term contracts are not possible. If we allow both parties to make full inter-temporal commitments, then the agency problem is mitigated, since the agent may no longer walk away when he becomes more pessimistic. However, our analysis of full commitment contracts shows that there are information structures where long term contracts do not improve on short term contracts, over and above the possibilities of optimal inter-temporal consumption smoothing that such contracts offer.

Finally, our model also shows that there could be a novel role for random effort in the absence of long term contracts. With random effort, the principal knows that he is less informed in the second period, i.e. there is true asymmetric information. This allows him to commit to leave rents in the second period, thereby relaxing both incentive and participation constraints in the initial period.

This paper is related to several strands of literature. The most immediate connections are the career concerns model, models of dynamic moral hazard and models of dynamic adverse selection. We consider each of these in turn.

Holmstrom's (1999) career concerns model is our point of departure. This early paper set out a model where the non-observability of effort gives rise to possible private information between employers and firms. By assuming a linear technology and normally distributed noise, Holmstrom was able to finesse many of the difficulties that arise due to the privateness of effort choice. In particular, optimal effort only depends upon calendar time, and not upon previous outputs or previously chosen efforts. Thus the agent's optimal continuation strategy does not depend upon his private information.

There is a substantial literature that has developed on Holmstrom's career concerns model, retaining the key assumptions of a linear technology and normally distributed noise. Gibbons and Murphy (1992) allow for linear contracts in the context of Holmstrom's model, and show that explicit incentive become more important over time, as uncertainty is reduced. Meyer and Vickers (1997) study the interaction between implicit incentives arising from career concerns and explicit incentives. Dewatripont et al. (1999a) analyze the implications of alternative information structures and technologies, including a multiplicative technology. Their companion paper (1999b) considers career concerns in public organizations. Prat and Jovanovic (2011) analyze long term contracts with full commitment in a setting similar to Holmstrom's. Their main finding is that as information accumulates, the contracting problem becomes easier. De Marzo and Sannikov (2011) analyze a continuous time contracting problem where the state follows a Brownian motion.

More recently, a series of papers on venture capital consider information structures and technologies that are different from Holmstrom's. Bergemann and Hege (1998, 2005) consider the problem of venture capital financing, where output is binary and depends upon the quality of the project as well as the effort of the agent. Horner and Samuelson (2009) analyze the same question under different economic assumptions (where the project rather than capital is the scarce factor). Manso (2011) considers the problem of motivating innovation, in a context where the outcome variable is binary. Kwon (2011) analyzes a limited liability moral hazard problem, where the probability of success depends upon an unobserved state variable that is partially persistent. In comparison with most of this literature, our work differs in two respects. First, our setting is classical: we allow complete contracting within the period, and our model incorporates the classical trade-off between incentives and optimal risk sharing. Second, our results are obtained under quite general informational assumptions. In particular, our main results allow for an arbitrary signal structure and hinge on an informational assumption that has an economic interpretation (essentially, we assume that effort and the nature of the job have similar effects upon the probability of a signal). Finally, a critical role in our analysis is played by the fact that the agent and the principal have different beliefs after a deviation, giving rise to potential information rents for the agent.

Our work is also related to the work of Lambert (1983) and Rogerson (1985) on dynamic moral hazard without uncertainty or learning. These papers focus on risk sharing in dynamic context in the presence of incentive problems, and some of these issues also arise when we analyze long term contracts with full commitment. Malcomson and Spinnewyn (1988) and Fudenberg et al. (1990) show that the private information may arise in this context if the agent may save, and his consumption is not observed by the principal.

Finally, our work also relates to work on dynamic adverse selection and dynamic mechanism design. Our substantive findings, that long term interaction makes the incentive problem harder, bears some resemblance to the rachet effect (see Freixas et al. (1985) and Laffont and Tirole (1998)). Laffont and Tirole show that the inability to make inter-temporal commitments prevents the principal from inducing full separation of types, when types are persistent and are drawn from a continuum (or are sufficiently close to each other). One key difference is that our effects arise in the absence of any ex ante private information, and also when the set of job types is binary. Also, whereas commitment possibilities eliminate the rachet effect, commitment has a more limited role in our context.

The layout of the rest of this paper is as follows. Section 2 sets out the basic model. Section 3 analyzes the two period model without commitments. Section 4 considers many periods. Section 5 analyzes the case where the principal and the agent can make long term commitments. Section 6 shows that inducing random effort may reduce agency costs in the dynamic context. The final section concludes.

# 2 The model

Our model combines moral hazard with uncertainty regarding job difficulty. Specifically, the job is either good (easy) or bad (hard), i.e. the job type is $\alpha \in \{G, B\}$. Let $\lambda \in (0,1)$ denote the common prior that $\alpha = G$. The agent chooses effort $e \in \{0,1\}$. Let $y \in Y = \{y_1, y_2, .., y_n\}$ denote the signal that is realized following effort choice. This depends, stochastically, on both the type and the effort chosen. Let $p^k_{e\alpha}$ be the probability of signal $y^k$ given effort $e$ and type $\alpha \in \{G, B\}$. Thus for each signal $y^k$, we have a 4-tuple $(p^k_{0B}, p^k_{1B}, p^k_{0G}, p^k_{1G})$. With a slight abuse of notation, we may also define $p^k_{1\mu}$ (resp. $p^k_{0\mu}$) to be the probability of signal $k$ when effort level 1 (resp. 0) is chosen, given that $\mu$ is the probability that the agent is type $G$.

We shall distinguish two types of likelihood ratio, the likelihood ratio on efforts for a given type (or belief over types) and the likelihood ratio over types for a given effort choice. The former is relevant for providing effort incentives, while the latter determines Bayesian learning. Let $\ell^k_\alpha = \frac{p^k_{1\alpha}}{p^k_{0\alpha}}$ be the likelihood ratio for signal $k$ for type $\alpha$. Generalizing this, $\ell^k_\mu = \frac{\mu p^k_{1G} + (1-\mu) p^k_{1B}}{\mu p^k_{0G} + (1-\mu) p^k_{0B}}$ denote the likelihood ratio for signal $k$ when $\mu$ is the probability that the agent is type $G$. Let $\ell^k_e = \frac{p^k_{eG}}{p^k_{eB}}$ be the likelihood ratio for signal $k$ for effort level $e$.

Our main assumption, that is maintained throughout this paper, is as follows:

**A1** All probabilities belong to $(0,1)$. For some $y^k$, $p^k_{1G} \neq p^k_{0B}$ i.e. there exists some informative signal. For any informative signal $y^k$, $p^k_{1B}$ and $p^k_{0G}$ lie in the interior of the interval spanned by $p^k_{1G}$ and $p^k_{0B}$, i.e. $p^k_{1B}, p^k_{0G} \in (\min\{p^k_{1G}, p^k_{0B}\}, \max\{p^k_{1G}, p^k_{0B}\})$.

To provide some intuition for this assumption, let $Y^H$ be the set of high signals, where $p^k_{1G} > p^k_{0B}$. Then this assumption implies that if $y^k \in Y^H$, $\ell^k_\alpha > 1$ for $\alpha \in \{G, B\}$ and $\ell^k_e > 1$ for $e \in \{0,1\}$.That is, if a signal is more likely when a given type of agent chooses high effort, it is also more likely for a given effort level when the job is the good type. This implies that signals that are indicative of high effort are also indicative of the agent being the good type. Similarly, let $Y^L$ be the set of low signals, where $p^k_{1G} < p^k_{0B}$. The assumption implies that if $y^k \in Y^L, \ell^k_\alpha < 1$ for $\alpha \in \{G, B\}$ and $\ell^k_e < 1$ for $e \in \{0,1\}$, so that a low signal indicates low ability as well as low effort. Finally, we may have some uninformative signals when $p^k_{1G} = p^k_{0B}$, where all likelihood ratios are one, but since there is at least one informative signals, both $Y^H$ and $Y^L$ are non-empty. Let $Y^U$ denote the set of uninformative signals, and let $\Pr(Y^U)$ denote the probability that an uninformative signal is realized – this does not depend upon effort choice or ability.

We shall assume that the agent's payoff in any period is given by $u(w) - c(e)$ where $u(.)$ is strictly concave, and unbounded, while $c(.)$ is increasing.

We begin our analysis by focusing on the principal's cost minimization problem. That is, we assume that the principal seeks to induce high effort in every period, and solve for the sequentially optimal dynamic contract that minimizes expected wage costs. Specifically, we study the dynamic game induced by this contracting problem, and solve for perfect Bayesian equilibria that satisfy sequential rationality, with beliefs given by Bayes rule. We do not have to deal with out of equilibrium beliefs, since there are no observable deviations. Since effort choice by the agent is private and public signals have full support, the principal does not see an out of equilibrium action, except when the game ends by the agent refusing the contract (at which point, beliefs are moot).

## 2.1 The static model

Suppose that the principal wants to induce $e = 1$.The principal's optimal contract depends upon second-order beliefs, i.e. his beliefs regarding the agent's beliefs regarding his own type. Let us suppose that the principal assigns probability one to the agent assigning probability $\mu$ to being the good type. Let $w_k$ denote the wage paid in the event that signal $y^k$ is realized. The incentive constraint corresponding to this belief is given by

$$\mu \sum_k (p^k_{1G} - p^k_{0G}) u\left(w_k\right) + (1-\mu) \sum_k (p^k_{1B} - p^k_{0B}) u\left(w_k\right) \geq c(1) - c(0). \qquad (1)$$

The individual rationality constraint given this belief is given by

$$\mu \sum_k p^k_{1G} u\left(w_k\right) + (1-\mu) \sum_k p^k_{1B} u\left(w_k\right) - c(1) \geq \bar{u}. \qquad (2)$$

The optimal contract that induces $e = 1$ minimizes expected wage payments subject to these constraints, and is standard, as in Holmstrom (1979) or Mirrlees (1999). Let $\mathbb{W}(\mu) = (w_k(\mu))^n_{k=1}$ denote the profile of wages corresponding to this optimal contract. The important thing that matters for our purpose is that wages are increasing in $\tilde{\ell}_{\mu}^{k}$, the likelihood ratio corresponding to belief $\mu$. In particular, if we compare two signals $y^l \in Y^L$ and $y^h \in Y^H$, then $w_l(\mu) < w_h(\mu)$ for any belief $\mu$.

