Delegation and Nonmonetary Incentives[*]

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Abstract

In many contracting settings, actions costly to one party but with no direct benefits to the other (money-burning) may be part of the explicit or implicit contract. Some examples include bureaucratic procedures in an employer-employee relationship, activists boycotting corporations, or commitment devices for time-inconsistent consumers. We study a model of delegation with an informed agent, where the principal may impose money-burning on the agent as a function of the agent's choice of action, and show that money-burning may be part of the optimal contract. This result holds even if action-contingent monetary transfers are possible, as long as transfers from the principal to the agent are bounded from below (as in limited liability or minimal wage requirements). In fact, the optimal contract can involve a combination of both efficient monetary incentives and inefficient nonmonetary incentives through money burning. Our model delivers some results novel to the delegation literature. First, money-burning is more likely if the principal is more "sensitive" to the choice of action than the agent. This is consistent with the perception that there is more bureaucratization in large organizations. Second, money-burning is more likely if the agent's limited liability constraint is tighter relative to his participation constraint. This implies that a higher minimum wage distorts employment contracts towards using socially wasteful nonmonetary incentives, leading to a Pareto inferior outcome as the agent is still held down to his reservation value through increased money burning.

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1 Introduction

In many economic and political interactions, the use of monetary incentives is ruled out or limited. For example, explicit monetary incentives for politicians are often ruled out and considered unethical: members of the legislation receive salaries that compensate them financially for their work, but their payments do not depend on how they vote, or how many amendments they make. Similar restrictions apply to organ donation or allocating courses to students within a school. It is even more common that in an economic relationship financial incentives are possible, but they are bounded from some direction. For example, a minimum wage requirement bounds monetary transfers from employers to employees from below. Another common source of limitations on monetary transfers is that one or both parties might be liquidity-constrained. For this reason, cash penalties are hardly ever used in employment contracts, except for employees with very large incomes relative to the damage that their actions can cause.[1] Despite these restrictions on transfers, nonmonetary incentives are often available, and thus may be used to align the interests of participants. In many settings these incentives take the form of imposing activities that are costly for one agent and do not directly benefit any of the rest. Following the standard terminology, we refer to such activities as money burning.

To study such situations formally, we consider a scenario where an uninformed principal delegates the task of choosing the action from a unidimensional action space to an agent, who before making the decision receives private information about a state variable. The state variable affects the well-being of both parties in a way that a higher state is associated with a higher optimal action choice for both of them. As standard in the literature, we assume that at any state the agent's preferred action is higher than the principal's. The principal delegates by offering the agent a contract which for any possible action prescribes a nonnegative amount of money burning the agent has to make if he takes that action. We investigate this contracting problem in two different contexts: first, we assume monetary transfers conditional on the action choice are completely ruled out (only an ex ante wage payment is possible); second, we add the possibility of such transfers. Throughout the paper we assume that the principal has to satisfy a participation constraint for the agent, and that the amount of monetary transfer from the principal to the agent is bounded from below.[2] The latter can come from a nonnegativity constraint on transfers to the agent, liquidity constraints on the part of the agent, or a minimum wage requirement.

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[1] See p.249 of Milgrom and Roberts (1992).

[2] In some applications it is reasonable to drop one of these constraints, which corresponds to the limiting case of our model in which the corresponding lower bound is $-\infty$.

The model we describe is applicable to many different situations. One important example is organizations, where a primary form of providing such incentives is bureaucracy, typified by formal processes, standardization, hierarchic procedures and written communication. If managers of organizational units are biased towards requesting a higher budget for their units than what would be optimal for the organization, a manager applying for a higher budget might be required to fill out more paperwork, or get stamps of approval from different offices, which might require going to the headquarters and waiting in line for hours. In fact, it is a common perception that there is an excess of bureaucratic procedures. According to a recent study released by *MeaningfulWork.com*, the number one complaint workers had about their job was "too much workplace bureaucracy" (see Leonard (2000)).[3]

A different application of the model is an activist group targeting a corporation for violations of environmental or human rights standards. Suppose that the corporation has private information about the cost of reducing pollution and chooses its emissions accordingly. An activist can only observe the emissions and choose the intensity of the campaign (say, scope and duration of a boycott) accordingly; such actions directly hurt the corporation but are of no immediate benefit to the activists.[4] Baron and Diermeier (2007) study a model where the corporation may be one of two types, and the activist commits to a contract which promises a boycott (costly to the corporation but not to the activist) in case it refuses to meet its demands. In a recent paper, Abito et al. (2011) investigate a similar model, although not through a contract theory approach as our paper. Another paper that examines a dynamic game with the possibility of money burning outside the delegation/contract theory framework is Padro i Miquel and Yared (2012). They examine a game between a government and an enforcement agent, in which the government can exert force in each period that is costly for both participants, as a disciplinary device providing incentives. Our model, which in contrast assumes that the principal can ex ante commit to a contract that prescribes a money burning scheme, provides an alternative approach to investigate this situation.[5]

Within the delegation literature, the papers closest to ours are Athey et al. (2004), and

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[3]Indeed, most of the related economic literature takes this stance, looking for explanations for excessive bureaucratization as in Strausz (2006), or connect bureaucracy and corruption as in Banerjee (1997) and Guriev (2004). In contrast, in this paper we suggest that bureaucratic procedures, as well as other costly and wasteful activities, can improve the (ex-ante) efficiency of an organization.

[4]Large-scale campaigns may have long-term reputational benefits for activists, but at first approximation boycott may be thought of as money-burning.

[5]There is a complementary literature suggesting that signaling, even if costly for individual agents, may improve economic efficiency in allocation problems. For example, Chakravarty and Kaplan (2013) show that allocation of goods without transfers may be more efficient if agents are able to send costly signals to the mechanism designer (see also Condorelli, 2012, and Yoon, 2011; the latter paper allows for the possibility that part of money-burning is not socially wasteful and is thus a form of transfer).

Amador et al. (2006). Athey et al. (2004) examine collusion in a repeated price-setting oligopoly game, through a symmetric contract that in particular can specify triggering a price war costly to all participants. Their stationary environment allows them to transform the problem to a static optimal contracting situation with the possibility of money-burning. Amador et al. (2006) investigates optimal savings contracts for time-inconsistent consumers, who also face preference or liquidity shocks, in a two-period model.[6] In this situation the ex-ante self (or a social planner) is the principal, and the present-biased ex-post self is the agent. The action is first-period consumption, and money burning corresponds to throwing away some of the budget. The above papers feature models that are similar to ours, but they impose restrictions that ultimately imply that money burning is not part of the optimal contract. In contrast, our paper suggests that the use of money burning can be widespread in many different types of situations. Moreover, in a setting canonical in the literature (quadratic utilities and uniform distribution over the state space) we analytically characterize optimal contracts that involve money burning, and examine when this is the case.[7]

Although, as argued above, our model is applicable in many different contexts, for ease of exposition in the remainder of the paper our terminology is based on an employer-employee relationship.

Our analysis reveals that money-burning is more likely to be used as an incentive device when the principal's utility, measured in monetary terms, is more sensitive to the implemented action than the agent's utility. This is consistent with the common perception that bureaucracy is more widespread in large organizations, as the agent's choices typically influence the well-being of more other agents (represented by the principal). To the best of our knowledge, ours is the first paper studying the effect of relative preference intensity of the principal and the agent on the optimal contract in the delegation literature.

Second, we find that money-burning is more likely to arise when there are more stringent restrictions on monetary transfers between the contracting parties, such as a higher minimum wage or other forms of wage control, and when outside option of the agent is low. Intuitively, if the minimum wage is so high or the outside option is so low that the agent is strictly willing to be employed, then the principal will have no problem using money-burning as an incentive device; we show, however, that money-burning is possible even if participation constraint is binding. This is interesting per se, but it also demonstrates an additional distortionary effect of increasing the minimum wage, besides the ones commonly discussed in policy debates. Namely,

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[6]See also Athey et al. (2005).

[7]In a more recent paper Amador and Bagwell (2012), building on our results, also address cases when the optimal contract involves money burning.

an increase in minimal wage makes it more likely that employers use socially inefficient non-monetary incentives, rather than efficient monetary incentives. This is because increasing the minimum wage (relative to the outside option) relaxes the participation constraint of an agent, hence the principal can cut down on positive (monetary) incentives, and in the meantime increase the negative (nonmonetary) incentives. The latter leads to two sources of inefficiency: a direct effect coming from wasteful money burning itself, and an indirect effect of distorting the implemented policy towards the principal's ideal point, relative to the socially optimal action. In particular, increasing minimum wage may lead to a Pareto inferior outcome, in which the agent is held down to the same reservation utility as before, with the increase in wage being offset by more money burning imposed, whereas the principal is strictly worse off.

To the best of our knowledge this last result regarding connecting the minimum wage and socially wasteful activities is novel in the literature, albeit Wessels (1980a, 1980b) makes an observation that is a counterpart of our point: If a firm can provide fringe benefits to its workers in a socially efficient way (namely the benefit for the workers is larger than the cost for the firm), for example by maintaining better working condiations, then an increase in the minimum wage can induce the firm to cut down in these socially efficient fringe benefits. Both the latter papers and our work points out a substitution effect between monetary and nonmonetary compensation schemes, effecting social surplus.

Our paper proceeds as follows. We first show some basic general results, such as existence of the optimal contract, that the implemented action scheme is monotonically increasing in the state, and that the action specified in the contract is never below the optimal action of the principal (no undershooting). Under some regularity conditions, we also show that both money burning and the action choice are continuous functions of the state, and that the implemented action is always between the optimal points of the principal and the agent. Under the same regularity conditions we transform the principal's problem to an intuitive and tractable form. We demonstrate the usefulness of this result by explicitly solving the transformed problem in the popular uniform-quadratic specification of the delegation model, with the extra parameter capturing the relative importance of the action chosen for the principal and the sender (in monetary terms).[8] We show that this parameter and the agent's outside option relative to the minimum bound on transfers are the two crucial parameters determining whether there is money burning in the optimal contract.

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[8]For motivation, consider for example a case in which the principal represents the government, so that the chosen action greatly influences the well-being of citizens, and the bias of the agent results from small private benefits from the chosen action, then the action choice matters much more for the principal, reflected by a large parameter value.

