Dynamic lending under adverse selection with limited borrower commitment: Can it outperform group lending?

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Abstract

We derive an optimal lending contract in a two-period adverse selection model with limited commitment on the borrower side. The contract involves "penalty" interest rates after default, and favorable rates after success. It also charges first-time borrowers higher rates than repeat borrowers, as in "relationship lending", because the lender is constrained to keep borrowing attractive while using revealed information to price for risk. We compare the efficiency of a group lending contract (of the kind popularized by the microcredit movement) to the dynamic, individual contract. Both types of contracts reveal the same information, but the contracts face different constraints on using the information to improve risk-pricing. As a result, each type of contract can outperform the other depending on specifics of the environment – opening the possibility that dynamic lending has played a role comparable to that of group lending in the success of microcredit. We discuss factors that push toward one contract form or the other, and a number of extensions including dynamic group contracts and the effect of competition.

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1 Introduction

Repeated lending relationships can be beneficial for a number of reasons, not least because they facilitate the resolution of information asymmetries.[1] When lenders do not know borrowers well initially, repeated lending allows the lender to gather information on the borrower over time as evidence from realized behavior accumulates. Use of this information can potentially allow the lender to tailor contract terms to eliminate inefficiencies stemming from unobserved borrower heterogeneity.

However, the lender can be constrained in its use of borrower information as it is revealed over time. For example, even if lenders can commit themselves to sequences of loan terms (potentially contingent), borrowers can typically drop out of the relationship at any time – at least after having repaid the current loan. Lenders may thus be constrained to keep borrowing attractive so that they can use the accumulating information. As another example, lenders may be limited in how much they can reward repayment, if risky borrowers can at least temporarily pretend to have done well.

One goal of this paper is to analyze efficiency and structure of optimal dynamic lending contracts in a simple two-period adverse selection setting. The setting is one where the lender learns about borrowers' fixed types (safe or risky) as repayment/default episodes accumulate, and seeks to use this information to overcome a lemons problem caused by unobserved risk heterogeneity in the credit market. However, the lender is constrained by borrower limited commitment and by a monotonicity constraint that limits the rewards to success.

A second goal of this paper is to explore further the reasons for success of the microcredit movement. Despite seeming impossibility several decades ago,[2] financial intermediation among the poor throughout the world has grown at unprecedented, rapid rates, to the ex-

[1] Dynamic lending models typically focus on moral hazard or strategic default, where threats of liquidation, capital market exclusion, or loan term deterioration provide incentives for effort, efficient risk-taking, or repayment. Less work has examined dynamic lending contracts in an adverse selection context, where a borrower's riskiness is exogenous but unknown to the lender. See section 2 for related literature.

[2] According to the 2006 Nobel Peace Prize press release: "Loans to poor people without any financial security had appeared to be an impossible idea" (www.nobelprize.org).

tent that now an estimated two hundred million people have borrowed from a microfinance institution ("MFI") (Maes and Reed, 2012). Many of these MFIs sustain operations while covering costs, raising a question of interest to economists: how has lending been successful in these environments where use of collateral is nearly impossible due to low wealth and/or weak institutions?

In answering this question, much research has naturally focused on the innovative techniques of MFIs, especially group lending. Group lending typically involves mutual co-signing of group members for each other's loans (or some other form of liability). It has been shown theoretically to increase efficiency relative to standard individual loans in a number of contexts, including adverse selection (Ghatak, 1999, 2000, Van Tassel, 1999). However, while group lending is still widely used by micro-lenders, there is speculation that its popularity is waning in favor of individual loan contracts. Also, microfinance is apparently possible in some contexts without group lending, since a number of successful MFIs have never used it.[3]

Though not as innovative and, perhaps consequently, not as well studied in the microcredit literature, dynamic lending is also a common strategy among MFIs. Hence, the second goal of this paper is to compare a dynamic lending strategy with a group lending strategy within a similar adverse selection framework. Is one type of contract more efficient than the other, or if not, how does context matter? This kind of theoretical exploration seems useful, not least because group lending has often been justified theoretically by comparison to one-shot individual lending, even though repeated individual lending seems more likely to be MFIs' main alternative. Further, if dynamic contracts are as efficient as group lending contracts, this raises the theoretical possibility that dynamic lending has played as important a role as group lending in the microcredit revolution.

We next preview our findings. Limiting attention first to a simple, standardized (pooling), two-period individual loan contract, which consists of three interest rates – an initial rate, and second period rates conditional on success or failure – we find that the dynamic contract

[3]See also the evidence of Gine and Karlan (2009).

achieves first-best efficiency over more of the parameter space than one-shot loans. This is because the information revealed over time allows the lender to price for risk more accurately, reducing the cross-subsidy from safe to risky borrowers. The lender does this by charging more in states of the world where risky borrowers predominate (after failure) then where they do not (after success).

We also find that contracts tend to be back-loaded from the borrower's perspective – rates for first-time borrowers are high, while second-time borrowers get better terms. Borrowers tend to make losses on the first loan (relative to their outside option), anticipating a better loan later; and vice versa for the bank. The model thus rationalizes a key feature of relationship lending, that terms get better for the borrower over time in lender-borrower relationships.[4] Here, the ability of borrowers to end the relationship pushes the lender to charge more upfront so that the later, information-differentiated rates can be low enough to promote continued borrowing.

We also show that in some cases when full efficiency is not attainable, *nearly* efficient lending can be achieved by giving up on unlucky safe borrowers, i.e. by charging such high rates for second period loans after failure that only risky agents borrow. Interestingly, safe borrowers often prefer this contract to all others, since it extracts more surplus from risky borrowers and results in lower rates for first-time borrowers. Thus there can be an equity-efficiency tradeoff, with equity favored by "punishing" failure more aggressively.

These results restrict attention to simple pooling contracts, which are a priori attractive in many microcredit settings where complex menus of contracts may be hard to implement. However, we show that focus on pooling contracts is without loss of generality in our setting of risk neutrality. The reason is that in any optimal menu, the risky borrower will be indifferent between his contract and the safe borrower's; thus offering *only* the safe borrower's contract is just as good for the risky borrower, and critically, just as good for the lender, since its payoff and the risky borrower's move one-for-one in opposite directions. The lender can also

[4] See further discussion in the literature review, section 2.

do no better with forced savings or collateral (given borrowers have no pledgeable wealth to start with), since it can replicate those contracts by adjusting interest rates. Hidden savings also pose no problem, given the back-loaded structure of the optimal contract.

We turn next to a comparison with group lending, studied in this context by Ghatak (1999, 2000). A first observation is that two-period dynamic lending and two-person group lending reveal the same amount of information to the lender. In the dynamic case, the lender gets two time series observations of project success/failure, from observing the same borrower twice. In the group case, the lender gets two cross sectional observations, from observing a borrower and his partner – both are equally informative about the borrower, given that groups form homogeneously (as Ghatak shows). Further, without any constraints on the contracts, the two kinds of contracts can achieve the same outcomes.

But with the imposed constraints they do differ: group lending achieves *fully efficient* lending over more of the parameter space than dynamic lending. The intuition is that dynamic lending is more constrained in its use of information to price for risk. Ideally, the second-period rate after failure should be high, to put more of the repayment burden on risky borrowers – but this "penalty" rate cannot be too high without inefficiently causing unlucky safe borrowers to drop out. Thus, limited borrower commitment constrains the lender's use of information by hampering its ability to vary the interest rate with repayment record.

However, when group lending fails to avert market breakdown, dynamic lending can outperform it, by pricing safe borrowers who fail out of the market. This strategy allows the lender to vary interest rates more freely based on information, which eliminates more of the cross-subsidy and attracts safe borrowers most of the time – i.e. initially and after success – in cases where group lending would exclude safe borrowers altogether.

In short, group lending or dynamic lending can be the more efficient contract, depending on context. The results thus raise the possibility that dynamic lending has played as significant a role as group lending in the success of microcredit.

Of course, the assumptions required for each kind of contract to operate are more plausible in some contexts than others. Group lending (as modeled) requires that borrower type is known within the community, and that borrowers form groups frictionlessly. Dynamic lending requires that the lender can commit to a two-period contract, and that borrowers are endowed with two similar projects over two periods. The analysis also points to correlated risk as important in these contexts. Where spatial correlation is significant, for example in some agricultural contexts, information revelation under group lending is reduced; serial correlation, on the other hand, which may be relatively more prevalent in small business contexts, hampers information revelation in a dynamic setting.

Finally, several extensions are discussed: the optimal dynamic group contract that could be offered when conditions for both types of contracts are met; and the effects of competition, risk aversion, and length of the relationship on the dynamic individual contract.

Section 2 discusses related literature. The basic model is presented in section 3. Optimal dynamic lending is derived in section 4 and compared with static group lending in section 5. Extensions are discussed in section 6, and proofs are in the Appendix.

2 Related Literature

There are two main contributions of the paper: the first is analysis of a dynamic adverse selection lending model, and the second is a comparison of dynamic lending with group lending in the adverse selection context.

The first contribution fits into a very large literature on dynamic adverse selection. Three distinguishing features are that here, agent types are fixed over time, an assumption that separates it from a large literature on insurance and risk-sharing (e.g. Townsend, 1982, Thomas and Worrall, 1990, Phelan, 1995, etc.); the principal (lender) can commit to the contract, which distinguishes it from literature focusing on the ratchet effect (e.g. Laffont and Tirole, 1988) and relationship lending with informational capture (see below); and the agent (borrower) cannot commit to remaining in the contract. To us, these three features seem applicable in many credit market contexts, especially microcredit.

Similar versions of one-sided commitment have been employed by a few other papers, including Harris and Holmstrom (1982) in the labor market context, Phelan (1995) and Cooper and Hayes (1987) in the insurance context, and Boot and Thakor (1994) in the lending context. These papers tend to find that contract terms improve over time for the agent, as is the case here.[5] Of these papers, only Cooper and Hayes examine a dynamic adverse selection model with fixed types,[6] and they also find that contract terms depend on revealed information, though only for safe customers. However, their focus is on screening heterogeneous risk-averse agents with insurance contracts, in constrast to our focus on solving a lemons problem caused by unobserved risk heterogeneity in the credit market. With the exception of Boot and Thakor (1994), discussed more below, we know of no other dynamic lending models that explore the implications of this one-sided commitment.

Ghosh and Ray (2001) find that loan sizes improve over time, though not necessarily interest rates. In their model, borrowers must be given incentives to repay through the prospect of future loans. However, a fraction of the population cares nothing for the future. This turns out for the best, under some assumptions. It leads to a costly "testing" period for new borrowers involving small loans, on which the myopic borrowers will always default; but the prospect of facing this testing period again with a new lender is what keeps good borrowers faithful to their current lenders. Their paper differs from ours in focusing on exclusion as a way to provide incentives for repayment, and how this can be aided by an adverse selection problem. Also, their lender is unable to commit, so the interest rate can rise as the relationship progresses, as in the "relationship lending" literature discussed next.

Much of the extensive theoretical and empirical literature on "relationship lending" – reviewed by Boot (2000) – has a different focus from ours: competition between asymmetrically

[5]Backloading has also been derived for reasons other than one-sided commitment of the kind we assume. Ray (2002) provides a general result on contract back-loading, in a hidden action setting where neither the principal nor the agent can commit to long-term contracts. Webb (1992) and Monnet and Quintin (2005) find that contract terms tend to improve over time in dynamic models of costly state verification.

[6]Harris and Holmstrom (1982) model imperfect but symmetric information; Phelan (1995) considers types that are i.i.d. over time rather than fixed; and Boot and Thakor (1994) study a hidden effort problem.

informed lenders. Representative, and closest to our paper, is Sharpe's (1990) two-period hidden-type model. There, the repeat lender has superior information in the second period and can charge some borrowers more than competitors. The result is then the opposite of ours, that loan terms get worse for the borrower over time as he becomes informationally captured. Key to the differences in our results is that in Sharpe, the lender develops superior information but cannot commit to a two-period contract.