Our first results concern the optimal strategy and utility of an agent who is offered contract $\mathbb{W}(\mu)$, but who in fact has belief $\nu$. If the agent accepts the contract, then he will choose effort optimally. Let $\Delta(\nu|\mu)$ denote the payoff difference between choosing $e = 1$ and $e = 0$ given belief $\nu$:

$$\Delta(\nu|\mu) = \nu \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k\right) + (1-\nu) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k\right) - c(1) + c(0). \quad (3)$$

Using the fact that the incentive constraint (1) binds at belief $\mu$, this can be re-written as

$$\Delta(\nu|\mu) = (\nu - \mu) \sum_k \left[(p_{1G}^k - p_{0G}^k) - (p_{1B}^k - p_{0B}^k)\right] u(w_k(\mu)). \quad (4)$$

Thus the agent's optimal effort choice is $e = 1$ if $\Delta(\nu|\mu) \geq 0$ and $e = 0$ if $\Delta(\nu|\mu) < 0$. Since $\Delta(\nu|\mu)$ is linear in $(\mu - \nu)$, there are three possibilities. If the term under the summation sign in (4) is zero, then $\Delta(\nu|\mu) = 0$ for all $\nu$ and the IC holds. Otherwise, either the IC binds strictly for all $\nu > \mu$ and is violated for all $\nu < \mu$ or vice versa.

**Lemma 1** *If $\nu > \mu$, the agent gets utility that is strictly greater than $\bar{u}$; he accepts the contract and chooses high effort if $\Delta(\nu|\mu) \geq 0$ and low effort if $\Delta(\nu|\mu) < 0$. If $\nu < \mu$, the agent rejects the contract since he gets a utility that is strictly lower than $\bar{u}$, regardless of his effort choice.*

The agent's optimal strategy if he accepts the contract, as a function of $\Delta(\nu|\mu)$ , has already been derived. Since the agent's IR constraint binds at belief $\mu$, his utility is the same regardless of whether he chooses $e = 1$ or $e = 0$, i.e. $U(e = 1, \mu) = U(e = 0, \mu)$.

$$U(e = 1, \nu) - U(e = 1, \mu) = (\nu - \mu) \sum_k (p_{1G}^k - p_{1B}^k) u\left(w_k(\mu)\right). \quad (5)$$

Now, $p_{1G}^k - p_{1B}^k > 0$ if $y^k \in Y^H$, and $p_{1G}^k - p_{1B}^k < 0$ if $y^k \in Y^L$. Since wages are uniformly higher for signals in $Y^H$ than for signals in $Y^L$, 5) has the same sign as

$(\nu - \mu)$. Similarly,

$$U(e = 0, \nu) - U(e = 1, \mu) = (\nu - \mu) \sum_k (p_{0G}^k - p_{0B}^k) u(w_k(\mu)). \tag{6}$$

Now, $p_{0G}^k - p_{0B}^k > 0$ if $y^k \in Y^H$, and $p_{0G}^k - p_{0B}^k < 0$ if $y^k \in Y^L$. Thus (6) has the same sign as $(\mu - \nu)$. We conclude that the agent's payoff from accepting the contract is strictly greater than $\bar{u}$ as long as $(\nu - \mu) > 0$, and strictly less if $(\nu - \mu) < 0$.

**Remark 2** *If assumption A1 is weakened, so that for any informative signal $y^k$, $p_{1B}^k$ and $p_{0G}^k$ lie in the closed interval spanned by $p_{1G}^k$ and $p_{0B}^k$, (rather than the open interval), this would imply that* either *(5)* or *(6) has the same sign as* $(\nu - \mu)$. *In this case, the agent would get a surplus when he is more optimistic, as in the proposition, but may get a payoff equal to his reservation utility by choosing effort optimally when* $(\nu - \mu) < 0$.

This result relates to some recent work in contract theory that examines that relaxes the common prior assumption, and allows the principal and the agent to hold different first-order beliefs regarding the relation between effort and output. For example, de la Rosa (2011) examines the implications of the agent being over-confident. In contrast with our analysis, de la Rosa assumes that the principal has correct second order beliefs, i.e. he knows that the agent is over-confident.

# 3 Short-term commitments: two periods

The focus of our paper is on the case where the principal may make commitments within the period, but cannot commit to future contracts. We shall also focus on the case the principal seeks to induce high effort with probability one, deferring to the end of this section, the choice of effort levels. We begin with the analysis of a two period model, where $t = 1, 2$, and there is an initial common prior probability $\lambda \in (0, 1)$ that the job is good. The agent lives for two periods, and discounts future payoffs at rate $\delta \in (0, 1]$, while principal discounts at rate $\beta \in [0, 1]$. We shall assume that neither the principal nor the agent can commit in period one regarding the contract in period two. One interpretation of the model is that there are two short term principals, one arriving in period one and the second arriving in period two, after consumption has taken place in period one. The principal in period two observes the public signal (output) in period one. This implies that wages paid have to satisfy incentive compatibility and individual rationality period by period.

### 3.1 The simple dynamic contract

One conjecture on optimal contracts without commitment is as follows: in period one, the optimal contract is the solution to the static problem with beliefs $\lambda$. In period 2, the optimal contract is the solution to the static problem, but with updated beliefs corresponding to the signal realizations and $e = 1$. Let us call this contract the *simple dynamic contract.* If effort in period one is observable ex post by the principal, before he offers the contract in period two, but that it is not verifiable, the simple contract is the optimal contract, since the agent cannot gain by deviating to low effort. However, since we assume that effort is not observable, the simple dynamic contract cannot be an optimal contract, since the agent has a profitable deviation in the game that this contract induces. Suppose that the agent deviates in period one and chooses $e = 0$. Since his IR binds in the simple contract, the utility he gets in period one remains unaffected, and is indeed equal to his reservation utility, $\bar{u}$. However, his period two beliefs are now different from the principal's beliefs about the agent's beliefs. In particular, there is at least one signal realization such that he becomes more optimistic about his ability. Since the agent suffers no penalty when he becomes more pessimistic – he quits and gets his outside utility, which is the same as under the simple contract, the agent has a profitable deviation.

The fact that the agent always becomes more optimistic at some signal realization after deviating is a consequence of the martingale property of beliefs. For any effort level $e$ that the agent chooses, the expectation of his posterior must equal his prior, $\lambda$. Thus his expected beliefs under $e = 0$ must equal his expected beliefs under $e = 1$. Since $e = 1$ makes signals in $Y^H$ more likely than when $e = 0$ is chosen the equality of expectations can only be satisfied if there is some signal realization $y$ such that $\mu_0^k > \mu_1^k$, where $\mu_e^k$ is the posterior probability that the agent is the good type given signal realization $y^k$ and effort choice $e$.

We now show this more formally. The agent's posterior beliefs at signal $y^k$ when he has chosen $e = 1$ in the first period are given by

$$\mu_1^k = \frac{\lambda p_{1G}^k}{\lambda p_{1G}^k + (1-\lambda) p_{1B}^k} = \frac{\lambda \ell_1^k}{\lambda \ell_1^k + (1-\lambda)}.$$

His posterior belief at $y^k$ after deviating to $e = 0$ are given by

$$\mu_0^k = \frac{\lambda \ell_0^k}{\lambda \ell_0^k + (1-\lambda)}.$$

Thus the agent is more optimistic about his ability after deviating on observing signal $y^k$ if $\mu_1^k < \mu_0^k$, i.e. if $\ell_1^k < \ell_0^k$.

**Lemma 3** *There exists some $k$ such that $\mu_0^k > \mu_1^k$.*

**Proof.** From the martingale property of beliefs, $\mathbf{E}(\mu_1^k | e = 1) = \mathbf{E}(\mu_0^k | e = 0) = \lambda$, i.e.

$$\sum_{k=1}^{n} p_{0\mu}^k \mu_0^k = \sum_{k=1}^{n} p_{1\mu}^k \mu_1^k.$$

This can be written as

$$\sum_{k=1}^{n} p_{0\mu}^k (\mu_0^k - \mu_1^k) = \sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k) \mu_1^k.$$

Since $\sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k) = 0$ (being the difference between two probability distributions), $\sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k) \lambda = 0$, so that

$$\sum_{k=1}^{n} p_{0\mu}^k (\mu_0^k - \mu_1^k) = \sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k)(\mu_1^k - \lambda).$$

Under assumption A1, for any $k$, $(p_{1\mu}^k - p_{0\mu}^k)$ has the same sign as $(\mu_1^k - \lambda)$ – i.e. a signal that has higher probability under high effort is also informative of the job being easier. Since there is some informative signal, we conclude that $\sum_{k=1}^{n} p_{0\mu}^k (\mu_0^k - \mu_1^k) > 0$, i.e. the expectation of the difference in beliefs under the experiment $e = 0$ is strictly positive. Thus there must be some signal $y^k$ such that $\mu_0^k > \mu_1^k$. ∎

We have therefore shown that the expectation of the "false belief" held by the principal, $\mu_1^k$, that is induced when the agent performs the experiment $e = 0$, is strictly smaller than the expectation of the true belief $\mu_0^k$. Thus there must be some signal realization for which $\mu_0^k > \mu_1^k$. This immediately proves that the simple dynamic contract is never incentive compatible.