In the version of the model that allows for monetary transfers, we show that the optimal contract can involve transfers only, or money burning only, or transfers in some states and money burning in other states, providing an explanation for why inefficient nonmonetary incentives are used even in settings where financial incentives are feasible. If the agent's outside option is high enough, there is no wasteful money burning, and the optimal contract achieves jointly efficient action choices. In this case, the principal and the agent essentially form a partnership. If the agent's outside option is very low, the agent receives exactly the minimum wage in each state, but money burning may be used. For an intermediate range of parameters, both positive and negative incentives can be used in the optimal contract, in a way that monetary transfers are used in low states, while money burning is used in high states. The intuition for this is that monetary transfers have to be decreasing in the state, while money burning has to be increasing in the state, provided that the regularity conditions hold. This makes monetary transfers a relatively expensive incentive device in high states (since it increases the required monetary transfers in all lower states), and money burning a relatively expensive incentive device in low states (since it increases money burning in all higher states, for which the agent needs to be compensated).

Lastly, we note that often the principal may choose between different costly activities to provide incentives to the agent, and some of these may well provide benefits to the principal. For example, more paperwork imposed on workers provides a more precise documentation of their activities, which may help the principal in the future. In this paper, we assume that the activity imposed on the agent is purely wasteful. However, all of the qualitative conclusions of the model would generalize to a setting in which the principal benefits from these activities, as long as this activity implies some total efficiency loss.

2 Related literature

Our work continues the literature on constrained delegation started by Holmstrom (1977).[9] Holmstrom, as well as Melumad and Shibano (1991) and Alonso and Matouschek (2007, 2008), considers deterministic delegation with no monetary transfers, in which the principal can restrict the action space of the agent, but cannot make different actions differentially costly.[10] In our framework the principal can always achieve such delegation schemes by setting some actions free

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[9]Dessein (2002) considers delegation in which restricting the agent's action space is not allowed, but the principal can potentially retain a veto power. See also Aghion and Tirole (1997) and Szalay (2005) for models of delegation less related to ours. There is also a literature in political science on delegation and control: see, for example, Bendor et al. (1987), and McCubbins et al. (1987).

[10]For a recent more detailed description of this line of literature, see Armstrong and Vickers (2010).

while the remaining ones prohibitively costly. This means that the principal has a larger set of feasible contracts and hence he is at least weakly better off.

Kováč and Mylovanov (2007) and Goltsman et al. (2009) investigate stochastic delegation mechanisms in the constrained delegation context, assuming that the principal and the agent have quadratic utility functions.[11] There is a mathematical connection between these works and our paper, for the following reason. Quadratic utilities imply that the utilities of both parties are additively separable to a term that only depends on the expectation of the induced action and another term that only depends on the variance of the induced action. In particular the latter variance term enters negatively in both parties' utility functions. In our model, money burning is only a direct cost for the agent. However, through the participation constraint, money burning is also costly for the principal, by increasing the amount of ex ante transfer. This exact correspondence between noise and money burning specified in the contract breaks down if the preferences are not quadratic: the effect of noise on the incentives and ex ante utility of the participants becomes complicated, while the money-burning term in our model is still additively separable, influencing the parties ex ante utilities symmetrically.

Ottaviani (2000), Krahmer (2004), and Krishna and Morgan (2008) investigate delegation with monetary transfers, although either not characterizing the optimal contract, or not incorporating a participation constraint for the agent. Since an important component of our model is the trade-off between the benefits of creating incentives and the costs that this induces, which operates through the participation constraint, this makes the results of the above papers difficult to compare to ours.[12] Koessler and Martimort (2012) consider a delegation problem in a 2-dimensional policy space, which is somewhat analogous to our 2-dimensional setting with money burning and a one-dimensional policy choice. However, their results are completely different than ours, in that the induced action in their model can never coincide with the agent's ideal point, and that there cannot be pooling in the optimal contract among different states.[13]

A major alternative of delegation is cheap talk communication between the informed and

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[11]Stochastic delegation implies that the principal can commit to different probabilistic action choices after different reports by the agent. That is, the agent cannot determine the action any more, but she can choose among probability distributions of actions after observing the state. See also Strausz (2006) on stochastic mechanisms in a principal-agent setting less related to ours.

[12]See the end of Section 4 for a partial comparison with the results in Krishna and Morgan (2008). The main qualitative difference is that while positive incentives (monetary transfers) are always used to some extent in the optimal contract, negative incentives (money burning) might be a too costly incentive device for the principal, and hence not used at all in the optimal contract - even when monetary transfers are completely ruled out.

[13]These differences stem from the assumptions Koessler and Martimort impose on the agents' preferences on the policy space, which in particular imply that an agent would always trade off increasing bias in a dimension where bias is small to decreasing bias in a dimension where bias is large. This drives the policy in the optimal contract away from the agent's bliss point. In our model money burning is typically too costly of an incentive device for the principal to use it to keep the optimal policy away from the agent's ideal point in every state.

the uninformed parties, as in Crawford and Sobel (1982) and a large literature building on it. In a cheap talk game the uninformed party cannot commit to let the informed party to choose an action (from a set of available choices), therefore her action choice is required to be sequentially rational. As opposed to this, delegation, or letting the informed party take an action, is equivalent to the uninformed party being able to commit to a message-contingent action scheme. The closest papers to our work in this literature are Austen-Smith and Banks (2000) and Kartik (2007), who consider communication with money burning by the informed party. In essence, these papers can be viewed as the "signaling" versions of our "screening" model. The focus of these papers is very different from ours: they investigate how money burning can expand the set of cheap talk equilibria in the Crawford and Sobel model.

The existing literature on both delegation and cheap talk only considers incentive compatibility constraints. A conceptual contribution of the current paper is incorporating a participation constraint for the agent.

The formal literature on procedural rules and organizational bureaucracy, despite its practical importance, is relatively scarce and not directly related to our paper.[14] At an informal level, Walsh and Devar (1987) claim that procedural rules might have a positive effect on administrative efficiency and organizational effectiveness because they provide a set of role expectations and reduce uncertainty, while Wilson (1989) argues that complex rules and regulations are imposed on bureaucracy to reduce discretion in order to restrict corruption.

3   The basic model

In this Section we set up the basic model, in which the principal can set costly procedural rules for the agent, but contingent monetary transfers are not possible. For the extension of the model which allows for contingent transfers, see Section 6.

We consider the following principal-agent problem. There is an uninformed principal, and an informed agent who observes the realization of a random variable $\theta \in \Theta = [0, 1]$. From now on we will refer to θ as the state. The c.d.f. of θ is $F(\theta)$, and we assume it has a density function f that is strictly positive and absolutely continuous on $[0, 1]$. The principal in our model *delegates* decision-making, hence the agent has to choose an action $y \in Y = [y_L, y_H]$, after observing the state. Both the state and the action affect the well-being of both parties. We assume that both the principal and the agent are von Neumann and Morgenstern expected utility maximizers. If action y is chosen at state θ, then the principal and the agent get utilities $u^p(\theta, y) = -l^p(\theta, y)$, while the corresponding utility for the agent is given by $u^a(\theta, y) = -l^a(\theta, y)$. We refer to l^p

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[14]See Tirole (1986), Garicano (2000), Prendergast (2007) and Crémer et al. (2007).

and l^a as the loss functions of the principal and the agent, and we assume that both functions are twice continuously differentiable and strictly convex in y. We assume that for fixed θ, $u^p(\theta, y)$ reaches its maximum value 0 at $y^p(\theta) = \theta$, while $u^a(\theta, y)$ reaches its maximum value 0 at $y^a(\theta) = \theta + b(\theta)$ for some $b(\theta) > 0$. We refer to $y^p(\theta)$ and $y^a(\theta)$ as the ideal points of the principal and the agent at state θ, and to $b(\theta)$ as the bias of the agent at state θ. We assume that Y contains the interval $[0, 1 + b(1)]$. We also assume the single-crossing condition $\frac{\partial^2 l^a(\theta, y)}{\partial \theta \partial y} < 0$; this implies, in particular, that $\theta + b(\theta)$ is continuous and strictly increasing.[15] Finally, we assume that all parameters of the model are commonly known to the two parties involved.

So far the model is just the standard workhorse model of the delegation literature, that builds on the framework provided in Crawford and Sobel (1982). The novel features of the model are the following:

(i) The principal can impose costs on the agent which may depend on his choice of action. Formally, the principal can specify a function $m : Y \to \mathbb{R}^+$. For any $y \in Y$, $m(y)$ is a non-recoverable loss for the agent, which does not directly affect the principal's utility, and we interpret it as the amount of paperwork needed to pick policy y. Following standard terminology for purely wasteful activities, we refer to $m(y)$ as the amount of money burning required when choosing action y. Money burning enters the agent's utility as a cost, in an additively separable manner. We note that delegation with differential costs encompasses standard delegation agreements considered in the existing literature, where the principal restricts the set of available policies for the agent to $D \subset Y$: in our framework this could be replicated by setting $m(y)$ to be zero if $y \in D$, and $m(y)$ to be prohibitively high if $y \in Y \setminus D$. Hence, a principal who can set differential costs is at least weakly better off than a principal who can only choose a set of feasible actions for the agent.

(ii) The principal has to hire the agent by offering an acceptable contract. We assume that contracting happens ex ante, i.e., before the agent observes the state. The contract specifies the cost function m (interpreted as the description of the paperwork requirements for all possible actions), and a constant transfer payment T (interpreted as a wage) that enters the agent's utility function in an additively separable manner. In our basic model, we assume that monetary transfers contingent on either θ or y are not possible (in Section 6 we relax this requirement). The agent has an outside option $\tilde{u}$, therefore we assume that he accepts any contract that gives

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[15] We therefore rule out the possibility that the agent's ideal point is state-independent. It is not unrealistic (the agent might always prefer to exert zero effort), but in this paper, we focus on the case where the interests of the principal and the agent are somewhat aligned. We want to thank an anonymous referee for pointing out that in this case, too, an optimal contract may involve money burning if the agent's willingness-to-pay depends on the state even if his ideal action does not.

him at least this much expected utility, given the ex ante distribution of θ. We assume that the unconditional transfer T must satisfy $T \geq \tilde{w}$ (which may or may not be a binding constraint). We allow $\tilde{w}$ to be either positive or negative: $\tilde{w} \leq 0$ would correspond to the case where the agent has liquidity constraints, with $\tilde{w} = 0$ meaning that no transfer from the agent is allowed, while $\tilde{w} > 0$ would naturally model a minimal wage requirement.

4 Properties of the optimal contract

In this section we derive some qualitative features of the optimal contract. We first establish properties that hold for the most general specification of the model that we introduced above. Then we derive additional properties that require certain regularity conditions on the loss functions and the prior distribution of states to hold.