Our model thus provides a new rationalization for relationship lending that does not revolve around the inside lender acquiring superior information. Here, what looks like a developing relationship with improving contract terms is simply the lender's best way of pricing for risk without driving away borrowers as information is revealed over time. In this, it is similar to Boot and Thakor (1994). They rationalize what look like relationship lending contracts in an infinitely repeated moral hazard lending situation where lenders can commit to long-term contracts but borrowers can leave. They find that borrowers face relatively expensive, collateralized loans until they repay a loan, and from then on relatively cheap, uncollateralized loans that induce efficient effort.[7,8] We view our papers as complementary since the fundamental problems being solved are quite different: inducing effort provision vs. pricing for risk as information is revealed over time.

Webb (1991) is perhaps the closest to our paper. He shows in a two-period adverse selection lending model that borrowers can be screened by the degree of conditionality of the second-period contract on first-period performance.[9] Our paper differs from his in imposing limited commitment and monotonicity constraints on the contracts, which limits what dynamic contracts can achieve and also results in the back-loaded contract structure. We also

[7]A number of empirical contributions test whether repeat loans from the same bank come with higher or lower rates. (The theoretical result that terms should get better over time is typically attributed to Boot and Thakor, 1994; that terms should get worse is often attributed to Sharpe, 1990, and Rajan, 1992.) More seem to find improvements in terms over time, (e.g. Berger and Udell, 1995, Bharath et al., 2011), though this is not without exception (Degryse and Van Cayseele, 2000).

[8]Stiglitz and Weiss (1983) study a related moral hazard model, which is a two-period extension of Stiglitz and Weiss (1981). They also find that interest rates are high initially and drop to the efficient level after success. Their main focus, however, is on exclusion from the credit market as an incentive device that is robust to competition but potentially inefficient.

[9]This is related to what Cooper and Hayes (1987) show in the insurance context.

explore in more detail the efficiency properties of the optimal contract and show that in this risk neutral setting, simple pooling contracts suffice.

In short, the current paper analyzes a new problem in dynamic lending under adverse selection, and derives new results on efficiency and contract structure. Some results have parallels elsewhere – back-loaded contracts have been derived in a number of settings – but the analysis of constrained credit-risk pricing as information is revealed over time is new, as best we can tell, and provides an alternative explanation for what look like relationship contracts.

The second contribution of the paper is the comparison of dynamic, individual lending contracts with static group lending contracts. Most of the group lending theory explores how group lending can outperform one-shot individual loan contracts; this includes the seminal contributions of Ghatak (1999, 2000), Van Tassel (1999), and Gangopadhyay et al. (2005). In contrast, our goal has been to understand a plausible *dynamic* alternative to group lending that does not rely on groups at all, and then to compare this dynamic contract to group lending in a standard framework (essentially, a simplified Stiglitz-Weiss, 1981, model). The hope is to be able to highlight how and when two potentially very different approaches work. To our knowledge, this paper is the first to point out the similarities in information revelation between group and dynamic lending, and to explore how constraints on use of information or differences in its quality allow one or the other kind of contract to perform better.[10]

[10]Several papers compare dynamic individual lending to dynamic group lending. Chowdhury (2007) shows that dynamic group lending is no better than dynamic individual lending at solving a hidden-type/hidden-action problem *unless* loans are made to group members sequentially rather than simultaneously. This is because, under some assumptions, sequential loans can harness social enforcement to raise repayment, while simultaneous group loans (and individual loans) cannot. Che (2002) also compares dynamic individual and dynamic group loans, under hidden effort. However, the individual loans are assumed to be repeated one-shot loans, which do not take advantage of information on borrower effort provision as it is revealed over time. The insights of these two papers are different but complementary to the work here.

3 Baseline Model

There is a continuum of risk-neutral agents, of measure one. Each is endowed with no capital, one unit of labor, an outside subsistence option, and a project. Both the outside option and the project require one unit of labor. In addition, the project requires one unit of capital. The outside option gives expected output $\overline{u} \geq 0$.

Agents' projects differ in risk, indexed by $p \in \mathcal{P}$. The project of a type-p agent pays R_p with probability p and 0 otherwise. Project risk is the agent's private information. As in Stiglitz and Weiss (1981), assume that all projects have the same expected value:

$$p \cdot R_p = \overline{R}, \quad \forall p \in \mathcal{P} \ . \tag{A1}$$

Agents require outside funding if they choose to carry out their projects. We assume limited liability, in particular that agents' exposure in any contract is limited to project returns. It follows that an agent who fails owes nothing to an outside financier. We also assume that output can be publicly verified, but only coarsely: anyone can distinguish between $Y = 0$ (fail) and $Y > 0$ (succeed), but not between different levels of $Y > 0$. This assumption along with limited liability makes debt contracts the only feasible financial contracts.[11] We also restrict attention to deterministic contracts.

Consider a single non-profit lender with access to capital at market gross rate $\rho > 0$. The lender's objective is to maximize total borrower surplus subject to earning the market expected rate of return ρ on all capital lent.[12]

Key results will depend on the net excess return to capital, $\mathcal{N}$, and the gross excess return to capital, $\mathcal{G}$, embodied in the agents' projects:

$$\mathcal{N} \equiv \frac{\overline{\mathbf{R}} - \overline{\mathbf{u}}}{\rho} \qquad \mathit{and} \qquad \mathcal{G} \equiv \frac{\overline{\mathbf{R}}}{\rho} \ . \tag{1}$$

[11]There are no enforcement issues by assumption: borrowers who can repay, do.
[12]Outcomes are similar to a competitive lending market; differences are discussed in section 6.

The numerator of $\mathcal{N}$ ($\mathcal{G}$) is the net (gross) return to a unit of capital invested in these projects; the denominator is the return to a unit of capital invested elsewhere. We assume

$$\overline{R} > \rho + \overline{u} \qquad \Longleftrightarrow \qquad \mathcal{N} > 1. \tag{A2}$$

This implies that all projects are expected to return more than the cost of their inputs, capital and labor. Thus, social surplus is strictly increasing in the number of projects funded, and fully efficient lending is equivalent to lending to all agents. If the lender exactly breaks even, borrower surplus is also strictly increasing in number of projects funded.

The analysis considers the two-type case: $\mathcal{P} = \{p_r, p_s\}$, with $0 < p_r < p_s < 1$. Let $\theta \in (0, 1)$ be the proportion of risky, and $\overline{f(p)}$ be the population average of $f(p)$: $\overline{f(p)} = \theta f(p_r) + (1-\theta) f(p_s)$. For example, $\overline{p}$ is the mean risk-type and $\overline{p^2}$ the mean squared-type.

Static individual lending. One-shot contracts are loans involving r paid after success and 0 paid after failure. As is well-known, a lemons problem can arise: loans are priced based on average risk in the pool, but this can be too expensive for safe borrowers.

An agent of type $\tau \in \{r, s\}$ will choose to borrow and undertake the project iff

$$\overline{R} - p_\tau r \geq \overline{u} \qquad \Longleftrightarrow \qquad r \leq \hat{\mathbf{r}}_\tau \equiv \frac{\overline{R} - \overline{u}}{p_\tau}\ , \tag{2}$$

where the left-hand side of the first inequality is the expected revenue of the project less the expected loan payment and the right-hand side is the subsistence payoff. The second inequality rearranges to obtain the reservation interest rate $\hat{\mathbf{r}}_\tau$, above which a type-τ agent will opt for subsistence. Clearly, $\hat{r}_s < \hat{r}_r$, i.e. safe agents are harder to attract, since they repay with higher probability. Hence, $r \leq \hat{r}_s$ is necessary for fully efficient lending.

If all agents borrow, the lender expects to earn $\overline{p}r$ on a unit of capital lent. Equating this with ρ gives the break-even interest rate $r = \rho/\overline{p}$. This rate attracts all borrowers iff

$$r \leq \hat{r}_s \qquad \Longleftrightarrow \qquad \frac{\rho}{\overline{p}} \leq \frac{\overline{R} - \overline{u}}{p_s} \qquad \Longleftrightarrow \qquad \mathcal{N} \geq \frac{p_s}{\overline{p}}\ .$$

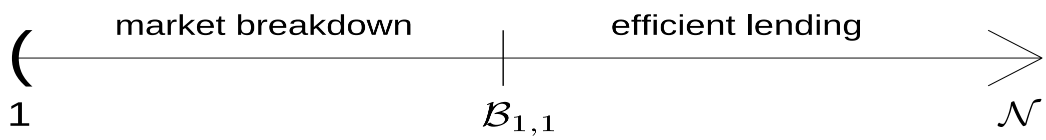

Figure 1: When $\mathcal{N}$ $((\overline{R}-\overline{u})/\rho)$ is at least $\mathcal{B}_{1,1}$ $(p_s/\overline{p})$, efficient lending is achieved by one-shot loan contracts; otherwise, the market breaks down and only risky borrowers are funded.

Thus, efficient lending can be achieved if the excess return to capital in this market, $\mathcal{N}$, is greater than the degree of asymmetric information, captured by $p_s/\overline{p}$. If so, the borrowers get all the surplus from the projects, but risky borrowers earn more than safe.

The lender cannot break even and attract safe agents simultaneously if instead[13]

$$\mathcal{N} < \mathcal{B}_{1,1} \equiv \frac{p_s}{\overline{p}} \ . \tag{A3}$$

The next best option is to give up on safe agents and lend only to risky; see Figure 1. Thus, under assumption A3, one-shot individual loans fund only a fraction θ of the efficient projects. Inefficiency arises from the lender's inability to price for risk. This gives rise to a cross-subsidy from safe to risky borrowers, since safe are more likely to repay; and the excess repayment burden can drive safe borrowers out of the market.[14]

4 Dynamic, Individual Lending

The model is now extended to assume agents receive the same endowment in each of two periods: a unit of labor, a subsistence option, and a project. The project type is fixed over time, and outcomes are independently distributed. To focus better on risk-pricing and efficiency, discounting and consumption smoothing motives are ignored, so that borrower payoffs are just the sum of their two-period expected payoffs.

In section 4.1, attention is restricted to simple two-period pooling contracts in which the

[13]Notationally, the subscripts of cutoff $\mathcal{B}_{1,1}$ are since the loan is for one agent and one period.

[14]The lender can perfectly price for risk with full information, charging ρ/p_s to safe borrowers and ρ/p_r to risky. This achieves efficient and equitable lending, with all surplus going to the borrowers.

lender offers one unit of capital in each period to borrowers, payment due after any failure is zero, and one-period contracts are not offered to new borrowers in period two. In section 4.2 we show that more complex contracts cannot generally improve on this simple one.

4.1 Simple pooling contracts

A pooling contract boils down to three parameters: $(r_\emptyset, r_1, r_0)$, where $r_\emptyset$ is the interest rate on the first loan and r_1 (respectively, r_0) is the interest rate on the second loan for a borrower who has succeeded (respectively, failed) in his first-period project.[15] (Subscripts refer to number of successes, with the empty set referring to the null history.)

In addition to borrower limited liability and the lender breaking even, several constraints are imposed on the contract. First, we assume the lender can commit to the two-period contract, but the borrower cannot commit to taking a second loan. Hence, borrowers drop out at period two if the loan terms are such that the outside option is more attractive. This assumption seems appropriate since typically borrowers can end a borrowing relationship at will, at least after settling existing debts, while lenders may often have the reputational incentives, mission-based integrity, and/or credible legal consequences not to renege from long-term commitments.[16]

Second, we follow Innes (1990), Che (2002), and Gangopadhyay et al. (2005) by imposing monotonicity constraints. Specifically, borrowers cannot be required to pay more when they fail than when they succeed. The argument is that feigning success might be relatively easy, e.g. via very short-term loans from relatives or moneylenders, so rewarding success with lower payments is not feasible.[17] The second-period monotonicity constraints are

$$r_0,\, r_1 \geq 0\ . \tag{3}$$

[15] This allows for credit market exclusion, which can be accomplished by a sufficiently high rate.