The following examples illustrate our arguments. Let output be binary, so that $y \in \{y^H, y^L\}$. Tables 1-3 give examples of information structures that satisfy our assumptions, where the entries show the probability of the signal $y^H$, and $0 < q < p < 1$. In all these examples, high output is most likely when $\alpha = G$ and $e = 1$, and least likely when $\alpha = B$ and $e = 0$. Let $\lambda = 0.5$, although any interior value will suffice.

| | $e = 1$ | $e = 0$ |
|---|---|---|
| G | $p$ | $p$ |
| B | $p$ | $q$ |

Example 1

Consider example 1, where high effort only makes a difference to output when the job is bad. Thus $e = 0$ is a more informative experiment than $e = 1$. Indeed, $e = 1$ is uninformative about the realization of $\alpha$, and if the principal induces high effort at $t = 1$, his posterior will equal the prior after either signal realization. Since $e = 0$ is informative, the agent becomes more optimistic after a success and more pessimistic after a failure. That is $\mu_0^H = \frac{p}{p+q} > \mu_1^H = \frac{1}{2}$ and $\mu_0^L = \frac{q}{p+q} < \mu_1^L = \frac{1}{2}$. The agent will quit after observing $y^H$. After $y^H$, he stays on the job and earns a surplus. In this case, he chooses $e = 0$, since his greater optimism implies that the incentive constraint is violated.

| | $e = 1$ | $e = 0$ |
|---|---|---|
| G | $p$ | $q$ |
| B | $q$ | $q$ |

Example 2

Example 2 is the polar opposite of the first example, since effort only makes a difference when the job is good, and $e = 0$ is uninformative. If he deviates to $e = 0$, he becomes more pessimistic than the principal after a success, and more optimistic after a failure. That is $\mu_1^H = \frac{p}{p+q} > \mu_0^H = \frac{1}{2}$ and $\mu_1^L = \frac{q}{p+q} < \mu_0^L = \frac{1}{2}$. Thus he quits after a success and earns a surplus after a failure. With greater optimism, he has more incentives to exert effort, and his incentive constraint is slack.[2]

| | $e$ =1 | $e = 0$ |
|---|---|---|
| G | $p$ | $\frac{p+q}{2}$ |
| B | $\frac{p+q}{2}$ | $q$ |

Example 3

In example 3 high effort raises the probability of success by the same magnitude, regardless of the nature of the job. In this case, it may be verified that $\mu_0^H > \mu_1^H$ and $\mu_0^L > \mu_1^L$, so that the agent is more optimistic than the principal after *either* output realization. Thus the agent always earns a surplus after deviating to $e = 0$. Since his incentive constraint is satisfied under his beliefs, it is optimal for him to exert high effort at $t = 2$. This example seems slightly paradoxical, since $\mu_0^k > \mu_1^k$ for all output realizations $y^k$. We label this phenomenon *uniform optimism* – as we shall see, many of our subsequent characterization results depend upon whether or not uniform optimism holds. Notice that uniform optimism does not violate the martingale property of beliefs – in our example, $y^H$ has lower probability under $e = 0$ than under $e = 1$, and thus one can have the equality of $\mathbf{E}(\mu_1^k|e = 1)$ and $\mathbf{E}(\mu_0^k|e = 0)$, even though $\mu_0^k > \mu_1^k$ for every value of $k$.

## 3.2 Characterizing optimal contracts

Suppose that the principal wants to induce $e = 1$ in both periods. Period 2 contracts are straightforward. Given that $e = 1$ is chosen, the principal's beliefs about the agent's beliefs are degenerate, and are given by $\mu_1^k$ after signal $y^k$. Thus the period two contract after signal $y^k$ is given by $w(\mu_1^k) \in \mathbb{R}^{|Y|}$. Let $w_j(\mu_1^k)$ denote the wage paid under the optimal second period contract after second period signal realization $y_j$ given that the principal has belief $\mu_1^k$.

Turning to period 1 contract, this must satisfy IR with the prior beliefs $\lambda$ and also a modified IC given these beliefs. We turn to deriving this modified IC.

[2]In examples 1 and 2, a weaker version of A1 holds since $p_{1B}^H$ and $p_{0G}^H$ are not necessarily strictly less than $p_{1G}^H$ or strictly greater than $p_{0B}^H$. As remark 2 makes clear, the agent necessarily gets a surplus when he is more optimistic, but he could get his reservation utility when he is more pessimistic, e.g. by choosing $e = 1$ in example 1, and $e = 0$ in example 2.

The agent's surplus, i.e. his continuation utility relative to $\bar{u}$, after signal $y^k$ and deviation $e = 0$ in the event that he stays on the job is denoted by $V(\mu_0^k, \mu_1^k)$ – it depends upon his belief $\mu_0^k$, and also upon $\mu_1^k$, since second period wages depend upon $\mu_1^k$. If $\mu_0^k < \mu_1^k$, the agent quits and $V(\mu_0^k, \mu_1^k) = 0$. If $\mu_0^k \geq \mu_1^k$, he stays on the job and chooses whichever effort level gives him a higher payoff. Thus $V(\mu_0^k, \mu_1^k)$ is given by

$$V(\mu_0^k, \mu_1^k) = \left[\mu_0^k - \mu_1^k\right] \max \left[ \sum_j (p_{1G}^j - p_{1B}^j) u\left(w_j(\mu_1^k)\right), \sum_j (p_{0G}^j - p_{0B}^j) u(w_j(\mu_1^k)) \right], \tag{7}$$

if $\mu_1^k \geq \mu_0^k$, and $V(\mu_0^k, \mu_1^k) = 0$ if $\mu_0^k < \mu_1^k$.

Therefore, the agent's expected increase in continuation utility from choosing $e = 0$ in the first period is given by

$$\mathbf{E}(\hat{V}(\lambda)) = \sum_k V(\mu_0^k, \mu_1^k)(\lambda p_{0G}^k + (1-\lambda) p_{0B}^k).$$

The modified IC for the first period is given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) \geq c(1) - c(0) + \delta \mathbf{E}(\hat{V}(\lambda)). \tag{8}$$

The IR constraint is unaffected and is given by

$$\lambda \sum_k p_{1G}^k u\left(w_k^1\right) + (1-\lambda) \sum_k p_{1B}^k u\left(w_k^1\right) - c(1) \geq \bar{u}. \tag{9}$$

Given an equilibrium where the agent chooses $e = 1$ at $t = 1$, the second period contract does not directly depend upon the wages chosen in the first period. This follows from the fact that the second period contract after any signal realization depends only on the beliefs, $\mu_1^k$, and not upon first period wages. This implies that $\mathbf{E}(\hat{V}(\lambda))$ is given, and also does not depend upon first period wages. Since first period wages do not affect the principal's continuation payoff, the optimal first period contract minimizes expected wages subject to the IR constraint and the modified incentive constraint. Thus it is the standard static optimal contract as in Holmstrom (1979), but with a more stringent incentive constraint. The characterization of the optimal contract in this case is standard (i.e. both constraints hold, and the marginal utility at any signal realization is related to the likelihood ratio. Inspecting the constraints (8) and (9), we see that it is as though the agent has a lower cost of low effort, i.e. $c(0)$ is reduced by an amount equal to $\delta\mathbf{E}(\hat{V}(\lambda))$, while the cost of high effort is unaffected. Thus the incentive scheme needs to be more high powered, in order to dissuade the agent from increasing his continuation value by deviating to low effort. The agent is subjected to greater risk, and in consequence, there is a *dynamic agency cost* that magnifies the static agency cost. Since the individual rationality constraint binds, the agent does not benefit in any way from this dynamic moral hazard. We summarize our results in the following proposition.

**Proposition 4** *The optimal dynamic contract that induces $e = 1$ in both periods is follows: i) in period 1, the contract wages minimize expected wage payments given the modified IC (8) and the IR (9), which hold with equality. ii) in period 2, the contract after signal realization $y^k$ is given by the static contract $\mathbb{W}(\mu_1^k)$,corresponding to common beliefs $\mu_1^k$. Since the incentive constraint is more stringent in the dynamic contract, the first period contract has more high powered incentives and the principal incurs a dynamic agency cost since the agent must bear more risk.*

One question that arises is, can the principal not do better by screening at the beginning of the second period? The answer is no; in equilibrium, the probability that the agent chooses $e = 0$ is zero, and thus the screening problem is one where the probability assigned by the principal to the agent having a belief $\mu_0^k$ equals zero. We shall see this more explicitly in section 6, where we discuss optimal screening when there is private information on the equilibrium path due to the agent's random choice of effort.

The dynamic agency cost that arises in period one is due to the fact that the agent can increase his continuation value by $\delta\mathbf{E}(\hat{V}(\lambda))$, by choosing low effort. We turn now to some of the factors that influence the size of the dynamic agency cost.

One immediate observation is that the dynamic agency cost increases if the agent becomes more patient, i.e. if $\delta$ is larger. That is the shadow of the future looms larger if the agent values it more. Notice that the discount factor of the principal plays no role in the analysis, in the absence of commitment.

More subtle is the fact that the dynamic agency cost is an increasing function of the static agency cost, i.e. $\mathbf{E}(\hat{V}(\lambda))$ is an increasing function of $\Delta c = c(1) - c(0)$. We do not provide a formal proof of this claim here since it can be proved along the same lines as proposition 7 in the following section. The intuition is as follows: a larger value of $\Delta c$ implies that incentives have to be more high powered in the final period. This increases the payoff difference between having optimistic as opposed to pessimistic beliefs, i.e. the absolute value of $V(\mu_0^k, \mu_1^k)$ is larger for any signal realization $y^k$. Thus, $\mathbf{E}(\hat{V}(\lambda))$ is also greater.

### 3.3 The choice of effort

Our focus hitherto has been on the principal's cost minimization problem – assuming that the principal seeks to induce $e = 1$ in both periods, we find that dynamic agency cost increases the cost of inducing high effort at the beginning of the relationship. In consequence, the principal may find it too expensive to induce high effort at the beginning, and may give up on this. In section 6 we discuss the rationale for inducing random effort in the first period.