We start the analysis by writing the delegation problem in the direct mechanism interpretation:[16] the principal's task is to set a transfer T and a pair of measurable functions $y(\theta)$, the action that the agent takes in state θ, and $m(\theta)$, the amount of paperwork or money burning in this state, that solve the following problem:

$$\max_{T,\{y(\theta),m(\theta)\}_{\theta \in \Theta}} \int_{\Theta} u^p(\theta, y(\theta)) \, dF(\theta) - T \tag{1a}$$

$$\text{s.t.} \quad \int_{\Theta} (u^a(\theta, y(\theta)) - m(\theta)) \, dF(\theta) + T \geq \tilde{u}, \tag{1b}$$

$$\forall \theta, \theta' \in \Theta : u^a(\theta, y(\theta)) - m(\theta) \geq u^a(\theta, y(\theta')) - m(\theta'), \tag{1c}$$

$$\forall \theta \in \Theta : m(\theta) \geq 0, \tag{1d}$$

$$T \geq \tilde{w}. \tag{1e}$$

In other words, the principal maximizes his payoff subject to the agent's individual rationality and incentive compatibility constraints (equations (1b) and (1c), respectively), as well as exogenous constraints on money burning and transfers.

First we observe that in an optimal contract either the agent's participation constraint (1b) binds, or the transfer is minimal, so (1e) binds: otherwise the principal could reduce the ex-ante transfer without violating the participation constraint (and not affecting the IC constraints) and achieve a higher expected payoff. Denote the total principal's loss from the contract by L^p and

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[16]Standard arguments establish that any deterministic mechanism is equivalent to a direct deterministic mechanism of the form described below the statement. As discussed above, in this paper we only consider deterministic delegation.

the agent's loss, conditional on θ, by $L^a(\theta)$:

$$L^p(y(\cdot), m(\cdot), T) = \int_\Theta l^p(\theta, y(\theta)) \, dF(\theta) + T, \tag{2}$$

$$L^a(\theta) = L^a(\theta, y(\theta), m(\theta)) = l^a(\theta, y(\theta)) + m(\theta). \tag{3}$$

We find it convenient to rewrite the problem in the following way (we denote the principal's loss from contract $(y(\cdot), m(\cdot))$ by $L^p(y(\cdot), m(\cdot))$:

$$\min_{T, \{y(\theta), m(\theta)\}_{\theta \in \Theta}} L^p(y(\cdot), m(\cdot), T) = \min_{T, \{y(\theta), m(\theta)\}_{\theta \in \Theta}} \int_\Theta l^p(\theta, y(\theta)) \, dF(\theta) + T \tag{4}$$

$$\text{s.t.} \int_\Theta (l^a(\theta, y(\theta)) + m(\theta)) \, dF(\theta) \leq T - \tilde{u}, \tag{5}$$

$$\forall \, \theta, \theta' \in \Theta : l^a(\theta, y(\theta)) + m(\theta) \leq l^a(\theta, y(\theta')) + m(\theta'), \tag{6}$$

$$\forall \theta \in \Theta : m(\theta) \geq 0, \tag{7}$$

$$T \geq \tilde{w}. \tag{8}$$

Our model is not a standard principal-agent model with monetary transfers, since money burning enters parties' utility functions differently than monetary transfers. Furthermore, as opposed to most principal-agent models, the agent's ideal action is a nontrivial function of the state.[17] For these reasons, existing results from the above literature cannot be directly used in our setting. However, some basic results can be derived in an analogous manner to the standard model. Claims 1–3 states these results. Since the proofs are straightforward and similar to proofs of analogous results in the literature, they are relegated to the Supplementary Appendix.[18]

Claim 1 *If a pair of functions* $\{y(\theta), m(\theta)\}_{\theta \in \Theta}$ *satisfies* (6), *then* $\theta_2 \geq \theta_1$ *implies* $y(\theta_2) \geq y(\theta_1)$. *Moreover, if* $y(\theta_1) = y(\theta_2)$, *then* $m(\theta_1) = m(\theta_2)$.

Claim 2 *If there exists a solution to the problem* (4)-(8) *then there is a solution* $\{y^*(\theta), m^*(\theta)\}_{\theta \in \Theta}$ *such that* $\inf_{\theta \in \Theta} m^*(\theta) = 0$.

Claim 3 *If a pair of functions* $\{y(\theta), m(\theta)\}_{\theta \in \Theta}$ *satisfies* (6), *then for agent's loss function* $L^a(\theta)$ *the following is true:*[19]

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[17] For this reason, the optimal contract that we derive below does not satisfy some standard results like "no distortion at the top." The agent never chooses his ideal action for $\theta = 1$, and it is possible that he does not choose his ideal action for any θ.

[18] We remark that one additional difficulty that we face, relative to the most standard principal-agent problems is that the $m(\cdot) \geq 0$ condition is difficult to translate into a condition on payoffs. For this reason we cannot follow the standard approach of solving for the optimal contract in payoffs space.

[19] We could apply Theorem 1 in Milgrom and Segal (2002) to obtain absolute continuity of $L^a(\theta)$. However, we need the stronger results that require a separate proof.

(i) $L^a(\theta)$ is Lipschitz continuous with parameter $\Delta_\theta = \max_{\theta \in \Theta, y \in Y} \left| \frac{\partial l^a(\theta, y)}{\partial \theta} \right|$.

(ii) $L^a(\theta)$ has left derivative for each $\theta_0 > 0$ and has right derivative for each $\theta_0 < 1$, given by:

$$\frac{d^l L^a(\theta_0)}{d\theta} = \frac{\partial l^a(\theta_0, \lim_{\theta \to \theta_0-} y(\theta))}{\partial \theta},$$
$$\frac{d^r L^a(\theta_0)}{d\theta} = \frac{\partial l^a(\theta_0, \lim_{\theta \to \theta_0+} y(\theta))}{\partial \theta}.$$

(iii) $L^a(\theta)$ is differentiable at $\theta_0 \in (0,1)$ if and only if $y(\theta)$ is continuous at θ_0, and then

$$\frac{dL^a(\theta_0)}{d\theta} = \frac{\partial l^a(\theta_0, y(\theta_0))}{\partial \theta}. \text{[20]} \tag{9}$$

Claim 3 implies that the IC constraint (6) pins down the amount of money-burning in each state $m(\theta)$ given an increasing function $y(\theta)$, up to a constant (which is itself pinned down by the condition $\inf_{\theta \in \Theta} m(\theta) = 0$). The reformulation (11) below is similar to known results in settings different from ours (see for example Appendix 1 in Jullien (2000) or Lemma 1 in Noldeke and Samuelson (2006)).[21] The proof of this and subsequent results are in the Appendix. Take a pair of functions $(y(\theta), m(\theta))_{\theta \in \Theta}$ satisfying (6) and define function $\tilde{m}(y)$ as the amount of money required to burn when action $y \in R$ is chosen (where R is the range of $y(\theta)$) by:

$$\tilde{m}(y) = m(\theta) \text{ where } \theta \in \Theta \text{ satisfies } y(\theta) = y; \tag{10}$$

notice that $\tilde{m}(\cdot)$ is well-defined (this follows from Claim 1). We now define $\tilde{\theta}(\cdot)$ as the inverse of $y(\cdot)$ if $y(\cdot)$ is continuous and strictly increasing; otherwise, we let $\tilde{\theta}(\cdot)$ be any monotone single-valued function $[y(0), y(1)] \to \Theta$ such that $y\left(\tilde{\theta}(y_0)\right) = y_0$ for any $y_0 \in R$.[22]

Claim 4 If $(y(\theta), m(\theta))_{\theta \in \Theta}$ satisfies (6), then for any $\hat{y} \in [y(0), y(1)]$,

$$\tilde{m}(\hat{y}) = \tilde{m}(y(0)) + \int_{y(0)}^{\hat{y}} \left(-\frac{\partial l^a\left(\tilde{\theta}(y), y\right)}{\partial y} \right) dy. \tag{11}$$

Conversely, if $y(\cdot) : \Theta \to Y$ is a monotone function and $\tilde{m}(y)$ satisfies (11), then $(y(\theta), m(\theta))_{\theta \in \Theta} = (y(\theta), \tilde{m}(y(\theta)))_{\theta \in \Theta}$ holds for (6).

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[21] Our analysis here is also similar to that in Goldman et al. (1984), who examine a context in which a monopolist can offer a menu of price-quantity pairs. Quantity (q) plays an analogous role to y in our model, while price (R) does to m in our model. Their $R(q)$ function is similar to $\tilde{m}(y)$ in our setting. However, both the assumptions they impose and the type of results they derive differ from ours, hence their results are not applicable in our investigation.

[22] Monotonicity on R must follow from Claim 1; to achieve monotonicity on the entire $\tilde{\theta}(y)$, we define $\tilde{\theta}(y)$ for $y \in [y(0), y(1)] \setminus R$ by $\tilde{\theta}(y) = \sup\{\theta : y(\theta) < y\} = \inf\{\theta : y(\theta) > y\}$. This last part is not necessary if $y(\cdot)$ is continuous, and thus $R = [y(0), y(1)]$.

This formula holds both for continuous actions schemes and for ones with jumps; in the latter case, for the purpose of integration, we pretend that there is a vertical segment connecting the two sides of the jump, and $\tilde{\theta}(y)$ is constant. It is obvious that money burning is (weakly) increasing as long as the prescribed action stays below the optimal point of the agent (that is, if there is no overshooting). This result has a convenient graphical representation when the agent has a quadratic utility function. In this case, $-\frac{\partial l^a(\tilde{\theta}(y),y)}{\partial y} = 2\left(\tilde{\theta}(y) + b - y\right)$, therefore the change in the amount of money burning is proportional to the area between the ideal points curve of the agent and the actions scheme (with negative sign if the action scheme increases above the agent's ideal curve $y = \theta + b$), as illustrated by the next figure.
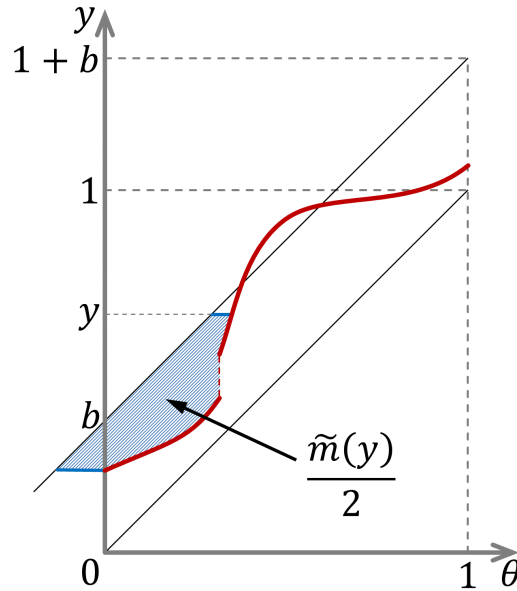

Figure 1: Representation of money burned as an integral

The problem is thus reduced to finding a monotone action scheme $y(\theta)$, as then $m(\theta)$ will be uniquely defined by (11) (also using $\inf_{\theta \in \Theta} m^*(\theta) = 0$ implied by Claim 2). This implies the following.