[16] See section 2 for other contributions assuming one-sided commitment of this form.

[17] This constraint can also be motivated as a reduced-form constraint from a costly state verification problem in which the lender only audits when a failure is reported. Since reports of success (with the required payments) go unverified, the constraint ensures there is no incentive to falsely report success.

These ensure that the amount due after second-period success (r_0 or r_1) is no less than the amount due after failure (0). Without these, the bank could reward success on a second-period loan with a negative gross interest rate, and a failed borrower would want to claim success in order to receive the reward. The first-period, "dynamic", monotonicity constraint ensures that claiming success after the first loan does not improve payoffs:

$$-r_\emptyset + \max\{\overline{R} - p_\tau r_1, \overline{u}\} \leq 0 + \max\{\overline{R} - p_\tau r_0, \overline{u}\}, \ \tau \in \{r, s\} \ . \tag{4}$$

The left-hand side is the payoff from claiming success: paying $r_\emptyset$ and enjoying the option of a second-period loan at rate r_1 (which will be exercised if it gives a better payoff than $\overline{u}$). The right-hand side is from claiming failure: paying nothing upfront but facing a second-period loan option at rate r_0. It is not obvious whether to impose this dynamic monotonicity constraint – even if claiming success and paying $r_\emptyset$ is worthwhile, a failed borrower may not be *able* to pay $r_\emptyset$.[18] As we show later, it turns out not to matter for efficiency, though imposing it can affect how surplus is split between types.

Define a type-τ agent's two-period payoff from taking the loan in period one and, only if optimal, in period two, as $\Pi_\tau(r_\emptyset, r_1, r_0)$; then

$$\begin{aligned} \Pi_\tau(r_\emptyset, r_1, r_0) &= \overline{R} - p_\tau r_\emptyset + p_\tau \max\{\overline{R} - p_\tau r_1, \overline{u}\} + (1 - p_\tau) \max\{\overline{R} - p_\tau r_0, \overline{u}\} \\ &= 2\overline{R} - p_\tau r_\emptyset - p_\tau^2 \min\{r_1, \hat{r}_\tau\} - p_\tau(1 - p_\tau) \min\{r_0, \hat{r}_\tau\} \ . \end{aligned} \tag{5}$$

The second equality uses the fact that $\overline{R} - p_\tau \hat{r}_\tau = \overline{u}$ (see equation 2).

In the dynamic case also, including safe borrowers is the hard part:

Lemma 1. *If safe agents choose to borrow in period one, so do risky agents.*

Hence, as usual, surplus maximization is highly related to attracting safe agents to borrow. Our approach will thus be to maximally tilt contract parameters in favor of safe borrowers.

[18] The bank cannot force a failed borrower to pay $r_\emptyset$, by assumption. However, borrowers may be able to come up with the money voluntarily, perhaps at some cost, e.g. from relatives or moneylenders.

Specifically, we maximize the safe-borrower payoff, subject to the lender breaking even, borrower limited liability, monotonicity, and borrower limited commitment. Several constraints are initially ignored and verified ex post: limited liability after success, and the dynamic monotonicity constraint. In the following analysis, we assume all agents borrow in period one, and find conditions under which this assumption holds true.[19]

We first establish that a contract that maximizes the safe borrower's two-period payoff (equation 5) subject to the imposed constraints sets $r_1 = 0$. Consider two ranges for r_1, first $r_1 \in (-\infty, \hat{r}_s]$. This r_1 is low enough to attract safe agents for a second loan after success. The safe borrower's payoff is

$$\Pi_s(r_\emptyset, r_1, r_0) = 2\overline{R} - p_s r_\emptyset - p_s^2 r_1 - p_s(1 - p_s)\min\{r_0, \hat{r}_s\} ,$$

while the lender's zero-profit constraint ("ZPC") is

$$\overline{p} \cdot r_\emptyset + \overline{p^2} \cdot r_1 = (1 + \overline{p})\rho + g(r_0) ,$$

where $g(r_0)$ captures the part of the lender's profits that relate to second-period failed borrowers, which depends on r_0 but not r_1 or $r_\emptyset$.[20] The safe borrower's indifference curve in $(r_\emptyset, r_1)$ space is less steep (slope: $-1/p_s$) than the bank's isoprofit line (slope: $-\overline{p}/\overline{p^2}$). Thus, raising $r_\emptyset$ and lowering r_1 along the bank's ZPC raises a safe borrower's payoff. Any contract that does not set r_1 as low as possible (from the monotonicity constraint 3), i.e. $r_1 = 0$, can be thus made better for safe borrowers.

Second, consider $r_1 \in (\hat{r}_s, \infty)$. Here, safe borrowers drop out and never pay r_1, so their preference is that the lender use r_1 to extract as much revenue as possible from risky borrowers, in order to lower $r_\emptyset$ and/or r_0. The bank clearly does this by charging the risky

[19]This is the relevant case since the ultimate question is whether safe borrowers will borrow, and if they do, risky do also, as Lemma 1 shows. The proof of Proposition 1 addresses this in more detail.

[20]Specifically, $g(r_0) = \rho[\theta(1-p_r)1\{r_0 \le \hat{r}_r\} + (1-\theta)(1-p_s)1\{r_0 \le \hat{r}_s\}] - r_0[\theta(1-p_r)p_r 1\{r_0 \le \hat{r}_r\} + (1-\theta)(1-p_s)p_s 1\{r_0 \le \hat{r}_s\}]$.

agents' reservation rate, $\hat{r}_r$. Thus, in this range safe borrowers uniquely prefer $r_1 = \hat{r}_r$.

In sum, to maximize the safe-borrower payoff r_1 should be set to either 0 or $\hat{r}_r$, offering either a free loan after success or maximally extracting risky borrowers' surplus. A direct algebraic comparison reveals that $\Pi_s(r_\emptyset, 0, r_0) > \Pi_s(r'_\emptyset, \hat{r}_r, r_0)$ for any value of r_0, where $r_\emptyset$ (respectively, $r'_\emptyset$) sets lender profits to zero given $r_1 = 0$ and r_0 (respectively, $r_1 = \hat{r}_r$ and r_0). Thus $r_1 = 0$ is better for safe borrowers: a contract tailored to safe borrowers charges as little as possible at the history where safe borrowers are most prevalent, i.e. after one successful loan.

Now let $r_1 = 0$ and consider r_0, first $r_0 \in (-\infty, \hat{r}_s]$. The safe borrower's payoff is

$$\Pi_s(r_\emptyset, 0, r_0) = 2\overline{R} - p_s r_\emptyset - p_s(1-p_s)r_0$$

and the lender's ZPC is

$$\overline{p} \cdot r_\emptyset + \overline{p(1-p)} \cdot r_0 = 2\rho \ . \tag{6}$$

The safe borrower's indifference curve in $(r_\emptyset, r_1)$ space is steeper (slope: $-1/(1-p_s)$) than the bank's ZPC (slope: $-\overline{p}/\overline{p(1-p)}$), implying that lowering $r_\emptyset$ and raising r_0 along the bank's ZPC raises a safe borrower's payoff. Thus safe borrowers prefer $r_0 = \hat{r}_s$ in this range.

Second, for $r_0 \in (\hat{r}_s, \infty)$, safe borrowers strictly prefer $r_0 = \hat{r}_r$, for the same reasons as above: they do not pay $\hat{r}_r$, and so prefer maximal extraction from risky borrowers. In sum, to maximize safe borrowers' payoff, r_0 should be set either to $\hat{r}_s$ or to $\hat{r}_r$.

One can show that under assumption A3, safe borrowers prefer $r_0 = \hat{r}_r$ to $r_0 = \hat{r}_s$. Either way they get their reservation utility in period two after a failure in period one. The comparison thus hinges on whether $r_0 = \hat{r}_r$ or $r_0 = \hat{r}_s$ extracts more profits for the bank, allowing it to lower the period-1 rate, $r_\emptyset$. Under assumption A3, $r_0 = \hat{r}_s$ loses money for the bank, so it is $r_0 = \hat{r}_r$ that earns the bank higher profits on borrowers with one failure. Thus safe borrowers prefer to be priced out of the market when they fail – either way they get their reservation payoff, but at the higher rate the bank earns more and can charge lower rates to first-time borrowers.

Thus the simple pooling contract with the greatest chance of attracting safe borrowers, given that risky also borrow, involves $r_1 = 0$, $r_0 = \hat{r}_r$, and $r_\emptyset$ from the bank' ZPC:

$$\overline{p} \cdot r_\emptyset + \theta(1-p_r) \cdot p_r \hat{r}_r = \rho[1+\overline{p}+\theta(1-p_r)] \quad \Longleftrightarrow \quad r_\emptyset = \frac{\rho[2-(1-\theta)(1-p_s)-\theta(1-p_r)\mathcal{N}]}{\overline{p}} \tag{7}$$

both left- and right-hand sides of the ZPC reflect the fact that safe borrowers who fail do not borrow in period two. This contract attracts safe borrowers in period one iff $\Pi_s(r_\emptyset, 0, \hat{r}_r) \geq 2\overline{u}$. Some algebra shows this is equivalent to

$$\mathcal{N} \geq \mathcal{B}^*_{1,2} \equiv \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{\theta+(1-\theta)p_s}{1+\theta+(1-\theta)p_s}}. \tag{8}$$

This is weaker than the condition needed for one-period individual loans to include safe borrowers (the reverse of condition A3), but is not guaranteed to hold: i.e. $1 < \mathcal{B}^*_{1,2} < \mathcal{B}_{1,1}$.[21]

This contract gives safe borrowers the best possible payoff, and thus provides the weakest conditions for attracting them in period one. But it falls short of achieving maximal borrower surplus, since it causes safe borrowers who fail to choose the inefficient outside option. The lender is thus interested in a contract that creates higher borrower surplus, if one exists.

Any higher-surplus contract must attract safe borrowers who fail, since this is the only source of borrower surplus not exhausted by the above contract; that is, it must involve $r_0 \leq \hat{r}_s$. We next add this constraint to the above maximization of safe borrower payoffs. The above logic gives $r_1 = 0$ as best for safe borrowers regardless of r_0, and $r_0 = \hat{r}_s$ as best when $r_0 \in (-\infty, \hat{r}_s]$. In short, the best contract for safe borrowers with $r_0 \leq \hat{r}_s$ involves $r_1 = 0$, $r_0 = \hat{r}_s$, and $r_\emptyset$ from the zero-profit constraint 6 given these values:

$$r_\emptyset = \frac{\rho\left[2 - \frac{\overline{p(1-p)}}{p_s}\mathcal{N}\right]}{\overline{p}}\ . \tag{9}$$

[21]The subscript $(1,2)$ is used since the contract is for one borrower over two periods.

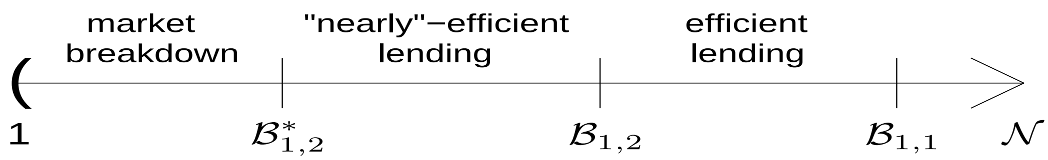

Figure 2: Market efficiency under standardized (pooling) two-period dynamic contracts. Recall that one-shot loans result in market breakdown when $\mathcal{N} < \mathcal{B}_{1,1}$.