Of course, the principal also learns about the relationship between effort and output in our model, and thus the benefit to the principal from inducing $e = 1$ as opposed to $e = 0$ also depend upon his beliefs. Let $R(e, \mu)$ denote the revenue of the principal, as a function of the effort level, $e$ :

$$R(1, \mu) = \sum_k \left[\mu p_{1G}^k + (1-\mu) p_{1B}^k\right] y^k.$$

$$R(0, \mu) = \sum_k \left[\mu p_{0G}^k + (1-\mu) p_{0B}^k\right] y^k.$$

The difference in revenue from inducing high effort and inducing low effort, is given by

$$R(1, \mu) - R(0, \mu) = \mu \sum_k (p_{1G}^k - p_{0G}^k) y^k + (1-\mu) \sum_k (p_{1B}^k - p_{0B}^k) y^k.$$

If we assume that the distribution of output given $e = 1$ first order stochastically dominates the distribution given $e = 0$, then $R(1, \mu) - R(0, \mu) > 0$. Indeed, our implicit assumption so far is that it is large enough that the principal desires to induce high output. However, its behavior as a function of $\mu$ is ambiguous, since $R(1,\mu) - R(0,\mu)$ is linear in $\mu$, and can be either increasing or decreasing. If $R(1,\mu) - R(0,\mu)$ varies a lot as a function of $\mu$ and becomes small at some values, then the principal's choice of effort in the second period will depends upon his beliefs. Then it is possible that the dynamic agency problem can be mitigated. Suppose that the principal wants to induce $e = 0$ in the second period, when beliefs are below some threshold, i.e. when he is sufficiently pessimistic about the project. Suppose also that $\mu_0^k - \mu_1^k$ is positive precisely when $\mu_1^k$ is below the threshold. However, the agent cannot benefit from his optimistic beliefs since the optimal contract for inducing $e = 0$ is a flat wage scheme.[3] However, as we shall see shortly, this is an artefact of the binary effort model – if there are multiple effort levels, and the principal always wants to induce something greater than the lowest effort level, he must always provide incentive pay. This implies that there is always an increase in the agent's continuation value when he deviates downwards. Thus the dynamic agency cost of inducing any non zero first period effort level is always strictly larger than the static cost, as long as zero effort is never optimal in the future.

We may flesh out these arguments in the context of our three examples with binary signals. The revenue difference between inducing high and low effort equals

$$R(1,\mu) - R(0,\mu) = \left[\mu(p_{1G}^H - p_{0G}^H) + (1-\mu)(p_{1B}^H - p_{0B}^H)\right]\Delta y,$$

where $\Delta y = y^H - y^L$. In example 1, the difference in revenue equals $(1-\mu)(p-q)\Delta y$, i.e. it i s decreasing as a function of $\mu$. In this example, if the principal finds it optimal to induce $e = 1$ at $t = 1$, his posterior equals the prior regardless of the signal, and thus she will induce $e = 1$ after both signals. In example 2, the difference in revenue equals $\mu(p - q)\Delta y$, and the principal may find it optimal to induce $e = 0$ if she is sufficiently pessimistic, e.g. after $y^L$ is realized. In this case, the agent cannot raise his continuation value at $t = 1$ by shirking, since he is more optimistic than the principal only after $y^L$. In example 3, the difference in revenue equals $(p - q)\Delta y/2$, and is independent of $\mu$. Thus if the principal finds it optimal to induce $e = 1$ at $t = 1$, he will also do so at $t = 2$.

The argument in proposition 4 generalizes beyond the binary effort case. Con-

[3] A similar argument applies if the principal finds it optimal to induce $e = 0$ above some threshold of beliefs.

sider an arbitrary finite ordered set of efforts, $\{e_i\}_{i=0}^m$, with a cost function $c(.)$ that is increasing. Assume that a version of assumption A1 holds for any adjacent pair of efforts, $e_i, e_{i+1}$. That is, for any signal $y^k$, $p_{i+1B}^k$ and $p_{iG}^k$ lie in the interior of the interval spanned by $p_{i+1G}^k$ and $p_{iB}^k$. Consider the two period problem and suppose that the principal seeks to implement a fixed effort level greater the minimum level at $t = 2$, and at $t = 1$, he seeks to implement $e_i, i > 0$. Our arguments imply that for any downward deviation by the agent to $e_j$, his increase in continuation value $\delta\mathbf{E}(\hat{V}(\lambda; i, j))$ is strictly positive. On the other hand, if the agent deviates upwards, to $e_j$, the change in his continuation value is non-negative, so $\delta\mathbf{E}(\hat{V}(\lambda; i, j)) \geq 0$. Thus the cost of implementing any effort level $e_i$ that is greater than the smallest one is strictly larger in the dynamic case as compared to the static one. If uniform optimism applies, one may make a stronger claim, since there is no benefit from upward deviations – when the agent deviates upwards, he will always be more pessimistic than the principal, and will therefore quit in the next period, regardless of the signal realization.

# 4 Short term commitments: many periods

We now consider the case where there are finitely many periods. Our main substantive finding is that as the period of interaction increases, the dynamic agency problem becomes harder, in the sense that it is increasingly difficult to provide incentives. Similarly, as the agent becomes more patient, the incentive problem becomes harder. We should stress here that both these effects do not arise only because of the fact that the agent values the future more. In addition, the problem is aggravated due to a compounding effect – since the agent will value the future more tomorrow, the incentive scheme tomorrow will be more high powered. This increases the continuation value today from having more optimistic beliefs, since high powered incentives imply higher informational rents.

We now consider the case where there are $T$ periods, and call this game $\Gamma^T(\lambda)$, to emphasize its dependence on the prior. The public history at date $t, h^t$, is an element of $(Y)^{t-1}$. The private history at date $t$, $\tilde{h}^t$ is an element of $(Y \times \{0,1\})^{t-1}$. Let $h^1 = \tilde{h}^1$ be a singleton set.

We focus on equilibria where the principal seeks to induce $e = 1$ in every period, and solve for the optimal dynamic contract. Since $T$ is finite, this is done recursively, solving backwards from the last period. It will be convenient to organize the discussion in terms of the number of periods remaining, which we denote by $\tau$. That is, if we are in period $t$ of a $T$ period game, $\tau = T - t + 1$.

We shall assume that if any period, the agent refuses the contract proposed by the principal, the game ends, and the agent gets his outside option $\bar{u}$ in every period thereafter. This ensures that the game we study has "no observable deviations" by the agent, i.e. when the principal becomes aware that the agent has chosen an out of equilibrium action, the game ends. If the agent quits, he principal also gets some payoff that may depend upon his belief, but this will not be relevant since we assume that the principal always wants to hire the agent, and in equilibrium, this happens with probability one.

Suppose that in some period, the principal has an initial belief $\mu$. Since he believes that the agent chooses $e = 1$ for sure, his posterior on observing signal realization $y^k$ is given by the belief operator $\phi^k(\mu, 1)$ :

$$\phi^k(\mu, 1) = \frac{\mu p^k_{1G}}{\mu p^k_{1G} + (1 - \mu)p^k_{1B}}.$$

Let $\nu$ denote the belief of the agent at the beginning of the period, and let $e$ denote his actual effort choice. The agent's posterior on observing signal realization $y^k$ is given by the belief operator $\phi^k(\nu, e)$ :

$$\phi^k(\nu, e) = \frac{\nu p^k_{eG}}{\nu p^k_{eG} + (1 - \nu)p^k_{eB}}.$$

The wages offered by the principal, as a function of the current period signal, depend upon history only via the beliefs $\mu$ that he has, and also depends upon the number of remaining periods. We denote the (optimal) contract by $w^k(\mu, \tau)$.

The value function of the agent at the beginning of the period depends upon his belief, the principal's belief and the number of periods remaining, and is denoted by $V(\nu, \mu, \tau)$. Recall that the value is the agent's discounted surplus, i.e. the sum of his payoffs in every period relative to his outside option. If $\nu < \mu, V(\nu, \mu, \tau) = 0$ and the agent's optimal strategy is to quit. If $\nu \geq \mu, V(.)$ is defined recursively by

$$V((\nu,\mu,\tau)=\max_{e\in\{0,1\}}\sum_k p^k_{e\nu}\left[u(w^k(\mu,\tau)-\bar{u}-c(e)+\delta V(\phi^k(e,\nu),\phi^k(1,\mu),\tau-1)\right]. \tag{10}$$

The recursion is initialized by setting $V(\nu,\mu,0)=0$.

The value of $e$ that maximizes the right hand side of (10) defines the agent's optimal pure strategy at $(\nu,\mu,\tau)$. If $e=1$ and $e=0$ are both optimal, we assume that the agent chooses $e=1$.