Theorem 5 *There exists a solution to problem* (4)-(8).

This result follows from Theorem 2.2 in Balder (1996).[23] It is not immediately applicable as T is not taken from a compact set. However, clearly, very high T is suboptimal, and by restricting T from above we can make this result applicable (see the proof in the Appendix for the formal argument).

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[23] An earlier draft of the paper contained an explicit proof of existence for this model and is available from the authors upon request.

We next establish some qualitative properties of the optimal contract. The first one, that the implemented action is never below the ideal point of the principal, holds in general. However, the other two, namely that the implemented policy is a continuous function of the state and that the implemented action is never above the ideal point of the agent, hold only under additional regularity conditions, discussed below. Most importantly for us, if there is no overshooting $(y^*(\theta) \leq \theta + b(\theta))$, then money-burning has to be monotonically increasing in the state. This follows from Claim 4 and means that it is enough for us to ensure that $m(0) \geq 0$. (If there is overshooting, then money-burning is *not* monotonically increasing in the state, provided that $y^*(\theta)$ is non-constant.) In the Supplementary Appendix we provide an example which shows that without the regularity conditions, the latter properties need not hold in the optimum. Intuitively, if money-burning has to be monotone, then it is very costly for the principal to impose it in low states, because he will have to increase money-burning everywhere. Overshooting in an interval of intermediate states would allow him to decrease money-burning in higher states. This sacrifices ex-post utility in the intermediate states, but decreases the level of money-burning in higher states. In the example we provide, the density function of the state takes high values in low and high states, but low values for an intermediate range of states. This violates regularity conditions and features overshooting in the optimum.

Theorem 6 *Suppose $(y^*(\cdot), m^*(\cdot))$ solves the problem (4)-(8) and $y^*(\cdot)$ is continuous at $\theta = 0$ and $\theta = 1$.*[24] *Then:*

1. *For every $\theta \in \Theta$, $y^*(\theta) \geq \theta$ (i.e., there is no undershooting);*

2. *If $\frac{\partial l^p(\theta_0, y)}{\partial y} / \left(-\frac{\partial^2 l^a(\theta_0, y)}{\partial \theta \partial y}\right)$ is increasing in y on (θ_0, y) for any $\theta_0 \in \Theta$ and any $y > \theta_0$, then y^* is continuous on Θ;*

3. *If, in addition, $f(\theta)\frac{\partial l^p(\theta, \theta+b(\theta))}{\partial y} / \left(-\frac{\partial^2 l^a(\theta, \theta+b(\theta))}{\partial \theta \partial y}\right)$ is non-decreasing in θ, then for every $\theta \in \Theta$, $y^*(\theta) \leq \theta + b(\theta)$ (i.e., there is no overshooting).*

For symmetric loss functions and constant agent bias (which is assumed in most of the literature), that is when $l^p(\theta, y) = l(y - \theta)$ and $l^a(\theta, y) = l(y - \theta - b)$, the condition in part 2 simplifies to requiring that $\frac{l''(x-b)}{l'(x)}$ is decreasing in x for $x > 0$. Furthermore, for any loss function of the principal that satisfies our basic assumptions (including ones with state-dependent bias),

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[24] We can always adjust $y(\theta)$ and $m(\theta)$ for $\theta = 0, 1$ so that $y(0) = \lim_{\theta \searrow 0} y(\theta)$, $y(1) = \lim_{\theta \nearrow 1} y(\theta)$, $m(0) = \lim_{\theta \searrow 0} m(\theta)$, $m(1) = \lim_{\theta \nearrow 1} m(\theta)$, and this will preserve (6) and other constraints and will not affect the objective function as the changes are made on the set of measure 0 only. In what follows, we restrict our attention on such contracts only. However, we cannot assume that $y^*(\cdot)$ is continuous in general, as there are examples (see Supplementary Appendix) where this is not the case.

the condition is satisfied whenever the agent's loss function is quadratic, that is when $l^a(\theta, y) = A(y - \theta - b)^2$ for some $A > 0$. To see this, note that in this case the denominator in the relevant expression is constant, hence the strict convexity of l^p implies that the condition holds. This, together with the subsequent results, suggests that the important assumption for the qualitative conclusions from the popular uniform-quadratic example to remain valid is that the agent's utility function is quadratic (while the principal can have any strictly convex loss function).

A sufficient condition for the condition in part 3 to hold is that $\dfrac{\frac{\partial l^p(\theta, \theta + b(\theta))}{\partial y}}{-\frac{\partial^2 l^a(\theta, \theta + b(\theta))}{\partial \theta \partial y}}$ is non-decreasing in θ and $f(\theta)$ is non-decreasing in θ. For a constant bias b, the first condition holds automatically, therefore the condition is equivalent to the simple requirement that $f(\theta)$ is non-decreasing in θ.

We prove each part by contradiction, i.e., by assuming the contrary and finding another feasible contract which decreases principal's loss function. For Part 1, we consider increasing the action $y(\theta)$ if $y(\theta) < \theta$; this relaxes the IR constraint of the agent and decreases the loss of the principal, while simultaneously decreasing the amount of money-burning needed to keep (11) fulfilled. However, such decrease in $m(\theta)$ may violate the nonnegativity constraint (7), and we need to adjust $y(\theta)$ so that this does not happen.

The idea of the proof of Part 2 is to take a candidate optimal contract $y^*(\cdot)$ that contains a discontinuity, say from y_1 to y_2 at $\theta = \theta_0$, and making it "less discontinuous" by adding an in-between action, with an amount of money-burning that attracts a small interval of types around θ_0. This on the one hand is beneficial for the principal since types above θ_0 who choose the new action now induce an action closer to the principal's ideal point. On the other hand, types below θ_0 who choose the new action now induce an action farther away the principal's ideal point. We derive a condition for this modification of the contract to be profitable for the principal close to the limit when the interval of types attracted to the new action goes to zero (by increasing the implied money burning). It turns out that this condition always holds for strictly convex loss functions if y_1 is below the agent's ideal point (if the jump involves no overshooting). Moreover, we show that Assumption 1 is sufficient for the inequality to hold for any kind of jump. This means that by making the jump in actions more "gradual", the principal could improve her welfare, contradicting that the optimal contract involves discontinuity.

Regarding Part 3, it is straightforward to show that it is suboptimal for the principal to specify an overshooting action at state 0: a deviation lowering the prescribed action on an interval around 0 to the ideal curve of the agent would be in the common interest of the players and hence unambiguously increase the well-being of the principal. Let now θ_0 be the infimum of states with overshooting. We consider a deviation which keeps the implemented action on the agent's ideal curve for a small interval on the right of θ_0. The direct effect of this would be an

increase in the welfare of the principal, from the implemented action getting closer to her ideal point over the interval. However, this action would negate the decrease in money burning that the original contract would induce over the interval. We show that the condition in part 3 of Theorem 6 implies that for small enough deviations like the one specified above the deviation is beneficial for the principal, contradicting that the original contract is welfare-improving.

Note that Claim 4 and Part 3 of Theorem 6 together imply that money burning is monotonically increasing as a function of state θ in the optimal contract (provided that the conditions in Part 3 of Theorem 6 hold). Consequently, we may assume $m(0) = 0$. This allows us to rewrite the maximization problem, in this case, in the following simpler way. Taking into account (9), we have

$$
\begin{aligned}
\int_{\Theta} L^a(\theta) f(\theta) d\theta &= \int_0^1 \left(L^a(0) + \int_0^\theta \frac{\partial l^a(\xi, y(\xi))}{\partial \theta} d\xi \right) f(\theta) d\theta \\
&= L^a(0) + \int_0^1 \int_\xi^1 \frac{\partial l^a(\xi, y(\xi))}{\partial \theta} f(\theta) d\theta d\xi = L^a(0) + \int_0^1 \frac{\partial l^a(\theta, y(\theta))}{\partial \theta} (1 - F(\theta)) d\theta.
\end{aligned}
$$

Now $m(0) = 0$ implies $L^a(0) = l^a(0, y(0))$, and thus the optimization problem is equivalent to the following one:

$$
\min_{T, \{y(\theta)\}} \int_0^1 l^p(\theta, y(\theta)) dF(\theta) + T \tag{12}
$$

$$
\text{s.t. } l^a(0, y(0)) + \int_0^1 \frac{\partial l^a(\theta, y(\theta))}{\partial \theta} (1 - F(\theta)) d\theta \leq T - \tilde{u}
$$

$$
y(\cdot) \text{ is non-decreasing and continuous,}
$$

$$
y(\theta) \leq \theta + b(\theta)
$$

$$
T \geq \tilde{w}.
$$

The rewritten form of the optimization problem has the advantage that the incentive constraints are incorporated in a simpler integral constraint. The reformulation indicates that there can be a trade-off between decreasing the first term $l^a(0, y(0))$, which is minimized at $y(0) = b(0)$, and the second integral term, which can be minimized pointwise with the minimizing $y(0)$ being possibly strictly below $b(0)$. The trade-off is caused by the requirement that $y(\cdot)$ is non-decreasing and continuous. Intuitively, this reflects the tension between minimizing the agent's loss (from money burning and from the implemented policy being away from the agent's ideal point), which serves the purpose of decreasing the ex-ante transfer to the agent, and the principal's loss from the implemented policy being away from the principal's ideal point.

5 The optimal contract in uniform-quadratic settings

The reformulated problem is tractable enough that we can explicitly solve for the optimal contract in a class of games that are generalizations of the canonical uniform-quadratic specification of the delegation problem. We will derive economic insights from how the optimal contract and in particular money burning imposed on the agent depends on various parameter values of the problem.

Concretely, we assume that θ is distributed uniformly on $[0, 1]$ and restrict ourselves to the quadratic loss functions

$$
\begin{aligned}
l^p(\theta, y) &= A(y - \theta)^2, \\
l^a(\theta, y) &= (y - \theta - b)^2,
\end{aligned} \tag{13}
$$

where $A, b > 0$. These loss functions imply that the agent has a constant bias $b(\theta) = b$. Parameter value $A = 1$ corresponds to the uniform-quadratic example frequently used in the literature. The extra parameter A allows us to change the sensitivity of the loss function of the principal relative to the sensitivity of the loss function of the agent, independently of the size of bias. Values $A < 1$ imply that the principal is less sensitive to policy choice than the agent (the same deviation from the ideal point means a smaller loss); values $A > 1$ imply the opposite.[25] Alternatively, if we interpret the principal and the agents as firms/organizations rather than individuals, then A may be interpreted as the ratio of people in the principal organization to that in the agent organization. In other words, a large A may be thought of as a large corporation (or the public sector) employing an individual, and a small A corresponds to an individual delegating the task to a large organization. As we show below, the qualitative features of the optimal contract, including whether money burning is used in equilibrium, depend crucially on this parameter. In particular, money-burning will *always* be part of the optimal contract if $A > 1$.