This contract attains full efficiency iff safe borrowers prefer to borrow under this contract, i.e. $\Pi_s(r_\emptyset, 0, \hat{r}_s) \geq 2\overline{u}$. Some algebra shows that this is true iff

$$\mathcal{N} \geq \mathcal{B}_{1,2} \equiv \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r}{2}}. \tag{10}$$

It is clear that $\mathcal{B}_{1,2} < \mathcal{B}_{1,1}$, that is, two-period dynamic lending can achieve full efficiency in some cases where static individual lending cannot. Intuitively, the dynamic contract better prices for risk by varying the second-period interest rate so as to shift the repayment burden toward risky borrowers ($r_1 < r_0$). On the other hand, it is also clear that $\mathcal{B}^*_{1,2} < \mathcal{B}_{1,2}$, i.e. it is sometimes possible to achieve near-efficiency (attracting safe borrowers except after they fail) when full efficiency is not possible. Building on this analysis, the following result can be shown; see Figure 2 for a graphical representation.[22]

Proposition 1. *Under assumptions A1-A3 and provided $\mathcal{G}$ is high enough, a simple two-period pooling contract that maximizes borrower surplus subject to borrower limited liability, lender breaking even, monotonicity, and limited borrower commitment:*

- *when $\mathcal{N} \in [\mathcal{B}_{1,2}, \mathcal{B}_{1,1})$, achieves full efficiency – i.e. funds all borrowers.*
- *when $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{1,2})$, achieves near efficiency – i.e. funds all but failed safe borrowers.*
- *when $\mathcal{N} \in (1, \mathcal{B}^*_{1,2})$, fails to avert market breakdown – i.e. funds only risky borrowers.*

[22] The conditions in the Proposition divide up the model's parameters into two types: the technological return parameters $(\overline{R}, \overline{u}, \rho)$, which fully determine $\mathcal{N}$ and $\mathcal{G}$; and the risk parameters (p_s, p_r, θ) which exclusively determine the cutoffs for $\mathcal{N}$ and $\mathcal{G}$ (including $\mathcal{B}_{1,1}$, $\mathcal{B}_{1,2}$, and $\mathcal{B}^*_{1,2}$).

The analysis in the text ignored limited liability after success – specifically, that $r_\emptyset$ be affordable from project returns. Satisfying this constraint necessitates the condition in Proposition 1 that gross return $\mathcal{G}$ be high enough.[23] If $\mathcal{G}$ is not high enough, the initial rate $r_\emptyset$ derived above is not affordable, so it must be lowered and another rate raised – namely, r_1. This limits variation in the second-period interest rate, the key advantage dynamic lending has in risk-pricing, and raises the $\mathcal{N}$ required to achieve efficient lending. In the limit, as $\mathcal{G} \searrow \mathcal{N} \Leftrightarrow \overline{u} \searrow 0$, it turns out that $\mathcal{B}_{1,2} \nearrow \mathcal{B}_{1,1}$, i.e. dynamic lending offers no improvement in achieving full efficiency. However, under some parameter values, even when $\overline{u} = 0$ there is a range of $\mathcal{N}$ for which nearly-efficient dynamic lending is attainable where one-shot loans would lead to market breakdown. Further analysis is possible here, but the basic point is clear: low enough gross returns limit the amount that can be charged upfront, and thus limit the ability of a dynamic contract to vary interest rates as information is revealed while still keeping customers.

The other ignored constraint is dynamic monotonicity (constraint 4). This is satisfied without further assumption by the fully efficient lending contract, but not always by the nearly-efficient contract. However, when it is violated it is because the initial loan is too cheap, which only happens when $\mathcal{N}$ is relatively high and safe borrowers are not close to being marginal. In these cases, the initial rate can be raised and the "penalty" rate lowered along the ZPC until dynamic monotonicity is satisfied, without driving away safe borrowers (see Proof for details). Thus, the dynamic monotonicity constraint may shift surplus from safe to risky borrowers, but does not affect overall efficiency attainable.

The contracts derived above tend to feature back-loaded incentives, with borrowing getting more attractive over time. In particular, with fully efficient lending under assumption A3, even failed borrowers pay a lower rate in period 2 than in period 1: $r_\emptyset > r_0 > r_1$. Safe agents borrow in period 1 even though they earn less than their outside option, in an-

[23]Note that with $\mathcal{N}$ fixed, $\mathcal{G}$ can take on any value such that $\mathcal{G} \geq \mathcal{N}$. This is because raising $\overline{R}$ and $\overline{u}$ in tandem raises $\mathcal{G}$ without changing $\mathcal{N}$. Put differently, a condition that $\mathcal{G}$ is high enough for fixed $\mathcal{N}$ can be written as a condition on the outside option $\overline{u}$ being high enough.

ticipation of cheaper future loans. Back-loading features also hold with the nearly-efficient lending contract derived.

The model thus provides a novel rationale for the feature of relationship lending that borrowers get better terms over time. Here, all borrowers face high rates upfront with the promise of more attractive rates in the future. This entices borrowers to keep borrowing while allowing the lender some ability to vary rates as information is revealed. By contrast, starting with a neutral rate and hiking it after failure runs the risk of driving away unlucky safe borrowers.

Interestingly, under the assumptions of Proposition 1 safe borrowers always prefer the "nearly-efficient" contract, which prices them out of the market after they fail. Thus when fully efficient lending is achievable, there is a tradeoff between equity and efficiency. The efficient, low "penalty" rate creates more surplus by funding all projects, but results in a larger cross-subsidy from safe to risky; the high "penalty" rate sacrifices some projects' surplus but reduces the cross-subsidy (which is never fully eliminated). Exclusion from the credit market turns out to be a good thing here for those excluded.

4.2 More complicated contracts

Given the constraints under which many micro-lenders operate and their apparent preference for simple products – standardized contracts seem quite commonplace, perhaps because both borrowers and bank officers often lack significant quantitative expertise – the pooling contracts analyzed thus far may be interesting in their own right. Here we argue that the simple pooling contracts cannot be improved upon by a range of more complicated contracts.

First, consider forced savings, collateral, and self-financing. In general, each of these instruments could be valuable in this context by raising the amount paid by borrowers after failure, thus shifting more of the repayment burden onto risky borrowers. However, here they are only available after period-1 success, since the borrower has no pledgeable wealth initially and after period-1 failure. In the end, these instruments offer nothing that cannot be accomplished by a simple contract of the form $(r_\emptyset, r_1, r_0)$.

In the case of collateral, consider a contract with rates $r_\emptyset$ and r_1, with required collateral $\kappa > 0$ on the period-2 loan after success.[24] The monotonicity constraint requires $\kappa \leq r_1$; otherwise a borrower who failed would be tempted to claim success to pay the lower amount.[25] Thus the successful borrower pays $r_\emptyset$ at the end of period one; pays r_1 if he succeeds in period two; and pays κ if he fails in period two. In terms of total two-period payoffs, this is equivalent to the simple contract $r'_\emptyset = r_\emptyset + \kappa$, $r'_1 = r_1 - \kappa \geq 0$, and nothing due after failure in period two. This modified contract satisfies the same constraints as the previous one.[26] Thus the bank can mimic any collateral policy with a simple contract already analyzed, by using the period-1 rate essentially to collect the collateral upfront.[27]

Nearly identical arguments show that a forced savings policy, where the savings is used as collateral, or a self-financing requirement can be replicated by a simple pooling contract. The idea is the same: the lender can collect the savings or self-financing amount upfront through the period-1 interest rate without adversely affecting any constraints. It is also clear that hidden savings poses no problem for the optimal pooling contract: after succeeding, the borrower gets a free loan, which he will take whether or not he saves.

In sum, savings, collateralization, and self-financing are allowed for by simple contracts. Key questions remain. First, can a menu of contracts do better by screening? Second, we disallowed negative interest rates: can they improve surplus? Finally, we assumed agents with no history could not borrow in period two: is this restrictive?

Proposition 2. *Under assumptions A1-A3 and provided $\mathcal{G}$ is high enough, the simple pooling*

[24] We assume no deadweight loss associated with pledging collateral.

[25] Without the monotonicity constraint here, collateral could indeed improve on the simple contract derived earlier, but only by stipulating more-than-full collateralization of loans. This would make borrowers pay more after failure than success, further shifting repayment burden toward risky borrowers. However, the optimal contract would have the successful borrower (on the period-2 loan after success) paying nothing and getting κ refunded, and the failed borrower losing κ; thus it would be vulnerable to the failed borrower claiming success in order to have κ refunded without paying anything.

[26] It may appear to raise $r_\emptyset$, potentially making it unaffordable; but note that under the contract with collateral, one successful project must also be able to cover $r_\emptyset + \kappa$.

[27] The same is also true in reverse: one can interpret the optimal simple pooling contract derived earlier, which involves $r_\emptyset$ high and $r_1 = 0$ as instead a more moderate $r_\emptyset$ plus a collateral or forced savings stipulation fully guaranteeing the second loan, which comes with a strictly positive rate $r_1 > 0$.

contract that maximizes borrower surplus subject to borrower limited liability, lender breaking even, monotonicity, and limited borrower commitment cannot be improved upon by any menu of contracts that satisfies the same constraints, including contracts that involve negative interest rates and/or first-time borrowing in period two.

It is mainly due to the monotonicity constraints that negative interest rates do not help. The lender would like to subsidize success by charging a negative interest rate on the period-2 loan after success – but monotonicity forces it to subsidize failure at least as strongly. Thus negative interest rates offer no increased scope for differentially subsidizing success. Further, a refund that is neutral across success and failure can be replicated without negative interest rates by eliminating the refund and lowering earlier payments.

More interestingly, screening contracts offer no improvement. This is due to the fact that borrower and lender payoffs are diametrically opposed (with readily dealt-with exceptions having to do with agents opting out of borrowing). That is, a risky borrower's indifference curve coincides exactly with an isoprofit curve of the lender for risky borrowers.[28] It is also true that the binding incentive constraint is the risky borrower's: at the optimum he is indifferent between his contract and the safe borrower's. Applying these two facts to a pair of optimal screening contracts, note that the risky borrower does equally well if instead he is given the safe borrower's contract; and since his payoff and the lender's payoff move one for one, the lender also does equally well. Thus, giving both risky and safe the safe borrower's contract – i.e. a simple pooling contract – does as well as the menu.

Finally, consider contracts that induce first-time period-2 borrowing. The same payoffs can be achieved by giving the would-be first-time period-2 borrowers the same deal in period one and an unattractive deal in period two. Thus, contracts that induce first-time period-2 borrowing offer nothing that cannot be accomplished without allowing it.

[28]This would be different if borrowers were risk-averse. Payoffs of the two parties could both move in the same direction, due to the gains from selling insurance. Diametrically opposed preferences are the key feature here making the standard problems with pooling inapplicable.

5 Comparing Group Lending and Dynamic Lending

The second goal of the paper is to compare group lending with dynamic lending in the same basic framework. First, existing results on group lending in this context are reviewed. Second, the efficiency properties are formally compared. Third, additional factors that could affect the comparison are discussed.

5.1 Group lending results

Here we review results from the model of Ghatak (1999, 2000) and Gangopadhyay et al. (2005). The environment is the one studied above, except that the setting is static and agents are now assumed to know each other's risk; risk is still unobservable to the lender.

The theory considers contracts written with pairs of borrowers, now with two parameters: r being the amount paid by an agent who succeeds, and c being the additional amount paid by an agent who succeeds and whose partner fails. For the sake of continuity, and without loss of generality, we restrict attention to pooling contracts.[29] The expected payoff of a borrower of type i paired with a borrower of type j, $(i, j) \in \{r, s\}^2$, is then

$$\overline{R} - p_i r - p_i(1 - p_j)c \;=\; \overline{R} - p_i[r + (1 - p_j)c] \;.$$

Ghatak (1999, 2000) shows that the unique stable match involves groups that are homogeneous in risk-type when $c > 0$. Thus in equilibrium the payoff above involves $p_j = p_i$.