Consider the situation where the principal and the agent have the common belief $\mu$, and there are $\tau$ periods to follow, after the current one. If the agent deviates in this period to $e=0$, his beliefs will differ from the principal's tomorrow, and will equal $\phi^k(0,\mu)$ rather than $\phi^k(1,\mu)$ when signal $y^k$ is realized. He will quit whenever $\phi^k(0,\mu)<\phi^k(1,\mu)$. Thus the increase in his expected continuation value following a deviation today equals

$$\mathbf{E}\hat{V}(\mu,\tau)=\sum p^k_{0\mu}V(\phi^k(0,\mu),\phi^k(1,\mu),\tau).$$

**Lemma 5** *If $v<\mu$, then $V(\nu,\mu,\tau)=0$, and the agent's optimal strategy is to quit.*

**Proof.** The proof of this mirrors that of proposition 1. If the agent stays and chooses

$e=1$ today, then his payoff in period $t$ is less than reservation value since he is more pessimistic. Furthermore, since $\phi^k(1,\nu)<\phi^k(1,\mu)$ if $v<\mu$, the agent's strategy asks him to to quit tomorrow. Thus the one-step deviation principle implies that quitting today is strictly better. If he stays and chooses $e=0$ today, then his payoff today is less than the payoff from choosing $e=0$ with belief $\mu$. Since $\phi^k(1,\nu)<\phi^k(1,\mu)$ if $v<\mu$ for any $k$, his continuation value $\mathbf{E}(\hat{V}(\nu,t))<\mathbf{E}(\hat{V}(\mu,t))$. Thus his overall payoff from choosing $e=0$ at $\nu$ is strictly less than his overall payoff from $e=0$ at $\mu$. Since the latter has the same overall payoff as getting $\bar{u}$ in every period, it is strictly optimal to quit. ■

Consider now the principal's optimal contract in period $\tau$ for inducing $e=1$, given the public state $(\mu,\tau)$. Since the agent can increase his continuation value by deviating to $e=0$, the modified IC for this period is given by

$$\mu\sum_k(p_{1G}^k-p_{0G}^k)u\left(w_k(\mu,\tau)\right)+(1-\mu)\sum_k(p_{1B}^k-p_{0B}^k)u\left(w_k(\mu,\tau)\right)\geq c(1)-c(0)+\delta\mathbf{E}(\hat{V}(\mu,\tau)). \tag{11}$$

The IR constraint is unaffected, and is given by

$$\mu\sum_k p_{1G}^k u\left(w_k(\mu,\tau)\right)+(1-\mu)\sum_k p_{1B}^k u\left(w_k(\mu,\tau)\right)-c(1)\geq\bar{u}. \tag{12}$$

Note that wages in period $\tau$ do not directly affect the principal's continuation value, conditional upon $e=1$ being chosen. Thus the principal minimizes current wages subject to the modified incentive constraint and participation constraint set out above. In other words, if we have solved for the optimal contract $w(\tau,\mu)$ and the agent's optimal strategy and value function $V(\nu,\mu,\tau)$ for $\tau\in\{1,2,..,\tau-1\}$ and $\mu\in(0,1)$, this defines $\mathbf{E}(\hat{V}(\mu,\tau))$ and the modified incentive constraint for this period (11). Thus the optimal contract that induces $e=1$ in period $\tau$ at belief $\mu$ has been reduced to the solution of a static problem. The solution to this, $w(\mu,\tau)$, is as in Holmstrom (1979), where both constraints bind, and where marginal utilities of consumption of the agent at $y^k$ are related to the likelihood ratio of this signal given belief $\mu$.

**Proposition 6** *The optimal dynamic contract that induces $e=1$ in every periods is solved for recursively, as follows: i) in period $\tau$ given the principal's belief $\mu$, the contract wages $w_k(\mu,\tau)$ minimize expected wage payments given the modified IC (8) and the IR (9), which hold with equality.*

We are now in a position to set out the main economic insight, that a longer period of interaction makes the incentive problem worse. Our comparison across games with different time horizons is as follows. Consider an arbitrary period $t$ in the $T$ period game with initial prior $\lambda,\Gamma^T(\lambda)$, and the $T+1$ period game with the same initial prior, $\Gamma^{T+1}(\lambda)$,where $t\leq T$. The set of possible $t$ period histories is identical across these games. Furthermore, if the equilibrium effort sequence $(e^t)_{\tau=1}^{t-1}$ is the same, then $\mu(h^t)$ is the same across these games for the same realized public history. Our main result shows that the incentive problem is strictly more difficult and inducing high effort is more costly when there is one additional period. That is, the expected wage cost of inducing $e = 1$ is greater when there are $\tau + 1$ periods remaining than when there are $\tau$ periods remaining, for any value of $\tau$, and for any belief $\mu$ :

$$\sum_k p_{1\mu}^k w_k(\mu, \tau + 1) > \sum_k p_{1\mu}^k w_k(\mu, \tau), \forall \tau, \forall \mu \in (0, 1). \tag{13}$$

To provide some intuition for the proposition that follows, let us compare the first period of the three-period game with the first period of the two-period game, both having the same prior $\lambda$. Assume that the principal seeks to induce $e = 1$ in all periods in both these games. Suppose that signal $y^k$ is realized in the first period. In the two-period game, since only one period remains, the wages correspond to $w_k(\mu_1^k, 1)$, i.e. those in the static contracting problem. In the three-period game, the continuation game is a dynamic contracting problem, as studied in section 3, and the wages correspond to $w_k(\mu_1^k, 2)$. As proposition 4 establishes, the incentive scheme $w_k(\mu_1^k, 2)$ is more high powered than $w_k(\mu_1^k, 1)$. However, higher powered incentives also increase the informational rents from more optimistic beliefs. This means that the agent can, by deviating in the first period, raise his second period payoff more in the longer game as compared to the shorter game. In addition, he can enjoy this informational rent for one period more.

**Proposition 7** *In any period $t \leq T$ and after any public history $h^t$, the $t$-period incentive constraint is strictly more severe in $\Gamma^{T+1}(\lambda)$ than in $\Gamma^T(\lambda)$. Thus, the cost of inducing high effort in period $t$ at $h^t$ is strictly greater in $\Gamma^{T+1}(\lambda)$ than in $\Gamma^T(\lambda)$, i.e. inequality (13) holds. In the $T$ period game $\Gamma^T(\lambda)$, the cost of inducing high effort is strictly greater in any period $t \leq T$ if the agent is more patient.*

**Proof.** The proof is by induction. Formally, we wish to show that for any value of $\tau$ and for any belief $\mu \in (0, 1), \mathbf{E}( (w_k(\mu, \tau + 1))) > \mathbf{E}( (w_k(\mu, \tau)))$, i.e. (13) holds. Proposition 4 has already proved this statement when $\tau = 1$. Assume now that the statement is true for any $\tau \in \{1, 2, ..., \hat{\tau}\}$. Fix an equilibrium strategy of the $T$ period game, and suppose that the agent deviates when there are $\hat{\tau} + 1$ periods remaining, i.e. in period $t = T - \hat{\tau}$, at some history $h^t$ and chooses $e = 0$. Let $\hat{s}^T$ denote an optimal continuation strategy for the agent in the continuation game, given that he has deviated at date $t$. Suppose now that the agent deviates at the same history $h^t$ in $\Gamma^{T+1}$ and chooses $e = 0$. Define his continuation strategy $\hat{s}^{T+1}$ as follows: it agrees with $\hat{s}^T$ at all $h^{t'}$ such that $t' \in \{t+1, .., T\}$, and in period $T+1$ it plays optimally. We now show that at every history where the agent makes a deviation gain using $\hat{s}^T$ in $\Gamma^T$, he makes a strictly larger deviation gain by using $\hat{s}^{T+1}$ in $\Gamma^{T+1}$.

Let $\mu$ denote the public belief at $h^t$, and et $w_k(\mu, \hat{\tau})$ denote $t+1$ period wages in $\Gamma^T$ and let $w_k(\mu, \hat{\tau}+1)$ denote $t+1$ period wages in $\Gamma^{T+1}$. These wages coincide with the solution to the static contracting problem, but with different effort costs in the incentive constraint, where the effort cost is strictly greater in $w_k(\mu, \hat{\tau}+1)$ as compared to $w_k(\mu, \hat{\tau})$, by an amount $b$. This implies that

$$\sum_j \left[\mu\left(p^j_{1G} - p^j_{0G}\right) + (1-\mu)\left(p^j_{1B} - p^j_{0B}\right)\right]\left[u\left(w_j(\mu, \tau+1)\right) - u\left(w_j(\mu, \tau)\right)\right] = b > 0.$$

The payoff gain in period $t+1$ from having a belief $\nu > \mu$ and there are $\hat{\tau}$ periods remaining can be written as

$$U(\nu, \mu, t+1, \hat{\tau}) = [\nu - \mu]\sum_j (p^j_{\tilde{e}G} - p^j_{\tilde{e}B}) u\left(w_j(\mu, \hat{\tau}\right),$$

where $\tilde{e} \in \{0, 1\}$ is the optimal effort choice at belief $\nu$.

The payoff gain in period $t+1$ from having a belief $\nu > \mu$ and there are $\hat{\tau}+1$ periods remaining, using the mimicking strategy, equals

$$U(\nu, \mu, t+1, \hat{\tau}+1) = [\nu - \mu]\sum_j (p^j_{\tilde{e}G} - p^j_{\tilde{e}B}) u\left(w_j(\mu, \hat{\tau}+1\right).$$

Thus if $\nu - \mu > 0$,the difference in rent at this history in $\Gamma^{T+1}$ and $\Gamma^T$ in period $t+1$ equals

$$[\nu - \mu]\sum_j (p^j_{\tilde{e}G} - p^j_{\tilde{e}B})\left[u(w_k(\mu, \hat{\tau}+1)) - u\left(w_k(\mu, \hat{\tau})\right)\right].$$

We now show that the above expression is strictly positive. Let $\mathbf{\Delta u}$ denote the vector $\left[u\left(w_j(\mu, \hat{\tau}+1)\right) - u\left(w_j(\mu, \hat{\tau})\right)\right]^n_{j=1}$, and let $\mathbf{\Delta p}_\mu$ denote the vector $[\left[\mu\left(p^j_{1G} - p^j_{0G}\right) + (1-\mu)\left(p^j_{1B} - p\right.\right.$
Thus the inner product $\mathbf{\Delta u}.\mathbf{\Delta p}_\mu = b > 0$. Let $\mathbf{\Delta p}_{\tilde{e}}$ denote the vector $(p^j_{\tilde{e}G} - p^j_{\tilde{e}B})^n_{j=1}$. Since $\mathbf{\Delta p}_{\tilde{e}}$ is the difference between two probability distributions, its components sum to zero, i.e. $\mathbf{1}.\mathbf{\Delta p}_{\tilde{e}} = 0$, where $\mathbf{1}$ denotes a vector where every component is one.