Under these assumptions on the functional form, both conditions in Theorem 6 are satisfied. Therefore, the principal's problem (12) becomes:

---

[25]Martimort and Semenov (2007) introduce a similar multiplicative parameter for a policymaker in an unrelated model of lobbying.

$$\min_{\{y(\theta)\}_{\theta \in \Theta}, T} \int_0^1 A\left(y\left(\theta\right) - \theta\right)^2 d\theta + T, \tag{14}$$

$$\text{s.t. } \left(y\left(0\right) - b\right)^2 - \int_0^1 2\left(y\left(\theta\right) - \theta - b\right)\left(1 - \theta\right) d\theta \leq T - \tilde{u}, \tag{15}$$

$$y\left(\cdot\right) \text{ is non-decreasing and continuous,} \tag{16}$$

$$y\left(\theta\right) \leq \theta + b, \tag{17}$$

$$T \geq \tilde{w}. \tag{18}$$

Theorem 7 *There exists a unique solution to the problem* (14)–(18).

Existence follows from Claim 5, while uniqueness is an immediate corollary of the strict concavity of problem (14)–(18). The Kuhn-Tucker theorem then implies that there exist $\lambda, \mu \geq 0$ such that $(T, \{y(\theta)\})$ solves

$$\min_{\{y(\theta)\}_{\theta \in \Theta}, T} \int_0^1 \left(A\left(y\left(\theta\right) - \theta\right)^2 - 2\lambda\left(y\left(\theta\right) - \theta - b\right)\left(1 - \theta\right)\right) d\theta + \lambda\left(y\left(0\right) - b\right)^2 + \lambda\tilde{u} + \mu\tilde{w} + \left(1 - \lambda - \mu\right)T, \tag{19}$$

$$\text{s.t. } y\left(\cdot\right) \text{ is non-decreasing and continuous,} \tag{20}$$

$$y\left(\theta\right) \leq \theta + b. \tag{21}$$

Inspecting (19) reveals two observations. First, $\lambda + \mu = 1$ (otherwise we could increase or decrease T to increase the Lagrangian indefinitely). This implies that $\lambda \in [0, 1]$ and μ is uniquely determined from λ, so we have to deal with only one parameter λ. Second, since $A > 0$ and $\lambda \geq 0$, the optimal action schedule $y(\theta)$ depends on A, $\tilde{u}$ and $\tilde{w}$ only through the ratio λ/A (where λ is of course determined endogenously). We find it instructive to start with characterizing $y(\theta)$ for any ratio $\lambda/A \geq 0$, and then study how it (and T) changes if any of A, b, $\tilde{u}$, $\tilde{w}$ change.

The following heuristics is helpful to find the optimal contract. Consider the problem

$$\min_{\{y(\theta)\}_{\theta \in \Theta}} \int_0^1 \left(A\left(y\left(\theta\right) - \theta\right)^2 - 2\lambda\left(y\left(\theta\right) - \theta - b\right)\left(1 - \theta\right)\right) d\theta + \lambda\left(y\left(0\right) - b\right)^2.$$

It is minimized if $y(0) = b$, and for $\theta > 0$, $y(\theta) = z(\theta)$, where $z(\theta) = \theta + \frac{\lambda}{A}(1 - \theta)$. This need not be the solution to (19)–(21), because such $y(\cdot)$ need not be non-decreasing (either because $z(\cdot)$ is decreasing or because $z(0) < b$), nor it needs to satisfy (21). Nevertheless, the following is true.

Claim 8 *Suppose* $y(\cdot)$ *solves problem* (19)–(21). *Then if* θ *satisfies* $y(0) < y(\theta) < y(1)$, *then* $y(\theta) = \min\{z(\theta), \theta + b\}$.

In other words, if the agent does not take an extreme action, optimality requires him to take the action $z(\theta)$, with the additional constraint that he cannot overshoot. The reason is simple: If $y(\theta_0) < \min\{z(\theta_0), \theta_0 + b\}$, then we could increase $y(\theta)$ on some interval containing θ_0 so that the conditions (20)–(21) are preserved and (19) is decreased further, because $y(\theta)$ has become closer to $x(\theta)$ (notice that $y(0) < y(\theta)$ implies $\theta > 0$). Similarly, if $z(\theta_0) < y(\theta_0) \leq \theta_0 + b$, then $y(\theta)$ could be decreased on some interval so that (19) decreases. With this claim in mind, it is easy to charaterize the optimal contract (explicit formulas are in the Supplementary Appendix, but graphical illustrations are included below). For simplicity, we focus on the case $b < 1$.

The qualitative properties of the optimal contract crucially depend on b and λ/A. If the ratio $\lambda/A \geq 1$, which is likely if outside option is high or the principal is less sensitive to the action choice then the agent, then the contract takes the familiar "cap" form, where the agent is allowed to take any action he wants up to certain cap. Interestingly, this cap may exceed the highest possible action that the principal may prefer; the principal will do this if $\lambda > A$ to minimize the ex-ante transfer while keeping the agent willing to participate. As λ/A decreases below 1, the principal still allows the agent to choose any action at no cost as long as the action is sufficiently low; for higher actions, money-burning is required. Intuitively, when participation constraint is less of an issue, the principal may revert to money-burning to align incentives; the same is true if the principal becomes more sensitive to the action choice. If λ/A decreases further and becomes less than b, the principal never allows the agent to choose his preferred action; in this case, all actions, except for the lowest one (which is never the agent's ost preferred) require money-burning. This happens when participation constraint is easy to satisfy. These cases are depicted on Figure 2. Notice that at the extreme, when the agent's outside option is very low and even the minimal transfer makes him willing to participate in contracts with a lot of money-burning ($\lambda = 0$), the principal will simply implement his preferred schedule $y(\theta) = \theta$.
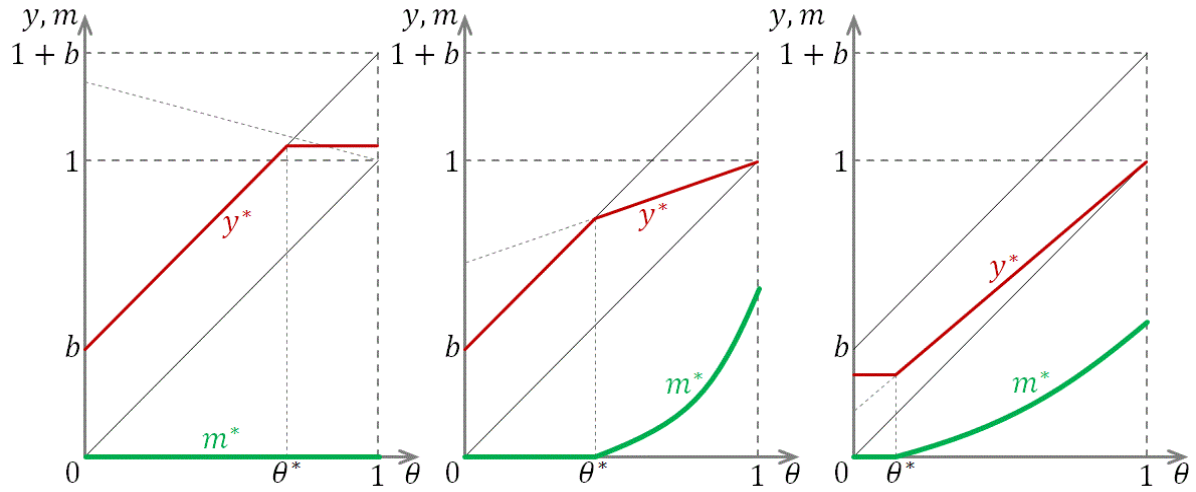
Figure 2: Optimal contracts if $b < 1$: (left) $\lambda/A > 1$, (center) $\lambda/A \in (b, 1)$, (right) $\lambda/A < b$.

The description above dealt with a fixed λ for the purpose of illustration. In fact, λ is an endogenous variable. The Theorem below provides a formal statement of the properties of the optimal contract.

The following result summarizes the comparative statics results.

Theorem 9 *Money-burning is more likely for lower b than for higher b. If the bias is not too large ($b < 1$), then the amount of money-burning (both in terms of probability that money-burning will be used and the expected amount of money-burning) is increasing in A, increasing in $\tilde{w}$, decreasing in $\tilde{u}$. Moreover, there exists $A^*(\tilde{u}, \tilde{w}) \leq 1$ that is (weakly) increasing in $\tilde{u}$ and decreasing in $\tilde{w}$ such that:*

(1) The optimal contract features money-burning if and only if $A > A^*(\tilde{u}, \tilde{w})$;

(2) For any $\tilde{u}$, there exists $\tilde{w}^* = \tilde{w}^*(\tilde{u})$ such that for $\tilde{w} \leq \tilde{w}^*$, $A^*(\tilde{u}, \tilde{w}) = 1$ and for $\tilde{w} > \tilde{w}^*$, $A^*(\tilde{u}, \tilde{w}) < 1$. This $\tilde{w}^*$ is increasing in $\tilde{u}$.

(3) If $\tilde{u}$ is low enough and $\tilde{w}$ is high enough, then $A^*(\tilde{u}, \tilde{w}) = 0$; in other words, for such $\tilde{u}$ and $\tilde{w}$, money-burning will be used for any A.

To understand the intuition, assume first that the agent has a good outside option ($\tilde{u}$ is high), and the restriction $T \geq \tilde{w}$ is not binding. Theorem 9 suggests that money-burning is used if and only if $A > 1$. Indeed, this corresponds to the situation where choosing the preferred action is more important for the principal than for the agent, and thus aligning incentives becomes more important than the need to compensate the agent for the loss from wasteful bureaucratic procedures. However, if the agent has low outside option ($\tilde{u}$ is low) or the restrictions on monetary transfers are stricter ($\tilde{w}$ is high), then the principal may use money burning under a wider

range of parameters, even if agent's utility is more important – simply because the two parties cannot increase efficiency, as the ability of the agent to compensate the principal is limited. In other words, if the agent's outside options are too few and the minimal wage requirements are too stringent, then excessive bureaucracy should be expected in all organizations.