As in Gangopadhyay et al. (2005), we impose the constraint that liability for one's partner cannot be more than his debt obligation, i.e. the group-level monotonicity constraint

$$r + c \leq 2r \qquad \Longleftrightarrow \qquad c \leq r \;.$$

If this were not true, a successful borrower would have the incentive to claim his partner

[29] Arguments as in those of Proposition 2 show that this is without loss of generality for surplus maximization. The focus on pooling contracts causes the analysis here to differ from the earlier papers'.

succeeded when he failed, in order to pay $2r$ rather than $r + c$.

In the group lending case also, including safe borrowers is the hard part, so we proceed by maximizing the safe-borrower payoff, subject to borrower limited liability, lender breaking even, and monotonicity. Limited liability after success is initially ignored and verified later. As before, the analysis assumes that all agents borrow in period one, and finds conditions for this to hold true; otherwise, only risky borrow.

If all agents borrow, the bank must set r and c to satisfy its ZPC

$$\overline{p} \cdot r + \overline{p(1-p)} \cdot c = \rho \ .$$

The safe borrower's payoff is

$$\overline{R} - p_s r - p_s(1-p_s)c \ .$$

The safe borrower's indifference curve in (r, c) space is steeper (slope: $-1/(1-p_s)$) than the bank's isoprofit line (slope: $-\overline{p}/\overline{p(1-p)}$). Thus, raising c and lowering r along the bank's ZPC raises a safe borrower's payoff. Since monotonicity requires $c \leq r$, the full liability contract $c = r$ is optimal. With $c = r$, the ZPC becomes

$$\overline{p(2-p)} \cdot r = \rho \qquad \Longleftrightarrow \qquad r = \frac{\rho}{\overline{p(2-p)}} \ .$$

All agents are willing to borrow under this contract iff safe borrowers are, i.e.

$$\overline{R} - p_s(2-p_s)\frac{\rho}{\overline{p(2-p)}} \geq \overline{u} \qquad \Longleftrightarrow \qquad \mathcal{N} \geq \mathcal{B}_{2,1} \equiv \frac{p_s(2-p_s)}{\overline{p(2-p)}} = \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r}{2-p_s}} \ . \quad (11)$$

By inspection, this is weaker than the condition needed for individual loans to include safe borrowers (the reverse of condition A3), but is not guaranteed to hold: i.e. $1 < \mathcal{B}_{2,1} < \mathcal{B}_{1,1}$.[30]

The previous discussion ignored the constraint of limited liablity after success – that is, whether $r + c$ is affordable. As in the previous section, this constraint is satisfied, for any $\mathcal{N}$,

[30]The subscript $(2, 1)$ is used since the contract is for two borrowers in one period.

if $\mathcal{G}$ is high enough. Summarizing, we have the result that under assumptions A1-A3 and provided $\mathcal{G}$ is high enough, the simple pooling contract that maximizes borrower surplus subject to borrower limited liability, lender breaking even, and monotonicity:

- when $\mathcal{N} \in [\mathcal{B}_{2,1}, \mathcal{B}_{1,1})$, achieves full efficiency – i.e. funds all borrowers.
- when $\mathcal{N} \in (1, \mathcal{B}_{2,1})$, fails to avert market breakdown – i.e. funds only risky borrowers.

5.2 Model-based comparison

We next compare maximal surplus achievable by the two types of contracts. The goal is to derive insight into the strengths and weaknesses of two seemingly popular contract forms in improving lending markets.

Of course, the environments needed for the two types of contracts are not identical. The dynamic contract assumes two periods of project endowment and lender commitment, while the group contract assumes borrowers know each others' types and match frictionlessly. Hence, one way to interpret the following comparisons is in terms of surplus attainable across different environments, one in which dynamic lending is feasible but group lending is not, and vice versa. This comparison sheds light on how effective in reviving credit markets dynamic lending can be (where it works) relative to group lending (where it works). A less rigorous way to interpret the comparisons is to imagine both types of lending are feasible, but for some reason (e.g. lack of information) lenders are not necessarily choosing the optimal contract.[31] The comparison would then give guidance as to which form of lending is preferable.[32]

Assume for now that $\mathcal{G}$ is high enough to support the best contracts derived. One can then compare the relative effectiveness of group lending and dynamic lending at achieving full efficiency by comparing conditions 10 and 11. By inspection, $\mathcal{B}_{2,1} < \mathcal{B}_{1,2}$. Thus

[31] In this vein, much empirical work implicitly assumes the lender is not optimizing the contract.

[32] However, if both types of lending are feasible, a combination of dynamic and group lending can do better – see next section.

Corollary 1. *Under assumptions A1-A3 and provided* $\mathcal{G}$ *is high enough, there exists a non-empty interval* $[\mathcal{B}_{2,1}, \mathcal{B}_{1,2})$ *such that for* $\mathcal{N} \in [\mathcal{B}_{2,1}, \mathcal{B}_{1,2})$*, two-person static group lending achieves fully efficient lending while two-period dynamic individual lending does not.*

In terms of achieving full efficiency, group lending thus dominates dynamic lending. Why? Note that both types of lending reveal the exact same information, namely two draws from the borrower's distribution. The two draws are time series observations in the dynamic case. In the group case, the two draws are cross-sectional observations, just as informative of each borrower's type since groups contain identical borrowers.[33]

Note also that absent the monotonicity and limited commitment constraints, the two types of contracts are equally effective at fully efficient lending. The group lending payoff is

$$\overline{R} - p_\tau r - p_\tau(1-p_\tau)c = \overline{R} - p_\tau[(r+c) - p_\tau\, \mathbf{c}],$$

while the average per-period dynamic lending payoff is

$$\frac{2\overline{R} - p_\tau r_\emptyset - p_\tau^2 r_1 - p_\tau(1-p_\tau)r_0}{2} = \overline{R} - p_\tau\left[\frac{(r_\emptyset + r_0)}{2} - p_\tau \frac{(\mathbf{r_0} - \mathbf{r_1})}{\mathbf{2}}\right].$$

All average payoffs (including the lender's) resulting from dynamic contract $(r_\emptyset, r_1, r_0)$ can be replicated by group contract $r = (r_\emptyset + r_1)/2$ and $c = (r_0 - r_1)/2$. Conversely, any group contract (r, c) can be replicated by dynamic contract $r_\emptyset = 2r$, $r_1 = 0$, and $r_0 = 2c$. Both contracts give payoffs quadratic in risk-type, and with two types, two contract instruments can position the quadratic so as to eliminate any cross-subsidy and attract all borrowers.

Thus, the contracts do not differ in information revelation and, unconstrained, in ability to tailor payoffs by risk-type. All differences come from constraints. As it turns out, group lending is less constrained in its ability to target discounts to safe borrowers. To see this,

[33] Thus the lender's Bayesian posterior assessment that a borrower is safe, e.g., would be the same after an individual succeeded in both projects under a dynamic contract as after both group members succeeded under a group contract ($\frac{(1-\theta)p_s^2}{\theta p_r^2 + (1-\theta)p_s^2}$). In general, its assessment would also be the same after k successes out of two in an individual, dynamic contract as in a static, two-person group contract.

note that the bracketed terms in the above payoffs can be considered the "effective interest rate" under each type of contract.[34] Further, embedded in each of the effective interest rates is a "discount" term (in bold), multiplying risk-type and thus differentially enjoyed by safe borrowers: c for group lending, $(r_0 - r_1)/2$ for dynamic lending.[35] Both contracts are constrained in how high this discount can be. With group lending, the monotonicity constraint forces $c \leq r$. With dynamic lending, the monotonicity constraint forces $r_1 \geq 0$ and the limited commitment constraint keeps $r_0 \leq \hat{r}_s$; the overall discount is then bounded by $\hat{r}_s/2$. It turns out that the dynamic lending bound is more restrictive, and thus the dynamic contract has a harder time targeting discounts to safe borrowers.

In sum, the need to keep safe borrowers in the market over time critically limits the amount by which the lender can vary the interest rate to target discounts to safe borrowers. When "penalty" rates need to attract unlucky safe borrowers, they cannot be too severe.

Corollary 1 implies that under parameter configurations where group lending cannot achieve full efficiency, dynamic lending cannot either. However, under group contracts, failure to achieve full efficiency implies market breakdown, while under dynamic contracts "nearly" efficient lending may still be possible. Indeed, comparing conditions 8 and 11, some algebra shows that $\mathcal{B}^*_{1,2} < \mathcal{B}_{2,1}$ iff[36]

$$p_r < \frac{(2 - p_s)[\theta + (1 - \theta)p_s]}{1 + \theta + (1 - \theta)p_s} \,. \tag{A4}$$

This gives rise to

Corollary 2. *Under assumptions A1-A4 and provided $\mathcal{G}$ is high enough, there exists a non-empty interval $[\mathcal{B}^*_{1,2}, \mathcal{B}_{2,1})$ such that for $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{2,1})$, two-period dynamic individual*

[34]The effective interest rate is the analog to the interest rate from a static individual loan contract, where the payoff is $\overline{R} - p_\tau r$.

[35]Thus, under the group contract the safe borrower's effective interest rate is lower than the risky's by $(p_s - p_r)c$: the safe borrower has a safer partner (by $p_s - p_r$) so, conditional on success, is less likely to owe c. The comparable figure under the dynamic contract is $(p_s - p_r)(r_0 - r_1)/2$: the safe borrower is more likely to succeed (by $p_s - p_r$) and get an interest rate discount $(r_0 - r_1)$ on half its loans.

[36]A sufficient condition for inequality A4 is $p_r \leq 1/2$.

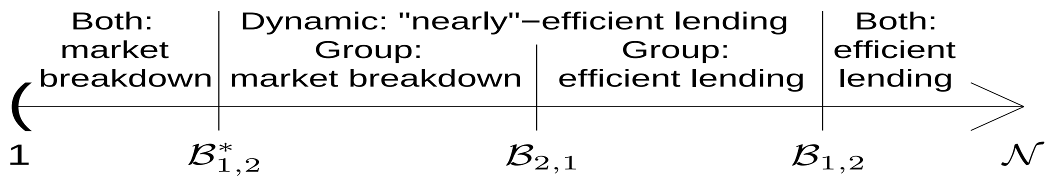

Figure 3: Comparison of group lending with dynamic lending under Assumption A4.

lending achieves "nearly"-efficient lending while two-person static group lending fails to avert market breakdown.

We thus find that dynamic lending can achieve greater efficiency than group lending, but only when it prices failed safe borrowers out of the market to lower the new-borrower rate. Giving up on failed safe borrowers allows the lender to vary the interest rate more greatly based on record, thus targeting greater discounts to safe borrowers.

In general – see Figure 3 – the efficiency comparison between group and dynamic lending is ambiguous, and neither style of lending dominates in every parameter configuration. Where net returns are relatively high, i.e. $\mathcal{N} \in [\mathcal{B}_{2,1}, \mathcal{B}_{1,2})$, group lending can achieve efficient lending while dynamic lending cannot, while under assumption A4 and with lower net returns, i.e. $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{2,1})$, dynamic lending is nearly efficient while group lending is inefficient.

5.3 Factors that alter the comparison

Beyond the basic point that no single contract form always dominates, the model can give guidance about what tilts the comparison in favor of one or the other type of contract. One obvious point, mentioned above, is that different assumptions are required for each type of contract. Group lending assumes good local information and low-friction, purposeful matching. Realistically, these are only available in degrees, and may be especially lacking in more anonymous urban contexts.[37] Dynamic lending assumes commitment by the lender to adjust rates in a pre-announced way based on past history. This may be easier said

[37] In favor of this assumption in one context, Ahlin (2009) rejects random risk-matching among rural/semi-rural microcredit groups in Thailand, in the direction of homogeneous risk-matching.

than done, especially when outreach is the main goal – lenders may not credibly price many borrowers out of the market, and they may not follow through with large discounts. Also, borrowers' need for capital may not extend over several periods, so that future loans cannot be used to price for risk after information has been revealed.