Write $\mathbf{\Delta u} = \mathbf{\Delta \tilde{u}} + k\mathbf{1}$, where $\mathbf{\Delta \tilde{u}}$ and the scalar $k$ are chosen so that every component of $\mathbf{\Delta \tilde{u}}$ has the same sign as the corresponding component $\mathbf{\Delta p}_{\mu}$.Assumption A1 implies that every component of $\mathbf{\Delta p}_{\tilde{e}}$ has the same sign as the corresponding component of $\mathbf{\Delta p}_{\mu}$, and so $\mathbf{\Delta u}.\mathbf{\Delta p}_{\mu} = b > 0$ implies $\mathbf{\Delta u}.\mathbf{\Delta p}_{\tilde{e}} = \mathbf{\Delta \tilde{u}}.\mathbf{\Delta p}_{\tilde{e}} > 0$.

The above argument, for period $t+1$, applies for every subsequent period up to period $T$. We have therefore shown that given any optimal deviation strategy at belief $\mu$ with $\hat{\tau}$ periods remaining in the game $\Gamma^T$, there exists a deviation strategy in $\Gamma^{T+1}$at belief $\mu$ with $\hat{\tau}+1$ periods remaining, that gives strictly higher payoffs in every succeeding period until period $T$ whenever the first strategy yields positive rents. Furthermore, the latter strategy also yields rents in period $T+1$. This proves the first part of the proposition.

The proof of the second part is along similar lines, and hence we do not repeat the details. Suppose that the agent is more patient, i.e. has a higher value of $\delta$. In period $T-1$, the expected increase in continuation value from deviating to $e=0$ is strictly larger, and thus incentives have to be more high powered in period $T-1$. Thus the payoff to the mimicking strategy in the longer game is strictly greater than the deviation gain in the shorter game. By the induction argument, the expected deviation gain is strictly larger at any period $t$ and at any public history, and thus the cost of inducing high effort is strictly greater. ■

We now use our three examples to illustrate how the agency cost escalates with the length of interaction. We compare the optimal contract in the first period of interactions of different lengths, ranging from one to six periods. In each case, the prior equals 0.5, and we choose $p = \frac{3}{4}, q = \frac{1}{4}$. We assume that $c(1) = 2$, $c(0) = 0$ and $\delta = 1$. The difference $(u_H - u_L)$ is a measure of how high powered incentives have to be. In the one-period case, $u_H - u_L = 8$ in all the three cases – given the prior, $p^H_{\lambda 1} - p^H_{\lambda 0} = \frac{1}{4}$ in all three cases, and since the additional cost of high effort equals 2, $u_H - u_L = 8$. With two periods, $u_H - u_L$, the difference in utilities in the first period is strictly greater in all three examples, ranging from 8.5 to 14. As we increase the number of periods, the first period difference in utilities increases steadily, and indeed, it increases at an accelerating rate. In all three cases, the effect of an additional period is larger when the game is longer, illustrating the importance of the cascading effect.

The table also illustrates that the dynamic agency cost is smallest in example 3 – intuitively, uniform optimism implies that the agent cannot benefit from being able to walk away. The dynamic agency cost is largest in example 2, and indeed, explodes as we increase the number of periods. The reason for the very large dynamic agency cost in example 2 is illuminating. In examples 1 and 3, $p^H_{\mu 1} - p^H_{\mu 0}$ is constant across histories and equals $\frac{1}{4}$ – in example 1 this is because $\mu = \lambda$ across histories, and in example 3, this is because $p^H_{\mu 1} - p^H_{\mu 0}$ does not depend upon $\mu$. Thus the Bayesian updating does not, in itself, give rise to differential needs for incentives across histories. In example 2 on the other hand, $p^H_{\mu 1} - p^H_{\mu 0}$ is increasing in $\mu$, and thus incentives need to be very high powered as the principal becomes more pessimistic. However, it is precisely at those histories where $\mu^k_0 > \mu^k_1$. The high powered incentives imply an even larger informational rent, which explains the very large agency cost in this example. This illustrates the interaction between static agency costs and dynamic agency costs.

Table 1: $(u(w_H) - u(w_L))$ at $t = 1$ and time horizon $T$

| | $T$ =1 | $T$ =2 | $T$ =3 | $T$ =4 | $T$ =5 | $T$ =6 |
|---|---|---|---|---|---|---|
| Ex. 1 | 8 | 10 | 12.5 | 16 | 20.5 | 26.3 |
| Ex. 2 | 8 | 14 | 26.8 | 69.4 | 174 | 462 |
| Ex. 3 | 8 | 8.5 | 9.6 | 11.0 | 13.0 | 15.4 |

Our focus in this section is quite different from that in the literature on repeated games, that establishes folk theorems for games with a long time horizon as players get arbitrarily patient (e.g. Wiseman (2005), who considers the case where players have to learn the stage game). If the principal is very patient, and the time horizon is very long, then she could simply induce low effort in the initial periods and only induce high effort, after learning about the nature of the job. Even though this is costly in terms of single period payoffs, cost in terms of long term average payoffs will be small. Thus our analysis is relevant to contexts where the initial periods matter to the principal, either because she is not arbitrarily patient, or because she has a short time horizon and will be replaced by a successor. It is also relevant to understanding the dynamics of the offered contracts over time, even when these initial periods have small effects on the principal's long run average payoff.

## 5 Long-term commitments

Our analysis has so far assumed that the principal and agent cannot commit to a long term contract. To understand better the implications of a lack of inter-termporal commitment, we now consider the case where both the principal and the agent can commit at date one to a long term contract. For simplicity, we focus on the two period case. That is, at date $t = 1$, the principal offers a contract that specifies period one wages as a function of output, and period two wages as a function of outputs in both periods. The agent must then decide whether or not to accept this contract, and if he does accept, he cannot quit at the end of period one. As before, we assume that the principal wants to induce high effort in both periods. We also assume that the principal and the agent have the same discount factor, one.[4] The contract must therefore minimize the sum of wage costs, while satisfying the incentive constraints and the overall participation constraint. Let $w_j^1$ denote the first period wage as a function of output $y_j$, and let $w_{jk}^2$ denote the second period wage given that $y_j$ is realized in period one and $y_k$ is realized in period two.

Consider the second period. The "on-path" incentive constraint in period two following $e = 1$ and signal realization $y^j$ in period one is given by

$$\mu_1^j \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_{jk}^2\right) + (1 - \mu_1^j) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_{jk}^2\right) \geq c(1) - c(0). \tag{14}$$

The individual rationality constraint at the beginning of period one is given by

$$\sum_j \left(\lambda p_{1G}^j + (1-\lambda) p_{1B}^j\right) u(w_j^1) + \sum_j \sum_k \left(\lambda p_{1G}^j p_{1G}^k + (1-\lambda) p_{1B}^j p_{1B}^k\right) u\left(w_{jk}^2\right) - 2c(1) \geq 2\bar{u}. \tag{15}$$

We now turn to the incentive constraint in period one. If the agent deviates to $e = 0$ in period one and signal $y^j$ is realized, then his second period incentive constraint may or may not hold, depending on whether $\Delta(\mu_0^j | \mu_1^j)$ is positive or negative. Thus the increment in his continuation utility relative to $\bar{u}$ is given by $\tilde{V}(\mu_0^j, \mu_1^j)$:

[4] With different discount factors, there would be incentives for inter-temporal borrowing or lending between the parties, raising issues that are orthogonal to our concerns in this paper.

$$\tilde{V}(\mu_0^j, \mu_1^j) = \max\left[\left[\mu_0^j - \mu_1^j\right]\sum_k (p_{1G}^k - p_{1B}^k) u\left(w_{jk}\right), \left[\mu_0^j - \mu_1^j\right]\sum_j (p_{0G}^k - p_{0B}^k) u(w_{jk})\right]. \tag{16}$$

In contrast to the no commitment case, his continuation utility $\tilde{V}(\mu_0^j, \mu_1^j)$ can well be negative, and this will be the case if $\mu_0^j < \mu_1^j$. Therefore, by deviating in period one, the agent gets an increment to second period expected utility equal to

$$\mathbf{E}(\tilde{V}(\lambda)) = \sum_j p_{0\lambda}^j \tilde{V}(\mu_0^j, \mu_1^j).$$

Thus the first period incentive constraint is given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) + \delta \mathbf{E}(\tilde{V}(\lambda)) \geq c(1) - c(0). \tag{17}$$

The optimal contract minimizes expected wage payments over the two periods subject to the first period IC, $n$ signal contingent (on-path) second period ICs, and the overall IR constraint.