The above result points out a downside of high minimal wage requirement (high $\tilde{w}$), all things equal. It is well known that minimal wage creates market distortions and leads to unemployment. Theorems 9 provides a new distortionary effect: if a higher minimal wage does not increase the agent's outside options and thus $\tilde{u}$ remains constant (which can be the case for example when the outside option is being unemployed), then increased bureaucratization in all organizations is the likely outcome.

It is instructive to compare these optimal contracts with the ones obtained in Krishna and Morgan (2008) – from now on KM – for the case of delegation with one-sided transfers, no IR constraint, and symmetric quadratic loss functions (corresponding to $u = -\infty$ and $A = 1$, without the possibility of money-burning). In this environment, the optimal transfer scheme sets a positive transfer to the agent when choosing low actions, and it is monotonically decreasing. This is parallel to our results that the money burning scheme specifies zero money burning at the lowest implemented action, and that it is monotonically increasing. Furthermore, the implemented action scheme is monotonically increasing in both models, with a possible cap on the highest action that can be chosen by the agent. An important qualitative difference is that while contingent monetary transfers are always used to some extent in the optimal contract, money burning might be a too costly incentive device for the principal and hence not used at all in the optimal contract. This results from the fact that as opposed to our model, there is no IR constraint in KM.[26]

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[26]Other relevant benchmarks include Melumad and Shibano (1991) and Kovac and Mylovanov (2009). Melumad and Shibano (1991) consider a communication setting without transfers and money-burning. In the setting with commitment (where the principal can commit to a certain action as a function of the agent's message, as in delegation but not cheap talk models), our models share a common particular case. If we assume uniform-quadratic setting with $A = 1$ in our model (and also that the individual rationality constraint is not binding), this would correspond to the case where parameter $a = 1$ and $k < 0$ in their model. In this case, our contract (for the interesting case $b < 1$) takes the form $y(\theta) = \min\{\theta + b, 1\}$, coinciding with their contract.

Kovac and Mylovanov (2009) do not have money-burning, but they explicitly allow for stochastic mechanisms. They assume the agent to have a quadratic utility function, which means that if the action is stochastic, the agent loses an extra term corresponding to the variance, which could be interpreted as money burning if the principal were not risk-averse as well and did not get disutility from non-deterministic contract herself. This means that the principal could do at least as well with money-burning as with stochastic contracts; moreover, once we allow money-burning, considering deterministic contracts, as we do, is without loss of generality. As expected, our contracts coincide if the principal also has a quadratic utility function with a constant bias, and $A = 1$, and there is no participation constraint. More generally, if the bias in the uniform-quadratic setting is not constant (but the parameter A still equals 1), the authors show that the optimal contract allows the agent to implement his ideal action, as long as it is above some floor and below some floor. We do not get a similar result because in our model, money-burning does not directly affect the principal's payoff, and in general he can choose among a

6 Delegation with both Conditional Transfers and Money Burning

So far, we have ruled out transfers other than a fixed wage to focus on money-burning. In this section, we allow for transfers conditional on the agent's report (and, by revelation principle, on the state θ). In other words, we will study the same problem, except that the transfer $t(\theta)$ may depend on the state θ rather than satisfy the restriction $t(\theta) \equiv T$. As before, this transfer must satisfy the minimum wage requirement, which we assume to hold in each state: $t(\theta) \geq \tilde{w}$ for all $\theta \in \Theta$. The principal therefore has two means to incentivize the agent: money-burning or contingent monetary transfers. In what follows, we show that even though the former is a less efficient way to create incentives for the agent than the latter, money-burning nevertheless can be used in the optimal contract (at different states). In fact, depending on parameter values, both means of providing incentives may be used, or only one of them. The primary factors determining which case applies are once again: (i) the outside option of the agent $\tilde{u}$, (ii) the minimum wage $\tilde{w}$, and (iii) the relative importance of the action choice for the principal versus the agent A.

Formally, in this Section the contract is given by a triple $(y(\cdot), m(\cdot), t(\cdot))$ consisting of policy $y(\theta)$, money burnt by the agent $m(\theta) \geq 0$, and transfer from principal to agent $t(\theta) \geq \tilde{w}$. The agent's loss function is now given by

$$L^a(\theta) = l^a(\theta, y(\theta)) + m(\theta) - t(\theta). \tag{22}$$

The principal solves the following problem (we immediately write it as a minimization problem, similar to (4)–(8)):

$$\min_{\{y(\theta), m(\theta), t(\theta)\}_{\theta \in \Theta}} \int_{\Theta} (l^p(\theta, y(\theta)) + t(\theta)) f(\theta) d\theta \tag{23}$$

$$\text{s.t.} \int_{\Theta} (l^a(\theta, y(\theta)) + m(\theta) - t(\theta)) f(\theta) d\theta \leq -\tilde{u}, \tag{24}$$

$$\forall \theta, \theta' \in \Theta : l^a(\theta, y(\theta)) + m(\theta) - t(\theta) \leq l^a(\theta, y(\theta')) + m(\theta') - t(\theta'), \tag{25}$$

$$\forall \theta \in \Theta : m(\theta) \geq 0, t(\theta) \geq \tilde{w}. \tag{26}$$

Many of the properties of the problem (23)–(26) are analogous to the case with fixed transfer. For example, under the same conditions as in Theorem 6, $y(\cdot)$ is continuous and satisfies $\theta \leq y(\theta) \leq \theta + b(\theta)$. Moreover, we can show that $m(\theta)$ is non-decreasing on Θ and $t(\theta)$ is non-increasing, and there is $\theta_0 \in \Theta$ such that $m(\theta_0) = 0$ and $t(\theta_0) = \min_{\theta \in \Theta} t(\theta)$; in other words,

---

broader set of contracts. However, whenever money-burning is not optimal (for $A < 1$), our optimal contract exhibits this property.

there is state θ_0 where there is no money-burning, and the transfer is minimal among all other states (it will typically, but not always, satisfy $t(\theta_0) = \tilde{w}$). If money burning is used, it will be used for high states ($\theta > \theta_0$), while conditional transfers will be made in low states ($\theta < \theta_0$). We can then rewrite the problem in a tractable way (details are available in Supplementary Appendix). The following is the result of these transformations for the uniform-quadratic case, which we focus on from now on:

$$\min_{\{y(\theta)\}_{\theta \in \Theta}, \theta_0, t(\theta_0)} \int_0^1 \left(A (y(\theta) - \theta)^2 - 2\lambda (y(\theta) - \theta - b)(1 - \theta) \right) d\theta \tag{27}$$

$$+ \int_0^{\theta_0} \left((y(\theta) - \theta - b)^2 + 2(\lambda - \theta)(y(\theta) - \theta - b) \right) d\theta + \lambda (y(0) - b)^2 + \lambda \tilde{u} + \mu \tilde{w} + (1 - \lambda - \mu) t(\theta_0),$$

$$\text{s.t. } y(\cdot) \text{ is non-decreasing and continuous,} \tag{28}$$

$$\forall \theta \in [0, 1] : y(\theta) \leq \theta + b. \tag{29}$$

The solution is explicitly characterized in the Supplementary Appendix. It takes a particularly simple form if $\lambda = 1$ (which will happen if $\tilde{w}$ is very small or $\tilde{u}$ is very high, so the agent has a high outside option and minimal wage is not a concern). In this case, there is no money-burning, and the optimal contract is supported by contingent transfers. In fact, the optimal contract will take the form

$$y^*(\theta) = \theta + \frac{1}{A+1} b = \frac{A}{A+1} \theta + \frac{1}{A+1} (\theta + b),$$

so it takes the form of a weighted average (with weights $\frac{A}{A+1}$ and $\frac{1}{A+1}$) of the principal's and the agent's ideal actions. In other words, if the minimal transfer constraint is not binding, the optimal contract will maximize the ex-ante utilitarian welfare, and money-burning will not be used. At the other extreme, if $\tilde{w}$ is very high or $\tilde{u}$ is very small, the principal will enforce his ideal action scheme $y^*(\theta) = \theta$ through the use of money-burning alone; this last result is similar to the previous case where conditional transfers were ruled out.

More generally, the principal may use contingent transfers, or money-burning, or both. One can prove that money-burning will take place in a positive mass of states if and only if $A > \lambda$ and $b < (1 - \lambda)\left(A - \lambda + 1 + \sqrt{(A - \lambda)(A - \lambda + 2)} \right)$ (notice that, analogously with the previous case, this happens if $A > \lambda$ and b is sufficiently small). Contingent transfers are used (i.e., $t(\theta)$ is not a constant) if $A \leq \lambda + 2A\lambda$ and $b < \frac{1}{2} \frac{(A+1)(A+\lambda)}{A(1-\lambda)}$, or $A > \lambda + 2A\lambda$ and $b < \lambda \frac{A+1}{A}\left(A - \lambda + 1 + \sqrt{(A - \lambda)(A - \lambda + 2)} \right)$. So, each ways of providing incentives to the agent are more likely to be used if b is low. In fact, if b is sufficiently low (and $\lambda < \min(A, 1)$), then both money-burning and conditional transfers are used. In this case, the optimal contract allows the agent to take his favorite action for intermediate states, while using money-burning

to align incentives at θ close to 1 and will make conditional transfers at θ close to 0. As before, for a fixed λ, money-burning is more likely if A is higher. Money-burning will not be used if the minimal transfer $\tilde{w}$ is low enough or the outside option $\tilde{u}$ is high enough: in this case, the agent needs a lot of compensation to satisfy his IR constraint, and it may well be made contingent on the action; this allows the principal to provide incentives without money-burning. Conversely, if either $\tilde{w}$ is high or $\tilde{u}$ is low, then money-burning will be used: if giving the agent the minimal transfer $t(\theta) = \tilde{w}$ for all θ makes his IR constraint slack, the principal may well introduce money-burning to align incentives.

With both money-burning and conditional transfers, the optimal contract may take different shapes. The next Figure 3 illustrates the most interesting situations ($b < 1$ is assumed throughout). If A is low (the action is not too important for the principal), but λ is not (the outside option of the agent is relatively high), then money burning is never used in the optimal contract. For these parameter values, there is a middle range of states where the agent chooses his ideal action; for higher states, his actions are capped, and for lower states, he is rewarded by a contingent transfer. If $A > \lambda$, money burning is possible. The central part of Figure 3 is similar to the previous case, except that the agent has less freedom to choose high actions, and he needs to burn money to choose some of them. The right part shows the case likely for even higher A, where the agent chooses a fixed action, where he neither burns money nor receives additional transfers, for a positive measure of states; he is rewarded with conditional transfers for choosing lower actions and punished with money-burning for choosing higher ones. In this last case, the agent never chooses his most preferred action.