The above discussion also brings out an interesting parallel between group lending and dynamic lending: the analog between size of group in group lending and number of periods in a dynamic lending relationship, both of which determine the amount of information revelation. Specifically, a k-person group contract reveals k draws from the distribution of a borrower's type (under homogeneous matching), as does a k-period dynamic contract. One might conjecture that larger groups and longer contracts reveal more information and are better able to price for efficiency. While a formal analysis is outside the scope of this paper,[38] this suggests that contextual limits on either group size or relationship length would work against one or the other type of contract.

Correlated risk also affects information revelation. Consider serial correlation. In the extreme case of perfect correlation, a two-period dynamic contract gives the lender not two draws from the borrower's distribution, but one. The dynamic contract is no better than a static one. Consider spatial correlation. Again under perfect correlation, a two-person group contract gives the lender only one draw from the borrower's distribution, and the group contract is no better than an individual contract. While a detailed analysis is beyond the scope of the paper, it appears that serial correlation works against dynamic lending while spatial correlation works against group lending. Plausibly, spatial correlation is more prevalent in agricultural endeavors, while serial correlation could be more relevant to small enterprise start-ups, if early shocks to a firm have long-term effects.

Finally, the level of gross payoffs, $\mathcal{G}$, can alter the comparison. The above results assume $\mathcal{G}$ is large enough so that the best-case contracts can be offered. If $\mathcal{G}$ is small, however, the

[38] Ahlin (2012) analyzes group size under static group lending and finds that larger groups generally allow for more efficient lending. Fully efficient lending is attainable for any $\mathcal{N} > 1$, under relatively moderate assumptions on $\mathcal{G}$, as long as group size is large enough. (This may no longer hold if the quality of local information deteriorates with group size.) Section 6 states a similar conjecture for relationship duration.

contracts must be modified: joint liability c must be reduced in the group case, and period-2 discounts for success must be reduced in the dynamic case (r_1 raised). Several points can be made while stopping short of a detailed analysis. First, under assumptions A1-A3, it is straightforward to show that limited liability is more binding in the efficient dynamic contract than in the group contract: i.e. the first-period interest rate in the efficient dynamic contract, $r_\emptyset$, is higher than the maximum obligation under the group contract, $r+c$. Thus, affordability concerns work in favor of group lending when we restrict attention to fully efficient lending. Second, the affordability comparison is ambiguous when we compare efficient group lending with *nearly*-efficient dynamic lending. One can show that under assumption A4, which implies that for $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{2,1})$ nearly-efficient dynamic lending is achievable while group lending fails, affordability can swing in favor of dynamic lending. There can exist values of $\mathcal{G}$ such that for $\mathcal{N}$ in a neighborhood *above* $\mathcal{B}_{2,1}$, dynamic lending dominates group lending because full liability is not affordable under group lending. In sum, affordability concerns can also tilt the comparison toward dynamic contracts.

6 Extensions

Dynamic group contracts. Of course, if conditions are met for both group lending and dynamic lending to be feasible – dynamic endowments, local knowledge, frictionless matching, lender commitment – there need not be a dichotomy between the two types of contracts.

Making both sets of assumptions and restricting attention to pooling contracts, we find that the optimal two-person, two-period dynamic group contract can achieve fully efficient lending over a larger parameter space than either dynamic individual lending or static group lending: $1 < \mathcal{B}_{2,2} < \mathcal{B}_{2,1}, \mathcal{B}_{1,2}$. The dynamic group contract gives the lender more information (cross-sectional and time series) and more instruments to improve risk-pricing.

Optimal contract structure, i.e. the one that minimizes the cross-subsidy from safe to risky, is a hybrid of group and dynamic lending: full joint liability on every loan, free loans after the first one is repaid (whether via a bailout or not[39]), and a more expensive loan after period-1 default. Again, terms get better over time. The dynamic aspect of the contract works against homogeneous risk-matching,[40] but does not overturn it.

More than two periods. As discussed in section 5.3, longer relationships allow for more information revelation and more opportunity for the lender to improve risk-pricing. We have not fully solved the T-period problem, but our conjecture is that efficient lending can be achieved over more of the parameter space the larger is T, and can be achieved for any $\mathcal{N} > 1$ for T and $\mathcal{G}$ large enough. This analysis is left for future work, including resolving whether the requirements for $\mathcal{G}$ are plausible.

Competition. The contracts derived assume a single non-profit lender. This was a common scenario in microcredit markets around the world, but for-profit lending and multiple lenders are on the rise as microcredit becomes more commonplace. A competitive environment raises the issue of whether a borrower's repayment history is known to outside lenders, but this issue turns out to be unimportant here.

The dynamic individual contracts derived are in most ways robust to competition. The two-period contracts are zero-profit by construction. Given back-loading, the period-2 loans are typically loss-making: certainly the free loan after success, and also the loan after failure in the fully efficient contract under assumption A3. Thus, no competitor would want to poach clients in these cases, even if repayment histories were public.

However, competition does have a significant impact, for two reasons. First, there is a profit-making loan in the case of nearly-efficient lending, when the lender lends to risky borrowers after failure at their reservation rate. Extracting all the risky borrowers' surplus at this history would clearly not survive competition; in a competitive market, risky bor-

[39]The contract would (typically) want to charge the high, "penalty" rate in the period after one partner bails out another, since this episode is (typically) more prevalent among risky borrowers. However, a group would then prefer to report two successes, pay $2r$, and enjoy a cheap period-2 loan rather than admit one failure, pay the same amount $2r$ (due to full liability), and face future "penalty" rates in addition. The dynamic monotonicity constraint then forces the contract to soften dynamic pricing or joint liability (or both), and joint liability survives as the more effective approach.

[40]See also Guttman (2008).

rowers could always get loans at the one-shot competitive full-information rate, by declaring themselves risky. Competition would thus force the lender to lower the "penalty" rate to the break-even level, $r_0 = \rho/p_r$, and to raise the initial rate, $r_\emptyset$. As a result, safe borrowers would be harder to attract. The cutoff $\mathcal{B}^*_{1,2}$ would rise, but still remain below $\mathcal{B}_{1,2}$. In terms of the comparison, dynamic lending would still dominate group lending under some parameters, but a stronger assumption on p_r than A4 would be required.

Second, competition would eliminate the fully efficient dynamic contract as an equilibrium, under assumption A3, even for values of $\mathcal{N}$ where it is possible under a single non-profit lender. This is because a competitor lender could offer the nearly-efficient contract (modified as discussed in the previous paragraph) and attract safe borrowers away, since they prefer the nearly-efficient contract. This contract is profitable for a lender that attracts only safe borrowers, since they repay more often. With only risky borrowers left in the efficient contract, it is no longer viable, since lenders lose money on risky borrowers. Thus, competition leaves the best contract for safe borrowers, the nearly-efficient contract, as the only equilibrium – full efficiency will no longer be attainable when static individual lending breaks down.

Risk Aversion. Risk aversion would change optimal contracting in two main ways. First, borrowers would disprefer variation in interest rates over time, so the lender would have to trade off varying interest rates by revealed risk against keeping borrower payoffs smooth. Risk aversion would thus work against the improvements in risk-pricing that are otherwise achievable. Second, pooling contracts would no longer be optimal. The risky borrower could be given a contract that gives him the same utility as the safe borrower's contract, but with less risk – this would result in greater revenue for the lender, which could then be used to lower interest rates. Risk aversion would similarly affect group lending, hampering risk pricing,[41] so it is not clear without further analysis how the comparison between the two contracts would be affected.

[41]See Stiglitz (1990). However, risk aversion may not adversely impact group lending if groups are very large; see Ahlin (2012).

7 Conclusion

When borrower types are unknown but fixed, dynamic loan contracts can effectively draw safe borrowers into the market. However, they can be limited in their effectiveness by borrowers' ability to drop out, which constrains the lender's ability to price for risk as information is revealed over time. As a result, initial loans from a lender will tend to come with higher rates, while repeat loans will often be cheaper – as in "relationship lending".

Dynamic loan contracts share some remarkable similarities in this context to the group lending contracts popularized by the microcredit movement. There is a close connection between number of periods in a dynamic contract and number of borrowers per group in a group contract – both determine the amount of information revelation. However, the contracts operate under different constraints governing the use of information. And, while the two kinds of contracts require different assumptions to be met, neither dominates the other in terms of overall ability to revive a dormant credit market, providing increased intermediation and lower interest rates. The theory is thus consistent with dynamic lending playing a role of similar magnitude to that of group lending in the recent unprecendented rise in financial intermediation among the poor.

The paper also points to features of the economic or institutional environment that might tip the balance in favor of one or the other kind of lending contract. Taking these predictions to data is left for future work.

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Proof of Lemma 1. One can derive the reservation interest rate in period 1 given r_1 and r_0, call it $\hat{\hat{r}}_\tau$ for a type-τ agent, from the condition $\Pi_\tau(\hat{\hat{r}}_\tau, r_1, r_0) = 2\overline{u}$. Using payoff 5 and the definition of $\hat{r}_\tau$ (see equation 2), $\hat{\hat{r}}_\tau$ can be written

$$\hat{\hat{r}}_\tau = 2\hat{r}_\tau - p_\tau \min\{r_1, \hat{r}_\tau\} - (1 - p_\tau)\min\{r_0, \hat{r}_\tau\} \ .$$

We then have

$$\begin{aligned}\hat{\hat{r}}_r - \hat{\hat{r}}_s = \ & 2(\hat{r}_r - \hat{r}_s) + [p_s \min\{r_1, \hat{r}_s\} - p_r \min\{r_1, \hat{r}_r\}] \\ & - [(1 - p_r)\min\{r_0, \hat{r}_r\} - (1 - p_s)\min\{r_0, \hat{r}_s\}] \ .\end{aligned}$$

It can be verified that the bracketed term on the first line is positive, and that the bracketed term on the second line is positive and maximized at $r_0 = \hat{r}_r$. It follows that

$$\hat{\hat{r}}_r - \hat{\hat{r}}_s \geq 2(\hat{r}_r - \hat{r}_s) - (1 - p_r)\hat{r}_r + (1 - p_s)\hat{r}_s = \hat{r}_r - \hat{r}_s + p_r\hat{r}_r - p_s\hat{r}_s = \hat{r}_r - \hat{r}_s > 0 \ .$$

Thus, $\hat{\hat{r}}_s < \hat{\hat{r}}_r$, so if at some contract $(r_\emptyset, r_1, r_0)$ safe borrowers choose to borrow in period one ($r_\emptyset \leq \hat{\hat{r}}_s$), so do risky ($r_\emptyset \leq \hat{\hat{r}}_r$). ■

Proof of Proposition 1. Consider the claim for $\mathcal{N} \in [\mathcal{B}_{1,2}, \mathcal{B}_{1,1})$. The text derives a contract ($r_1 = 0$, $r_0 = \hat{r}_s$, $r_\emptyset$ from the ZPC) that satisfies most constraints, has the lender at zero profits, and funds all projects iff $\mathcal{N} \geq \mathcal{B}_{1,2}$, where $\mathcal{B}_{1,2} \in (1, \mathcal{B}_{1,1})$. Thus, if the remaining constraints are satisfied, maximal borrower surplus is attainable (along with full efficiency) for $\mathcal{N} \in [\mathcal{B}_{1,2}, \mathcal{B}_{1,1})$. The ignored constraints were limited liability after success and the dynamic monotonicity constraint 4. The dynamic monotonicity constraint boils down to $r_\emptyset \geq p_s\hat{r}_s = \overline{R} - \overline{u}$, which can be shown to hold under assumption A3. Limited liability after success is satisfied in period two, where the maximum interest rate is $\hat{r}_s$, which can be shown affordable for all borrowers. The potentially binding limited liability constraint is the affordability of $r_\emptyset$, which by assumption A1 is more binding for safe borrowers:

$$R_{p_s} \geq r_\emptyset \qquad \Longleftrightarrow \qquad \mathcal{G} \geq \frac{2p_s - \overline{p(1-p)}\mathcal{N}}{\overline{p}}.$$

The second inequality multiplies the first by p_s/ρ and uses equation 9 for $r_\emptyset$. This constraint is tighter the lower is $\mathcal{N}$, so if

$$\mathcal{G} \geq \mathcal{C}_{1,2} \equiv \frac{2p_s - \overline{p(1-p)}\,\mathcal{B}_{1,2}}{\overline{p}},$$

and[42] $\mathcal{N} \in [\mathcal{B}_{1,2}, \mathcal{B}_{1,1})$, fully efficient lending is achieved.