We now show that following any first period signal $y_j$, the profile of wages that follow that signal in period one and in period two, must satisfy the following condition

$$\frac{1}{u'(w_j^1)} = \sum_k \left[\mu_1^j p_{1G}^k + (1-\mu_1^j) p_{1B}^k\right] \frac{1}{u'\left(w_{jk}^2\right)}.$$

This condition is essentially the Lambert (1983)-Rogerson (1985) condition on the inverses of the marginal utilities. To prove that this condition must hold in the present context, consider a profile of wages and undertake the following experiment where the utility $u(w_j^1)$ is increased by $\varepsilon$, and the utility $u(w_{jk}^2)$ is increased by $-\varepsilon$, uniformly for every $k \in \{1, 2, .., n\}$. This does not affect the second period incentive constraint following signal $y_j$, (14). Furthermore, since the total utility, over the two periods, following signal $y_j$ is unchanged, it also does not affect the overall individual rationality constraint. Finally, since the change in total utility following $y_j$ is zero, independent of the probability distribution over second period signals, it also does not affect the first period incentive constraint. Since this change does not induce a violation of any of the constraints, it must be unprofitable at the optimum, and the standard argument shows that the martingale condition on the inverses of marginal utilities must be satisfied. Notice that this also implies that the full commitment contract is renegotiation proof. [5]

The long term contract obviously allows the principal to do better than a sequence of short term contracts. This arises due to two reasons. First, under the long term contract, the agent can no longer walk away if, upon deviating in period one, he finds that he is more pessimistic than the principal. This reduces the agent's incentive to deviate, and thereby slackens the incentive constraint in the first period. Second, the long term contract also allows for optimal consumption smoothing between the principal and the agent, as is well known in dynamic moral hazard (the Lambert-Rogerson findings), and allows the principal to appropriate the gains from consumption smoothing. It is illuminating therefore to consider a situation where the consumption smoothing is possible, even though long term commitments are not possible. To this end, we consider a model where the principal signs a one-period contract at $t = 1$. However, after output is realized, and first period wage payments are made, the principal can propose a contract for period 2, before the agent has consumed in the first period.[6]

## 5.1 A comparison: renegotiation without full commitment

Suppose the signal realization is $y_j$, and the agent has been paid $w_j^1$ by principal one. In this case, prior to consumption, the principal may propose a renegotiation consisting of consumptions $(\hat{w}_j^1, (\hat{w}_{jk}^2)_{k=1}^n)$. The first component, $\hat{w}_j^1$, is the consumption he offers the agent for $t = 1$. The second component is the vector of output contingent second period consumptions. We assume that the principal makes a take it or leave it offer to the agent, and that if the agent refuses, he takes the outside option.

[5] In the case where there are two signals, the martingale condition, the incentive constraints and the single IR constraint fully determine the contract wages. I am grateful to Francesco Squintani (personal communication), for pointing this out. More generally, the optimal contract is the solution to the general programing problem set out above.

[6] This argument can also be made in the case where there are two distinct principals, where the principal in period 2 arrives at the end of period one, i.e. before the agent has consumed his wage.

The renegotiation offered must satisfy the following constraints. The incentive constraint is given by

$$\mu_1^j \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_{jk}^2\right) + (1-\mu_1^j) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_{jk}^2\right) \geq c(1) - c(0).$$

The agent will accept the offered on the equilibrium path (i.e. contingent on having chosen $e = 1$ in the first period) only if the following individual rationality constraint is satisfied

$$u(\hat{w}_j^1) + \mu_1^j \sum_k p_{1G}^k u\left(\hat{w}_{jk}^2\right) + (1-\mu_1^j) \sum_k p_{1B}^k u\left(\hat{w}_{jk}^2\right) - c(1) \geq u(w_j^1) + \bar{u}.$$

The optimal second period contract minimizes

$$\hat{w}_j^1 + \left[\mu_1^j \sum_k p_{1G}^k w_{jk}^2 + (1-\mu_1^j) \sum_k p_{1B}^k w_{jk}^2\right]$$

subject to these constraints. Now, by the same argument as in Rogerson (1985), it follows first and second period consumptions must satisfy a martingale condition on the inverses of marginal utilities. That is, we must have

$$\frac{1}{u'(\hat{w}_j^1)} = \sum_k \left[\mu_1^j p_{1G}^k + (1-\mu_1^j) p_{1B}^k\right] \frac{1}{u'\left(\hat{w}_{jk}^2\right)}. \quad (18)$$

Let us denote the consumptions that follow renegotiation from wage $w_j^1$, $\mathbb{W}(w_j^1, \mu_1^j)$. Thus the optimal second period contract after any signal realization must satisfy IR and IC with equality, and must also satisfy the martingale condition on the inverses of the marginal utilities.

We now examine the implications for the first period. Consider first the individual rationality constraint. If the agent chooses high effort in period one, then his continuation payoff when signal $y_j$ is realized in period one is exactly equal to $u(w_j^1) + \bar{u}$, since the IR constraint binds in the second period. Thus the individual rationality constraint in period one is given by

$$\lambda \sum_k p_{1G}^k u\left(w_k^1\right) + (1-\lambda) \sum_k p_{1B}^k u\left(w_k^1\right) - c(1) + \bar{u} \geq 2\bar{u}.$$

That is, the IR constraint in the first period is exactly as in the previous analysis, where neither commitment nor renegotiation was possible.

Now let us consider the incentive constraint. If the agent deviates to low effort in period one, his continuation payoff after signal $y_j$ conditional on staying on the job is equal to

$$V(\mu_0^k, \mu_1^k)) = \begin{cases} \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{1G}^j - p_{1B}^j) u\left(\hat{w}_{kj}^2\right) & \text{if } \Delta(\mu_0^k|\mu_1^k) \geq 0 \\ \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{0G}^j - p_{0B}^j) u(\hat{w}_{kj}) & \text{if } \Delta(\mu_0^k|\mu_1^k) < 0. \end{cases}$$

This has the same qualitative form as in the case without renegotiation, except that the relevant wages are different. In particular, we see that the agent makes a positive rent when he is more optimistic, i.e. when $\mu_0^k > \mu_1^k$ and a negative rent when he is more pessimistic. Since the agent can always quit in latter instance, his actual rent is given by the above expression when $\mu_0^k \geq \mu_1^k$, and by 0 otherwise. Define

$$\mathbf{E}(\bar{V}(\lambda)) = \sum_k V(\mu_0^k, \mu_1^k))(\lambda p_{0G}^k + (1-\lambda) p_{0B}^k).$$

We now show that the rent in the case of renegotiation has the same sign as the rent in the first model, without renegotiation. Note that the incentive constraint in the case with renegotiation has exactly the same form as in the first model, since neither the first period wage nor consumption enter. Since the second period incentive constraint holds with equality, we may re-write this as

$$\mu_1^j \sum_k (p_{1G}^k - p_{1B}^k) u\left(w_{jk}^2\right) - \mu_1^j \sum_k (p_{0G}^k - p_{0B}^k) u\left(w_{jk}^2\right) = c(1) - c(0) - \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_{jk}^2\right).$$

The modified IC for the first period is therefore given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) \geq c(1) - c(0) + \mathbf{E}(\bar{V}(\lambda)).$$

Thus, from the point of view of the principal in period one, the problem is formally similar to the case where there is no renegotiation. Thus exactly the same analysis applies, as far as period one is concerned. We therefore have the following proposition.

**Proposition 8** *Suppose the principal cannot commit at date 1 but can renegotiate after the realization of the signal at date 1. The optimal dynamic contract that induces* $e = 1$ *in both periods is follows: i) in period 1, the contract wages solve the modified IC and the IR with equality. ii) in period 2, the consumptions after signal realization* $y^k$ *are given by* $\mathbb{W}(w_j^1, \mu_1^j)$*,corresponding to the first period contingent wage* $w_j^1$ *and common beliefs* $\mu_1^k$*.The optimal consumptions under renegotiation coincides with the optimal contract with full commitment if and only if there is uniform optimism, i.e. if* $\ell_0^k \geq \ell_1^k \ \forall k$*.*

**Proof.** The only if part of the proposition is straightforward. If the first period incentive constraint binds in the full commitment contract, then it is violated in the case with renegotiation since the continuation value of the agent from deviating to $e = 0$ is $\mathbf{E}(\bar{V}(\lambda))$, which is strictly greater than $\mathbf{E}(\tilde{V}(\lambda))$. To prove if, let the consumptions following renegotiation be identical with those in the full commitment contract. After every signal $y^j$, we need to find a wage $w_j$ such that

$$u(\hat{w}_j^1) + \mu_1^j \sum_k p_{1G}^k u\left(w_{jk}^2\right) + (1 - \mu_1^j) \sum_k p_{1B}^k u\left(w_{jk}^2\right) - c(1) = u(w_j^1) + \bar{u},$$

Since $u$ is strictly increasing and continuous and has an unbounded range, there exists a unique value $\hat{w}_j^1$ that solves the above equation. Suppose now that the principal offers a first period contract $(\hat{w}_j^1)$, and his continuation strategy is to offer the optimal renegotiations after any signal $y_j$. We now that the agent's optimal strategy is put high effort at $t = 1$, and to stay on the job after all signal realizations, and choose $e = 1$ also in period 2, independent of his effort choice at $t = 1$. If the agent deviates to $e = 0$ at $t = 1$, then he becomes more optimistic after all signals, and thus his expected continuation value is greater if he stays than if he goes. Given that he does not quit, his overall payoff is given by the long term contract, which by satisfies incentive constraints in each period. Thus the long term contract can be implemented by a short term contract followed by renegotiation. ■

This proposition clarifies the precise role of commitment. It permits intertemporal risk sharing, as in Lambert-Rogerson, but this can also be done if the principal is able to renegotiate at the end of the first period. The key difference is that it relaxes the first period IC in the case where the agent becomes more pessimistic after some realizations of the signal, since the agent cannot now walk away. The proposition also has an intriguing implication. Suppose that there is uniform optimism. Then the principal gains no advantage from a long term contract over and above the ability to renegotiate and ensure optimal consumption smoothing, conditional on first period wages. However, as we have noted, under renegotiation, the first period contract suffers from the same dynamic agency cost that we uncovered in our basic model, without renegotiation. Thus long term commitments do not allow any mitigation of the dynamic agency cost when there is uniform optimism, even though they permit the principal to appropriate the gains from optimal consumption smoothing, conditional on the wages that have been paid in the first period. Intuitively, the non-observability of effort implies that the principal is unable to insure the agent against an adverse shock, i.e. finding that the job is more difficult than anticipated. Thus the benefits of long term contracts are also limited.