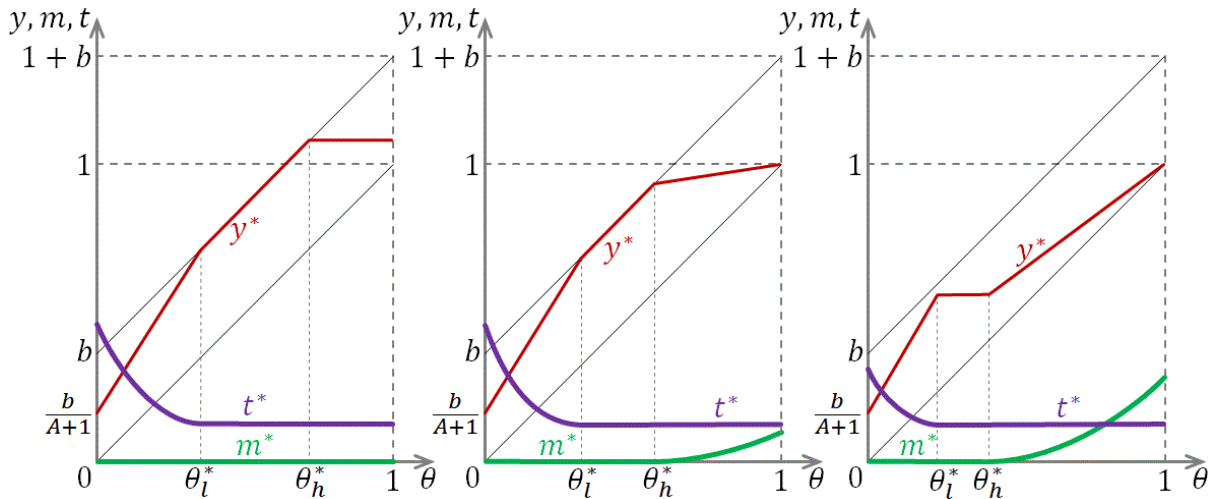Figure 3: Optimal contract if: (left) $A < \lambda$ and $b < \frac{(1-\lambda)(A+\lambda)}{A(A-\lambda+2)}$,

(center) $A > \lambda > 0$ and $b < \frac{\lambda(1-\lambda)}{A}$, (right) $A > \lambda > 0$ and $\frac{\lambda(1-\lambda)}{A} < b < \tilde{b}$.[27]

The following summarizes the key results about the optimal contract.

Theorem 10 *Money-burning is more likely to be used for low b. Moreover, if $b < 1$:*

1. *If the constraint $t(\theta) \geq \tilde{w}$ is not binding, then an efficient contract is implemented, and there is no money-burning.*

2. *If the constraint $t(\theta) \geq \tilde{w}$ is binding, then for almost all parameter values there is a positive mass of states $[\theta_1, \theta_2]$ for which the transfer is minimal and money-burning is not used: $t(\theta) = \tilde{w}$ and $m(\theta) = 0$ for $\theta \in [\theta_1, \theta_2]$.*

3. *Money-burning is more likely to be used for $\tilde{u}$ low and $\tilde{w}$ high.*

4. *Contingent transfers are more likely to be used for $\tilde{u}$ high and $\tilde{w}$ low.*

The economic lessons from the above results can be summarized as follows. The optimal contract achieves efficiency only if the agent has a high enough outside option (relative to the minimal wage). Lower levels of outside option result in two sources of inefficiency: (i) the implemented action scheme gets distorted from the jointly efficient scheme; (ii) the principal uses socially inefficient money burning (at least in some states) to distort the action choices of the agent. If the agent's outside option is very low (the minimum wage that the principal has to pay to the agent is very high relative to the utility the agent could obtain outside the relationship) then this inefficiency is particularly severe, and wasteful money burning is prescribed at almost every state. Furthermore, as the outside option of the agent gets worse, the optimal contract does not necessarily switch from one in which only monetary transfers are used to one in which only money-burning is used. If $A > \lambda$, then there is an intermediate range of outside options for which both positive (transfers) and negative (money-burning) incentive devices are used. The intuition behind this is that using monetary transfers is a relatively expensive incentive device in high states, because the regularity conditions imply that monetary transfers have to be non-increasing in the state. Hence specifying monetary transfers in high states increases the monetary transfers in all lower states. The opposite holds for money-burning: the regularity conditions imply that money-burning is non-decreasing in the state, which makes money-burning a relatively expensive incentive device in low states (but relatively cheap in high states).

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[27] The last requirement $b < \tilde{b}$ ensures that the optimal $y(\theta)$ has both conditional transfers and money-burning, so the graph has three linear parts. If $b > \tilde{b}$, only one instrument will be used. For b very large, neither conditional transfers nor money-burning will be used, and $y(\theta)$ will be a constant. See the Supplementary Appendix for details.

The above implies that the use of inefficient money-burning should be expected in delegation problems in which the relative importance of the action choice is high for the principal, and when the outside option of the principal is low relative to the minimum wage (or in general minimum transfer) requirement.

We conclude the section by pointing out a nonmonotonicity of the implemented action scheme in the optimal contract, as a function of the outside option of the agent. If the outside option is very low, then for any A, the implemented action scheme is equal to the principal's ideal line. As the outside option increases, the implemented action scheme shifts towards the principal's ideal points. As a result, for low values of A, there is an intermediate range of outside options in which the agent can choose his ideal action in a large set of states. However, a further increase in the outside option results in the jointly optimal action being implemented in all states, meaning that the agent cannot choose his ideal action in any of the states.

7 Conclusion

Our model of delegation with nonmonetary transfers may be developed in many different directions. Monetary versus nonmonetary incentives are extensively discussed in the economics of crime literature, starting from Becker (1968). An incomplete list of references on the topic includes: Shavell (1987), Mookherjee and Png (1994), and Levitt (1997). The models offered in this literature differ in many crucial aspects from ours: for example it is assumed that in the absence of any deterrents every criminal would choose the highest possible crime activity level, while our approach would assume that the optimal crime activity, from the criminal's viewpoint, is state-specific. Applying the delegation framework in this area might provide new insights on the structure of optimal monetary fines and prison sentences.

An intriguing question is that why bureaucratic procedures and paperwork seem to be the primary types of costly activity that organizations impose on their workers. A possible explanation for this, which we would like to investigate in future research, is that bureaucratic paperwork has the feature that the same level of activity is less costly in higher states. For example, when applying for a research grant requires turning in a long proposal, writing the proposal is less costly for an applicant who indeed has a good idea for a research project than for one who does not. Similarly, when an employee has to explain it in a report when taking a guest to an expensive restaurant from corporate budget, writing the report is less costly for employees who indeed had good reasons to select the expensive restaurant. This suggests incorporating costs of lying as in Kartik (2009) into our model.

8 Appendix

Proof of Claim 4. Consider $Z = [y(0), y(1)] \setminus R$ and let us extend the definition of function $\tilde{\theta}(\cdot)$ on the set Z. Take any $y \in Z$ (then $y(0) < y < y(1)$) and let $\tilde{\theta}(y) = \sup_{y^*(\theta) \leq y} \theta = \inf_{y^*(\theta) \geq y} \theta$ (these coincide because y^* is monotone). Let us now define, for $y \in Z$, $\tilde{m}(y) = \tilde{m}\left(y^*\left(\tilde{\theta}(y)\right)\right) + l^a\left(\tilde{\theta}(y), y^*\left(\tilde{\theta}(y)\right)\right) - l^a\left(\tilde{\theta}(y), y\right)$; in other words, we define $\tilde{m}(y)$ as an amount of money burning that would leave the type $\tilde{\theta}(y)$ indifferent between $(y, \tilde{m}(y))$ and $\left(y^*\left(\tilde{\theta}(y)\right), \tilde{m}\left(y^*\left(\tilde{\theta}(y)\right)\right)\right)$. We will have $\tilde{m}(y) \geq 0$ because l^a is convex in its second argument. Let us now prove that $\tilde{m}(\cdot)$ is continuous at any $y \in [y_1, y_2]$; its left derivative exists at y if $y > y_1$, and its right derivative exists at y if $y < y_2$, and they are equal to:

$$\frac{d^l \tilde{m}(y)}{dy} = -\frac{\partial l^a\left(\tilde{\theta}_{min}(y), y\right)}{\partial y},$$

$$\frac{d^r \tilde{m}(y)}{dy} = -\frac{\partial l^a\left(\tilde{\theta}_{max}(y), y\right)}{\partial y},$$

where $\tilde{\theta}_{min}(y) = \inf_{y^*(\theta)=y} \theta$ and $\tilde{\theta}_{max}(y) = \sup_{y^*(\theta)=y} \theta$.

Indeed, if $y > y_1$, take sufficiently small $\varepsilon > 0$. Let us prove that

$$l^a\left(\tilde{\theta}(y-\varepsilon), y-\varepsilon\right) + \tilde{m}(y-\varepsilon) \leq l^a\left(\tilde{\theta}(y-\varepsilon), y\right) + \tilde{m}(y);$$
$$l^a\left(\tilde{\theta}(y), y\right) + \tilde{m}(y) \leq l^a\left(\tilde{\theta}(y), y-\varepsilon\right) + \tilde{m}(y-\varepsilon). \tag{30}$$

We prove the first (the second is proved analogously). Applying (6) to type $\tilde{\theta}(y-\varepsilon)$, we get

$$l^a\left(\tilde{\theta}(y-\varepsilon), y^*\left(\tilde{\theta}(y-\varepsilon)\right)\right) + \tilde{m}\left(y^*\left(\tilde{\theta}(y-\varepsilon)\right)\right) \leq l^a\left(\tilde{\theta}(y-\varepsilon), \lim_{\delta \to +0} y^*\left(\tilde{\theta}(y) - \delta\right)\right) + \lim_{\delta \to +0} \tilde{m}\left(y^*\left(\tilde{\theta}(y) - \delta\right)\right);$$

indeed, he does not want to pretend to be any type arbitrarily close to $\tilde{\theta}(y)$. By construction, we have

$$l^a\left(\tilde{\theta}(y-\varepsilon), y^*\left(\tilde{\theta}(y-\varepsilon)\right)\right) + \tilde{m}\left(y^*\left(\tilde{\theta}(y-\varepsilon)\right)\right) = l^a\left(\tilde{\theta}(y-\varepsilon), y-\varepsilon\right) + \tilde{m}(y-\varepsilon)$$

(this inequality is trivial if $y - \varepsilon \in R(y^*)$). We also have that if $y \in R(y^*)$, then

$$l^a\left(\tilde{\theta}(y-\varepsilon), y^*\left(\tilde{\theta}(y)\right)\right) + \tilde{m}\left(y^*\left(\tilde{\theta}(y)\right)\right) = l^a\left(\tilde{\theta}(y-\varepsilon), y\right) + \tilde{m}(y),$$

and it remains to prove that for $y \in Z$, we must have

$$l^a\left(\tilde{\theta}(y-\varepsilon), y\right) + \tilde{m}(y) \geq l^a\left(\tilde{\theta}(y-\varepsilon), \lim_{\delta \to +0} y^*\left(\tilde{\theta}(y) - \delta\right)\right) + \lim_{\delta \to +0} \tilde{m}\left(y^*\left(\tilde{\theta}(y) - \delta\right)\right).$$

But the agent of type $\tilde{\theta}(y)$ must be indifferent between the contracts $(y, \tilde{m}(y))$ and $\left(\lim_{\delta \to +0} y^*\left(\tilde{\theta}(y) - \delta\right), \lim_{\delta \to +0} \tilde{m}\left(y^*\left(\tilde{\theta}(y) - \delta\right)\right)\right)$. As $y \geq \lim_{\delta \to +0} y^*\left(\tilde{\theta}(y) - \delta\right)$ by monotonicity,

and $\tilde{\theta}(y-\varepsilon) \leq \tilde{\theta}(y)$ the type $\tilde{\theta}(y-\varepsilon)$ must strictly prefer the latter. This establishes the first inequality of (30).