Consider next the claim for $\mathcal{N} \in (1, \mathcal{B}^*_{1,2})$. The text shows that the contract best for safe borrowers, assuming all agents borrower in period one and subject to most constraints, succeeds in attracting them iff $\mathcal{N} \geq \mathcal{B}^*_{1,2}$, where $\mathcal{B}^*_{1,2} \in (1, \mathcal{B}_{1,2})$. It follows that there is no way to include safe and risky borrowers in period one if $\mathcal{N} \in (1, \mathcal{B}^*_{1,2})$. (Adding the

[42] One can show that when $\mathcal{N} \in [\mathcal{B}_{1,2}, \mathcal{B}_{1,1})$, $[2p_s - \overline{p(1-p)}\mathcal{N}]/\overline{p} > \mathcal{B}_{1,1}(> \mathcal{N})$, so the constraint is not automatically satisfied, i.e. requires $\mathcal{G} > \mathcal{N} \Leftrightarrow \overline{u} > 0$.

omitted constraints could only weakly lower the maximum achievable safe payoff.) Hence, by Lemma 1, only risky borrowers can be included in period one, and thus in period two.

Finally, consider the claim for $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{1,2})$. The text derives a "nearly-efficient" contract ($r_1 = 0$, $r_0 = \hat{r}_r$, $r_\emptyset$ from the ZPC) that satisfies most constraints, has the lender at zero profits, and includes all safe borrowers except after failure (and all risky borrowers) if $\mathcal{N} \geq \mathcal{B}^*_{1,2}$, where $\mathcal{B}^*_{1,2} \in (1, \mathcal{B}_{1,2})$. Thus, if the remaining constraints are satisfied and if no higher borrower surplus is achievable for $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{1,2})$, then this contract is optimal and involves everyone borrowing except safe borrowers after failure.

First, is higher borrower surplus achievable? If so, a contract would exist that included failed safe borrowers, since all other sources of borrower surplus are exhausted by the nearly-efficient contract. But, the best contract for safe borrowers that includes them after failure – and by Lemma 1, would include all borrowers in period one – is the full efficiency contract, which fails to include safe borrowers when $\mathcal{N} < \mathcal{B}_{1,2}$. Thus, there is no way to include failed safe borrowers for $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{1,2})$, and thus no way to improve borrower surplus.

Second, are the remaining constraints satisfied? Limited liability after success is satisfied in period two, where the only non-zero rate is $\hat{r}_r$, faced only by risky and affordable for them. Again, the potentially binding limited liability constraint is the affordability of $r_\emptyset$:

$$R_{p_s} \geq r_\emptyset \qquad \Longleftrightarrow \qquad \mathcal{G} \geq \frac{p_s[2 - (1-\theta)(1-p_s) - \theta(1-p_r)\mathcal{N}]}{\overline{p}}.$$

This constraint is tighter the lower is $\mathcal{N}$, so if

$$\mathcal{G} \geq \mathcal{C}^*_{1,2} \equiv \frac{p_s[2 - (1-\theta)(1-p_s) - \theta(1-p_r)\mathcal{B}^*_{1,2}]}{\overline{p}},$$

limited liability is satisfied for all $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{1,2})$.[43] Finally, consider the dynamic monotonicity constraint 3, which given the contract boils down to

$$r_\emptyset \geq p_r\hat{r}_r = p_s\hat{r}_s = \overline{R} - \overline{u} \qquad \Longleftrightarrow \qquad \mathcal{N} \leq \frac{2 - (1-\theta)(1-p_s)}{1 - (1-\theta)(1-p_s)},$$

where the second inequality divides the first by ρ and uses equation 7 for $r_\emptyset$. This constraint is not always satisfied in this range of $\mathcal{N}$. If not, the contract would have to be modified to satisfy dynamic monotonicity, but not in any consequential way, because it fails exactly when $\mathcal{N}$ is high. The following modification would work: raise $r_\emptyset$ and lower r_0 along the ZPC until the dynamic monotonicity constraint binds, i.e. until $r_\emptyset = \overline{R} - \overline{u}$. The new contract involves $r_\emptyset = \overline{R} - \overline{u}$, $r_1 = 0$, and r_0 from the ZPC. It is straightforward to verify that both safe and risky borrowers choose to borrow in this case, and that this modified contract satisfies all constraints, including the limited liability constraint since $R_{p_s} > \overline{R} \geq \overline{R} - \overline{u} = r_\emptyset$. It is also clear that r_0 will still be higher than $\hat{r}_s$,[44] so the contract continues to include all borrowers except failed safe ones. With the bank at zero profits, borrower surplus is unchanged since

[43] Under some parameter values and for $\mathcal{N}$ high enough in this range, this constraint is not binding, i.e. is satisfied for any $\overline{u} \geq 0$ (i.e. $\mathcal{C}^*_{1,2} \leq \mathcal{N}$). But for $\mathcal{N}$ near $\mathcal{B}^*_{1,2}$, it requires $\mathcal{G} > \mathcal{N} \Leftrightarrow \overline{u} > 0$.

[44] This is clear since if not, this contract would achieve full efficiency, which we have shown is impossible for $\mathcal{N}$ in this range; it can also be verified algebraically.

the same agents borrow; the only difference is a shift in surplus from safe to risky borrowers. In summary, imposing the dynamic monotonicity constraint may shift surplus but does not change the Proposition's efficiency conclusions. ■

Proof of Proposition 2. A menu of contracts now involves several parameters, r^τ_σ and x^τ_σ, where σ represents the state/history and $\tau \in \{r, s\}$ represents the type. There are four histories, $\sigma \in \{\emptyset_1, 0, 1, \emptyset_2\}$: the null histories for first-time borrowers in periods one and two, and repeat borrowers in period two having 0 or 1 success. For each state and type, r^τ_σ represents the amount due after success from a loan taken by that type at that history, and x^τ_σ the amount due after failure. Both r^τ_σ and x^τ_σ can be negative. Thus, we are considering contract menus described by $(r^s_{\emptyset_1}, r^r_{\emptyset_1}, x^s_{\emptyset_1}, x^r_{\emptyset_1}, r^s_1, r^r_1, x^s_1, x^r_1, r^s_0, r^r_0, x^s_0, x^r_0, r^s_{\emptyset_2}, r^r_{\emptyset_2}, x^s_{\emptyset_2}, x^r_{\emptyset_2})$.

First, we show that the borrower surplus attainable by any menu involving first-time borrowing in period two can be achieved by a menu without first-time borrowing in period two. This will imply that menus that induce first-time borrowing in period two can be ignored without loss.

By the revelation principle we restrict attention to menus that induce each type to accept its intended contract. Consider now a menu that induces safe agents to borrow for the first time in period two, involving $r^s_{\emptyset_2}$ and $x^s_{\emptyset_2}$. Next consider substituting a different contract for safe borrowers that charges $\widetilde{r^s_{\emptyset_1}} = r^s_{\emptyset_2}$ and $\widetilde{x^s_{\emptyset_1}} = x^s_{\emptyset_2}$, as well as $\widetilde{r^s_\sigma} = \infty$ and $\widetilde{x^s_\sigma} = 0$ for $\sigma \in \{0, 1, \emptyset_2\}$. (Also fine would be $\widetilde{r^s_\sigma} = 2\hat{r}_r$, $\sigma \in \{0, 1, \emptyset_2\}$.) This gives the same payoff to safe agents – one loan at $r^s_{\emptyset_2}$ and $x^s_{\emptyset_2}$, one dropout period – so safe will choose the modified safe-borrower contract and borrow in period one only. It also ensures that risky borrowers do not switch to the new safe contract – the new safe contract gives one loan at $r^s_{\emptyset_2}$ and $x^s_{\emptyset_2}$ plus one dropout period, while the original safe contract, which was not preferred by risky to their own, gave one loan at $r^s_{\emptyset_2}$ and $x^s_{\emptyset_2}$ plus one period doing no worse than dropping out. Lender profits are also clearly unchanged. In sum, the alternate contract achieves the same borrower payoffs: safe agents borrowing in exactly one period at the same rates, risky choices and payoffs unchanged. Thus, the borrower surplus achieved by the original menu can be achieved by a menu that does not allow first-time borrowing by safe in period two.

The same argument shows that first-time period-2 borrowing by risky can also be ignored. Thus, without loss we can ignore menus that induce first-time borrowing in period two. An admissible menu becomes $(r^s_{\emptyset_1}, r^r_{\emptyset_1}, x^s_{\emptyset_1}, x^r_{\emptyset_1}, r^s_1, r^r_1, x^s_1, x^r_1, r^s_0, r^r_0, x^s_0, x^r_0)$.

Second, we show that there is no loss of generality in setting $x^\tau_\sigma = 0$ when choosing a menu of contracts to maximize the safe-borrower payoff subject to most of the imposed constraints – all except limited liability after success and dynamic monotonicity – *plus* risky borrower incentive compatibility.

Define $\kappa^{\tau'|\tau}_\sigma \equiv p_\tau r^{\tau'}_\sigma + (1 - p_\tau) x^{\tau'}_\sigma$ as the expected payment of a type-τ agent with a loan intended for type τ' at history σ. The two-period payoff of a type-τ agent who borrows – if and when optimal – under the contract intended for type τ' can be written $2\overline{R} - Z^{\tau'|\tau}$, where $Z^{\tau'|\tau} = \min\{M^{\tau'|\tau}, 2p_\tau\hat{r}_\tau\}$ and

$$M^{\tau'|\tau} = \kappa^{\tau'|\tau}_{\emptyset_1} + p_\tau \min\{p_\tau\hat{r}_\tau, \kappa^{\tau'|\tau}_1\} + (1 - p_\tau) \min\{p_\tau\hat{r}_\tau, \kappa^{\tau'|\tau}_0\} \ .$$

Here $M^{\tau'|\tau}$ is the two-period expected payment when the agent takes a loan in period one:

the expected payment for a loan at state $\emptyset_1$; if successful the choice between a loan at state 1 or dropping out, which is equivalent to a loan at rate $\hat{r}_\tau$; and if not, the choice between a loan at state 0 or dropping out. Also reflected in $Z^{\tau'|\tau}$ is the option not to borrow at all, which is payoff-equivalent to facing two loans at rate $\hat{r}_\tau$.