# 6 Random effort

Is there any advantage to the principal in inducing random effort? This question is a non-sequiter in the case of static moral hazard – it is sub-optimal for the principal to induce randomization, since such randomization is dominated.[7] Suppose that the principal offers a contract where the agent randomizes between $e = 1$ and $e = 0$. Since the principal can break the agent's indifference by an arbitrarily small change in wages, he must be indifferent between the agent's choices. Since $e = 1$ is optimal, the contract must satisfy the incentive constraint for $e = 1$, so wages must be non-constant. So in the event that the agent chooses $e = 0$, the principal pays higher expected wages than he would by inducing $e = 0$ for sure, implying that inducing $e = 0$ yields a higher payoff than the random contract.

Nevertheless, in a dynamic context, randomization may play a role by inducing asymmetric information at $t = 2$, and thereby reducing wage payments at $t = 1$.Suppose that the optimal contract for the agent induces him to choose $e = 1$ with probability $\pi \in (0, 1)$. This implies that at $t = 2$, there is asymmetric information,

[7]Random effort can arise if the principal has the inability to fully commit within the period, and can renegotiate the contract after effort is chosen but before output is realized – see Fudenberg and Tirole (1990). This inability to commit is necessarily costly for the principal.

since the agent knows his chosen effort, while the principal does not. In particular, if signal $y^k$ is realized, the principal believes that the agent has chosen $e = 1$ with probability

$$\theta^k = \Pr(e = 1|y^k) = \frac{\pi[\lambda p^k_{1G} + (1-\lambda)p^k_{1B}]}{\pi[\lambda p^k_{1G} + (1-\lambda)p^k_{1B}] + (1-\pi)\pi[\lambda p^k_{0G} + (1-\lambda)p^k_{0B}]}.$$

Therefore, the principal's second order belief assigns probability $\theta^k$ to the agent having first order belief $\mu^k_1$ and probability $1 - \theta^k$ to the agent having first order belief $\mu^k_0$. The principal therefore faces a classical mechanism design problem where the agent knows his "type" while the principal knows the probability distribution over these types, where a type is to be interpreted as the agent's belief about his own ability. Consider the mechanism design problem where agent has two possible beliefs, $\mu^k_1$ and with probabilities $\theta^k$ and $(1 - \theta^k)$ respectively. The principal has limited screening possibilities. He can offer a contract which is acceptable only to the more optimistic type, i.e. the type with belief equal to $\max\{\mu^k_1, \mu^k_0\}$, without being required to pay a rent to this type, in which case the more pessimistic type will refuse the contract. This will be optimal if the probability assigned by the principal to this type (i.e. $\theta^k$ or $1-\theta^k$ as the case may be) is sufficiently low. Alternatively, he can offer a contract which is acceptable to the pessimistic type, i.e. the type with belief equal to $\min\{\mu^k_1, \mu^k_0\}$. In this case, he must pay an informational rent to the optimistic type. If $\mu^k_0 > \mu^k_1$, the informational rent equals to the type with belief $\mu^k_0$ equals $V(\mu^k_0, \mu^k_1))$, as defined in (7). If $\mu^k_0 < \mu^k_1$, the informational rent equals to the type with belief $\mu^k_1$ equals

$$V(\mu^k_1, \mu^k_0)) = \begin{cases} \left[\mu^k_1 - \mu^k_0\right] \sum_j (p^j_{1G} - p^j_{1B}) u\left(w^2_{kj}(\mu^k_0)\right) \text{ if } \Delta(\mu^k_1|\mu^k_0) \leq 0 \\ \left[\mu^k_1 - \mu^k_0\right] \sum_j (p^j_{0G} - p^j_{0B}) u(w_{kj}(\mu^k)) \text{ if } \Delta(\mu^k_1|\mu^k_0) > 0. \end{cases}$$

Consider first the case where there is uniform optimism so that $\mu^k_0 \geq \mu^k_1$ for every signal $y^k$. In this case, one can show that the principal cannot gain by inducing random effort at $t = 1$. If $\theta^k_1$ is sufficiently close to one, then at $t = 2$, the principal will always want to ensure the participation of the belief type $\mu^k_1$ after every signal $y^k$. Thus he must pay an informational rent to type $\mu^k_0$ which equals $V(\mu^k_0, \mu^k_1)$ after every signal $y^k$. Thus the increase in continuation value of the agent from choosing $e = 0$ is exactly equal to $\mathbf{E}(\hat{V}(\lambda))$, just as in the case where $e = 1$ is induced with probability one. Now since $e = 1$ must be optimal at $t = 1$, this implies that the incentive constraint corresponding to this is exactly the same as before. Thus random effort does not reduce the cost of provision of high effort, and is sub-optimal.

Consider next the case where $\mu_0^k < \mu_1^k$ for some signal $y^k$. Suppose that $\theta^k$ is small enough that exclusion of the $\mu_0^k$ is not optimal after any signal $y^k$. In this case, the belief type $\mu_1^k$ gets an informational rent at $t = 2$ after signals $y^k$ such that $\mu_0^k < \mu_1^k$, which equals $V(\mu_1^k, \mu_0^k)$.Therefore the agent's expected continuation utility from choosing $e = 1$ in the first period is given by

$$\mathbf{E}(\hat{V}_1(\lambda))) = \sum_k \max\{V(\mu_1^k, \mu_0^k), 0\}(\lambda p_{1G}^k + (1-\lambda)p_{1B}^k).$$

This relaxes the incentive constraint for choosing $e = 1$ at $t = 1$, which is now given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) \geq c(1) - c(0) + \delta \mathbf{E}(\hat{V}(\lambda) - \delta \mathbf{E}(\hat{V}_1(\lambda)). \tag{19}$$

Furthermore, the participation constraint at $t = 1$ is also slackened by the term $\delta\mathbf{E}(\hat{V}_1(\lambda))$.Note that the first period revenue cost of randomization is linear and decreasing in $\pi$, the probability of high effort. In the second period, $\pi$ does not enter directly into the expressions for $\mathbf{E}(\hat{V}(\lambda))$ or $\mathbf{E}(\hat{V}_1(\lambda))$, since these depend only on the agent's beliefs ($\mu_0^k$ and $\mu_1^k$) and not upon the principal's second order beliefs, which depend upon $\pi$. However, the principal's second order beliefs must assign sufficiently high probability to $\mu_0^k$ when it is lower than $\mu_1^k$, or otherwise the principal will find it optimal to exclude belief type $\mu_0^k$. Thus $\pi$ must be sufficiently low such that after every signal $y^k$ such $\mu_0^k < \mu_1^k$, the principal finds it optimal not to exclude type $\mu_0^k$.

More generally, it may be the case that the principal induces sufficient randomization so that type $\mu_0^k$ is not excluded after some but not all signals $y^k$ such that $\mu_0^k < \mu_1^k$. In this case, the expected rent of the agent must be modified appropriately. That is, for every signal $y^k$ such that $\mu_0^k < \mu_1^k$, there is an associated maximum probability $\pi^k$ with which $e = 1$ must be chosen so that a rent can be paid to the high effort type after this signal. Thus, if one is permitted a slight abuse of notation, the expected rent of the agent from choosing high effort may be written as

$$\mathbf{E}(\hat{V}_1(\lambda,\pi)) = \sum_{k:\pi\leq\pi^k} \max\{V(\mu_1^k,\mu_0^k),0\}(\lambda p_{1G}^k + (1-\lambda)p_{1B}^k).$$

Thus the first period incentive constraint is as in (19), with $\mathbf{E}(\hat{V}_1(\lambda,\pi))$ replacing $\mathbf{E}(\hat{V}_1(\lambda))$. Since this is a decreasing function of $\pi$, reducing $\pi$ relaxes the incentive constraint. Revenue is a strictly increasing in $\pi$ since we have assumed that $R(1,\lambda) > R(0,\lambda)$. Since there are finitely many signals, the optimal contract can be computed by comparing revenues and wage costs corresponding to the finitely many values $\pi^k$. We summarize our results in the following proposition.

**Proposition 9** *With one period commitment, random effort is never optimal if there is uniform optimism. If $\mu_0^k < \mu_1^k$ for some $k$, random effort can relax the agent's incentive constraint, and may be part of the optimal contract.*

Randomization is particularly attractive in the case where the principal is short-lived and is replaced at the end of the period. If the signal structure does not satisfy uniform optimism, then the randomization allows the agent to earn rents in the second period in the event that he chooses $e = 1$ in the first period. This relaxes both the incentive constraint and the participation constraint for the principal in the first period, since he can pay lower and less high-powered wages.

# 7 Concluding comments

Models of repeated agency (e.g. Radner, 1985) have argued that repeated interaction alleviates the agency problem, especially if the principal is unable to make commitments. The present paper offers a different perspective. If there is significant uncertainty regarding the production technology, and this pertains to the nature of the job rather than the general talent of the agent, then long term interaction may well exacerbate the agency problem, by providing opportunities for the agent to earn future rents on the job. Although the agent never earns these rents in equilibrium, their potential existence means that agency contracts have to be more high powered, thus subjecting the agent to additional risk.

This suggests that organizations may well seek to limit the tenure of the agent on the job, even when this is sub-optimal from a production efficiency standpoint. Multi-plant firms often rotate their managers and employees frequently, even though this means that managers have to acquire plant or location specific knowledge, and may also be personally costly for their families. The largest business group in India, the Tatas, adopt such a policy – for example, managers of tea plantations are re-assigned after three years. While there may be alternative explanations for such phenomena, our model suggests that this policy may have the advantage of limiting the dynamic agency cost.

Conceptually, we have analyzed a model where the non-observability of effort gives rise to potential information on the part of the agent. While this feature was already present in Holmstrom's career concerns model, the specific structure analyzed by Holmstrom ensured that the agent's beliefs played no substantive role, since only the market's beliefs affected the agent's payoff. The phenomenon of belief manipulation is likely to arise more generally in any context where there is uncertainty and where the agent's action is privately chosen. An example would be dynamic models that combine hidden information and hidden actions. If the agent's information is partial and may be improved by learning, this may depend upon his action. Thus the considerations explored here would also arise in this context. We leave such extensions for future work.

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