Consider the problem of maximizing the safe-borrower payoff, i.e. minimizing $Z^{s|s}$, subject to the imposed constraints: limited liability after failure, $x^\tau_\sigma \leq 0$, $\forall \sigma \in \{\emptyset_1, 0, 1\}, \tau \in \{r, s\}$; period-2 monotonicity, $x^\tau_\sigma \leq r^\tau_\sigma$, $\forall \sigma \in \{0, 1\}, \tau \in \{r, s\}$; the risky IC constraint, $Z^{r|r} \leq Z^{s|r}$; and the ZPC, which can be written, letting μ_τ be the measure of type-τ agents,

$$0 \leq \sum_{\tau \in \{r,s\}} \mu_\tau \mathbf{1}\{M^{\tau|\tau} \leq 2p_\tau \hat{r}_\tau\} \cdot \left[\kappa^{\tau|\tau}_{\emptyset_1} - \rho + p_\tau \mathbf{1}\{\kappa^{\tau|\tau}_1 \leq p_\tau \hat{r}_\tau\}(\kappa^{\tau|\tau}_1 - \rho) + (1 - p_\tau)\mathbf{1}\{\kappa^{\tau|\tau}_0 \leq p_\tau \hat{r}_\tau\}(\kappa^{\tau|\tau}_0 - \rho)\right] .$$

Note that adjusting x^τ_σ up and r^τ_σ down keeping $\kappa^{\tau|\tau}_\sigma = p_\tau r^\tau_\sigma + (1 - p_\tau)x^\tau_\sigma$ fixed does not alter $Z^{s|s}$, $Z^{r|r}$, or the ZPC. This adjustment may affect $Z^{s|r}$, but if so it raises it, since

$$\frac{dx^s_\sigma}{dr^s_\sigma}\bigg|_{\overline{\kappa^{s|s}_\sigma}} = -p_s/(1 - p_s) < -p_r/(1 - p_r) = \frac{dx^s_\sigma}{dr^s_\sigma}\bigg|_{\overline{\kappa^{s|r}_\sigma}}.$$

Thus, without loss x^τ_σ can be set at their upper bounds defined by limited liability or monotonicity. The intuition is that minimizing the subsidy for failure minimizes the temptation for the risky borrower to pick the safe-borrower contract.

Next, consider x^τ_0 at some optimum. If $r^\tau_0 \geq 0$, then $x^\tau_0 = 0$ (the limited liability upper bound); otherwise, $x^\tau_0 = r^\tau_0 < 0$ (the monotonicity upper bound). In the latter case, consider lowering $x^\tau_{\emptyset_1}$ by $|x^\tau_0|$ and raising x^τ_0 and r^τ_0 to zero. This leaves $Z^{s|s}$, $Z^{r|r}$, $Z^{s|r}$, and the ZPC unchanged, and continues to satisfy the imposed limited liability and monotonicity constraints. Thus, without loss $x^\tau_0 = 0$. Similarly, $x^\tau_1 = 0$ without loss – if it were negative, then $x^\tau_1 = r^\tau_1 < 0$, but these could be raised to zero and $r^\tau_{\emptyset_1}$ lowered by the same amount without changing any key terms or violating any constraints. Finally, consider $x^\tau_{\emptyset_1}$. The argument of the previous paragraph established that without loss, it is at its upper bound, which is zero from limited liability.

In sum, for purposes of maximizing the safe borrower payoff subject to the above-mentioned constraints, we can safely assume $x^\tau_0 = x^\tau_1 = x^\tau_{\emptyset_1} = 0$. For these purposes, then, the menu of contracts simplifies to $(r^s_{\emptyset_1}, r^r_{\emptyset_1}, r^s_1, r^r_1, r^s_0, r^r_0)$.

Third, we show that a simple pooling contract can do as well as any menu of contracts in maximizing the safe payoff subject to the above imposed constraints. Fix a menu of contracts $(r^s_{\emptyset_1}, r^r_{\emptyset_1}, r^s_1, r^r_1, r^s_0, r^r_0)$ that maximizes the safe payoff subject to these constraints.

Consider two subcases, 1) in which at the optimum safe borrowers do not borrow in period one (and hence not in period two), and 2) in which at the optimum safe borrowers borrow in period one. In case 1), we know that at the optimum $Z^{s|s} = 2p_s\hat{r}_s$. One way to accomplish this is $r^\tau_\sigma = \hat{r}_r$ for all $\sigma \in \{\emptyset_1, 0, 1\}$ and $\tau \in \{r, s\}$. Thus, imposing that risky and safe have identical contracts is without loss in case 1) for maximizing safe-borrower payoffs.

In case 2), we argue that the risky borrower IC constraint must bind. If the opposite were true, then $Z^{r|r} < Z^{s|r} \leq 2p_r\hat{r}_r$, that is risky agents borrow in period one. But then the lender could raise $r^r_{\emptyset_1}$ and lower $r^s_{\emptyset_1}$ along the ZPC, lowering $Z^{s|s}$ without violating any constraint. Thus, in case 2) we must have $Z^{r|r} = Z^{s|r}$. We also argue that without loss, $r^\tau_\sigma \leq \hat{r}_r$ for $\tau \in \{r, s\}$ and $\sigma \in \{0, 1\}$. If not, i.e. if $r^\tau_\sigma > \hat{r}_r$ for some $\tau \in \{r, s\}$ and $\sigma \in \{0, 1\}$, consider lowering it to $\hat{r}_r$. One can show this does not affect $Z^{s|s}$, $Z^{r|r}$, $Z^{s|r}$, and the imposed limited liability and monotonicity constraints. Iff $\tau = r$, it can affect the ZPC, but then only positively, because the drop in interest rate attracts risky borrowers in period two and raises more money for the lender. Combining this period-2 interest rate cap of $\hat{r}_r$ with the binding IC constraint, $Z^{r|r} = Z^{s|r}$, and dividing both sides by p_r, we have

$$\min\{2\hat{r}_r, r^r_{\emptyset_1} + p_r r^r_1 + (1-p_r) r^r_0\} = \min\{2\hat{r}_r, r^s_{\emptyset_1} + p_r r^s_1 + (1-p_r) r^s_0\} \ .$$

Define

$$Q^{r|r} \equiv r^r_{\emptyset_1} + p_r r^r_1 + (1-p_r) r^r_0 \qquad \textit{and} \qquad Q^{s|r} \equiv r^s_{\emptyset_1} + p_r r^s_1 + (1-p_r) r^s_0 \ ;$$

thus $\min\{2\hat{r}_r, Q^{r|r}\} = \min\{2\hat{r}_r, Q^{s|r}\}$. The ZPC simplifies to

$$\theta \cdot \mathbf{1}\{Q^{r|r} \leq 2\hat{r}_r\}(p_r Q^{r|r} - 2\rho) + (1-\theta) \cdot [\ldots] \geq 0 \ .$$

The first term represents risky borrowers' expected total payment minus capital costs, if they borrow ($Q^{r|r} \leq 2\hat{r}_r$), while the second term represents safe borrowers and depends solely on the contract through r^s_σ terms, which are omitted for brevity.

Continuing case 2), we next show that $Q^{s|r} < 2\hat{r}_r$. Since $\min\{2\hat{r}_r, Q^{r|r}\} = \min\{2\hat{r}_r, Q^{s|r}\}$, this will guarantee that $Q^{s|r} = Q^{r|r}$. Note that since safe agents borrow in period one, by assumption, then $Z^{s|s} = p_s Q^{s|s}$, where

$$Q^{s|s} \equiv r^s_{\emptyset_1} + p_s \min\{\hat{r}_s, r^s_1\} + (1-p_s) \min\{\hat{r}_s, r^s_0\} \ .$$

Also, $Z^{s|s} \leq 2p_s\hat{r}_s$. Thus, $Q^{s|s} \leq 2\hat{r}_s$. Note that

$$\begin{array}{rcccccc} Q^{s|r} - Q^{s|s} = & p_r r^s_1 - p_s \min\{\hat{r}_s, r^s_1\} & + & (1-p_r) r^s_0 - (1-p_s)\min\{\hat{r}_s, r^s_0\} & & \\ \leq & 0 & + & (1-p_r)\hat{r}_r - (1-p_s)\hat{r}_s & = & \hat{r}_r - \hat{r}_s \ . \end{array}$$

The inequality is because the terms involving r^s_1 are together non-positive (for $r^s_1 \in [0, \hat{r}_r]$), while the terms involving r^s_0 are together maximized (over $[0, \hat{r}_r]$) at $r^s_0 = \hat{r}_r$. Thus,

$$Q^{s|r} \leq Q^{s|s} + \hat{r}_r - \hat{r}_s \leq \hat{r}_r + \hat{r}_s < 2\hat{r}_r \ .$$

Combining this with the earlier analysis, we have that $Q^{s|r} = Q^{r|r}$.

Now consider an alternate menu that simply replaces the risky contract with the safe one: $r^r_{\emptyset_1} = r^s_{\emptyset_1}$, $r^r_1 = r^s_1$, $r^r_0 = r^s_0$. Since $Q^{s|r} = Q^{r|r}$ under the original menu, the risky payoff does not change with the new menu (equals $2\overline{R} - p_r Q^{r|r}$ under the original and $2\overline{R} - p_r Q^{s|r}$ under the new). Again since $Q^{s|r} = Q^{r|r}$, inspection of the ZPC above gives that the lender's profits do not change.[45] Safe payoffs do not change, and all other imposed constraints are

[45] This is a key statement of the proof, and it relies on the lender's and borrower's payoffs being diametrically opposed. This implies there is no loss in putting the risky borrower on his IC constraint exactly where

satisfied.

Wrapping up, we have shown that a simple pooling contract does as well as any other menu of contracts at maximizing *safe borrower payoffs* subject to "most constraints" – all except limited liability after success and dynamic monotonicity – plus risky borrower incentive compatibility. Since a simple pooling contract also satisfies safe borrower incentive compatibility, we can say more: a simple pooling contract does as well as any other *incentive compatible* menu of contracts – and by the revelation principle, as well as any menu of contracts, incentive compatible or not – at maximizing safe payoffs subject to "most constraints".

Now consider maximizing *borrower surplus* subject to "most constraints". Clearly when $\mathcal{N} \in [\mathcal{B}_{1,2}, \mathcal{B}_{1,1})$, no menu of contracts can do better than the best simple pooling contract subject to these constraints, since the latter achieves maximal borrower surplus, as shown in the proof of Proposition 1. When $\mathcal{N} \in (1, \mathcal{B}^*_{1,2})$, no simple pooling contract can give safe agents high enough payoffs to induce them to borrow, as we have shown. No menu of contracts can attract safe borrowers either, for to do so it would need to provide a higher payoff for safe borrowers than any simple pooling contract – a contradiction. When $\mathcal{N} \in [\mathcal{B}^*_{1,2}, \mathcal{B}_{1,2})$, no menu of contracts can improve surplus, relative to the "nearly-efficient" simple pooling contract. To raise surplus, the menu would need to attract failed safe borrowers in period two, the only source of borrower surplus not exhausted by the nearly-efficient contract. But one can then repeat the above third part of the proof and show that no menu of contracts satisfying the same constraints *plus* $r^s_0 \leq \hat{r}_s$ can give safe borrowers higher payoffs than the simple pooling contract that maximizes safe-borrower payoffs subject to the same constraints *plus* $r^s_0 \leq \hat{r}_s$, which is the fully-efficient contract derived in the text. Thus for $\mathcal{N}$ in this range, no menu that has a chance at delivering higher efficiency than the nearly-efficient contract can provide high enough payoffs for safe borrowers to include them, since the fully-efficient simple pooling contract cannot.

Summarizing, no menu of contracts satisfying all constraints except dynamic monotonicity and limited liability after success produces higher borrower surplus than the optimal simple pooling contract satisfying the same constraints. Now consider imposing dynamic monotonicity and limited liability after success. The Proof of Proposition 1 establishes that if $\mathcal{G}$ is high enough, these constraints do not alter the borrower surplus attainable by simple pooling contracts. This guarantees that no menu of contracts can achieve higher borrower surplus than the optimal simple pooling contract, as long as $\mathcal{G}$ is high enough. ■

the safe borrower is, since anywhere on the IC constraint is equally good for both risky borrower and lender.