Now, (30) imply

$$l^a\left(\tilde{\theta}(y-\varepsilon), y-\varepsilon\right) - l^a\left(\tilde{\theta}(y-\varepsilon), y\right) \leq \tilde{m}(y) - \tilde{m}(y-\varepsilon) \leq l^a\left(\tilde{\theta}(y), y-\varepsilon\right) - l^a\left(\tilde{\theta}(y), y\right).$$

Since this holds for any function $\tilde{\theta}(\cdot)$ that satisfies $y^*\left(\tilde{\theta}(y)\right) = y$, by continuity, we have

$$l^a\left(\tilde{\theta}_{min}(y-\varepsilon), y-\varepsilon\right) - l^a\left(\tilde{\theta}_{min}(y-\varepsilon), y\right) \leq \tilde{m}(y) - \tilde{m}(y-\varepsilon) \leq l^a\left(\tilde{\theta}_{min}(y), y-\varepsilon\right) - l^a\left(\tilde{\theta}_{min}(y), y\right).$$

Dividing all parts by ε, we notice that the leftmost and the rightmost parts tend to $-\frac{\partial l^a\left(\tilde{\theta}_{min}(y), y\right)}{\partial y}$, because $\lim_{\varepsilon \to +0} \tilde{\theta}_{min}(y-\varepsilon) = \tilde{\theta}_{min}(y)$. This shows that $\frac{d^l \tilde{m}(y)}{dy}$ exists and is given by the formula. The same argument works for the right derivative; in either case, $\tilde{m}(y)$ is continuous at y. This also implies that $\tilde{m}(\cdot)$ is differentiable at y if and only if $(y^*)^{-1}(y)$ is a singleton (in particular, $\tilde{m}(\cdot)$ is differentiable in any point in Z).

Given that $\tilde{m}(\cdot)$ is almost everywhere differentiable with bounded derivative, it satisfies Lipschitz conditions and is thus absolutely continuous. It can then be reconstructed from its derivative. ∎

Proof of Theorem 5. To make Theorem 2.2 in Balder (1996) applicable, we need to make the space of feasible contracts compact. First of all, we can assume that $T \leq \bar{T}$ for some $\bar{T}$ sufficiently high, so T is taken from a compact set $[\tilde{w}, \bar{T}]$ (indeed, the contract $y(\theta) = \theta + b$, $m(\theta) = 0$, $T = \max\{\tilde{u}, \tilde{w}\}$ is feasible and yields $L^p = Ab^2 + \max\{\bar{u}, \bar{w}\}$, and thus any contract featuring $T > \bar{T} = \int_0^1 l^p(\theta, \theta + b(\theta))dF(\theta) + \max\{\bar{u}, \bar{w}\}$ is suboptimal). The set of monotonic mappings from Θ to Y is compact in sup-metrics, so the set of feasible $y(\theta)$ is compact. Finally, by Claim 4 we have

$$m(\theta) = \tilde{m}(y(0)) + \int_{y(0)}^{\hat{y}} \left(-\frac{\partial l^a\left(\tilde{\theta}(y), y\right)}{\partial y}\right) dy, \tag{31}$$

which means that the entire function $m(\theta)$ may be derived from $y(\theta)$ and $m(y(0))$. Since $m(y(0))$ may be assumed to be taken from some compact set $[0, \bar{m}]$, the set of feasible contracts $(T, y(\theta), m(\theta))$ may be assumed to be compact. With this modification of the space the conditions of Theorem 2.2 from Balder (1996) trivially hold, therefore there exists a solution to the problem. ∎

Proof of Theorem 6. Part 1. Assume $y^*(\theta) < \theta$ for some $\theta \in [0, 1]$. Below we show that the principal can offer a contract that improves his payoff.

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Since we are focusing on $y(\cdot)$ that are continuous at $\theta = 0$ and $\theta = 1$, there must exist $(\underline{\theta}, \overline{\theta})$ such that $y^*(\theta) < \theta$ for every $\theta \in (\underline{\theta}, \overline{\theta})$. Take $0 \leq \underline{\theta} < \overline{\theta} \leq 1$ such that (i) $y^*(\theta) < \theta$ for every $\theta \in (\underline{\theta}, \overline{\theta})$; (ii) either $y^*(\underline{\theta}) \geq \underline{\theta}$ or $\underline{\theta} = 0$; (iii) either $y^*(\overline{\theta}) \geq \overline{\theta}$ or $\overline{\theta} = 1$. We consider two situations.

Case 1. Suppose that for all $\theta > \overline{\theta}$, $y^*(\theta) \leq y^a(\theta)$. Then define action function $y'(\cdot)$ such that $y'(\theta) = \theta$ for $\theta \in [\underline{\theta}, \overline{\theta}]$ and $y'(\theta) = y^*(\theta)$ otherwise, and define $m'(\cdot)$ such that $m'(\theta) = m^*(\theta)$ for $\theta \leq \underline{\theta}$ and $m'(\cdot)$ and $y'(\cdot)$ together satisfy (1c). In this case, $m'(\theta) \geq 0$ for every $\theta \in [0,1]$ because it is increasing on $[\overline{\theta}, 1]$, and therefore $(y'(\cdot), m'(\cdot), T^*)$ is a contract that yields a strictly higher payoff to the proncipal than $(y^*(\cdot), m^*(\cdot), T^*)$. To see this, observe that given $(y'(\cdot), m'(\cdot))$ every type θ is weakly better off than given $(y^*(\cdot), m^*(\cdot))$, hence the participation constraint of the agent holds with transfer T^*. This means the contract is feasible. For $\theta \in (\underline{\theta}, \overline{\theta})$ the implemented action is strictly better for the principal (while at other states the implemented action remains the same), hence the principal is strictly better off ex ante.

Case 2. Suppose that for some $\theta < \overline{\theta}$, $y^*(\theta) > y^a(\theta)$. Take $\theta' = \inf_{\theta \geq \overline{\theta}} \{\theta : y^*(\theta) > y^a(\theta)\}$; then there is some $\theta'' > \theta'$ such that for all $\theta \in (\theta', \theta'')$, $y^*(\theta) > y^a(\theta)$. We can then take $y'(\theta)$ and $m'(\theta)$ such that $y'(\theta) = y^*(\theta)$ for $\theta \notin [\underline{\theta}, \overline{\theta}] \cup [\theta', \theta'']$, $y'(\theta) \in (y^*(\theta), \theta)$ for $\theta \in (\underline{\theta}, \overline{\theta})$ and $y'(\theta) \in (y^a(\theta), y^*(\theta))$ for $\theta \in (\theta', \theta'')$, and $m'(\theta) = m^*(\theta)$ for $m < \underline{\theta}$ or $m > \theta''$. Claim 4 ensures that this is feasible, and also that in this case, $0 \leq m'(\theta) \leq m^*(\theta)$ for $\theta \in [\underline{\theta}, \theta'']$. A similar argument shows that this contract is feasible and yields a strictly higher utility for the principal, which completes the proof. ∎

Part 2. We start by proving the following result. Suppose that $\dfrac{\frac{\partial l^a(\theta, y_1)}{\partial \theta} - \frac{\partial l^a(\theta, y_0)}{\partial \theta}}{l^p(\theta, y_0) - l^p(\theta, y_1)} > \dfrac{\frac{\partial l^a(\theta, y_1)}{\partial \theta} - \frac{\partial l^a(\theta, y_2)}{\partial \theta}}{l^p(\theta, y_2) - l^p(\theta, y_1)}$, for every $\theta \in (0, 1)$, and $\lim_{\theta' \searrow \theta} y^*(\theta') \geq y_2 > y_0 > y_1 \geq \theta$. Then $y^*(\theta)$ and $m^*(\theta)$ are continuous on $(0, 1)$.

To prove this, note that Claim 4 implies that m^* is continuous at θ if y^* is continuous at θ, hence it is enough to prove continuity of the latter. The proof below is by contradiction. Suppose that for some $\theta_0 \in (0, 1)$, y^* is discontinuous at θ_0. Denote

$$\hat{y}_1 = \sup_{\theta \in [0, \theta_0)} y^*(\theta),$$
$$\hat{y}_2 = \inf_{\theta \in (\theta_0, 1]} y^*(\theta).$$

Note that, since $y^*(\theta)$ is monotonic, it is true that $\hat{y}_1 = \lim_{\theta \to \theta_0-} y^*(\theta)$, $\hat{y}_2 = \lim_{\theta \to \theta_0+} y^*(\theta)$. Define $\hat{m}_1 = \lim_{\theta \to \theta_0-} m^*(\theta) \geq 0$ and $\hat{m}_2 = \lim_{\theta \to \theta_0+} m^*(\theta) \geq 0$; these limits exist by continuity of loss function $L^a(\theta)$: $\hat{m}_1 = \lim_{\theta \to \theta_0-} L^a(\theta) - l^a(\theta_0, \hat{y}_1)$, and similarly $\hat{m}_2 = \lim_{\theta \to \theta_0+} L^a(\theta) - l^a(\theta_0, \hat{y}_2)$. It is evident that an agent of type θ_0 is indifferent between contracts $(y^*(\theta_0), m^*(\theta_0))$, $(\hat{y}_1, \hat{m}_1)$ and $(\hat{y}_2, \hat{m}_2)$: otherwise, if, for instance, we had $l^a(\theta_0, \hat{y}_1) + \hat{m}_1 > l^a(\theta_0, \hat{y}_2) + \hat{m}_2$ instead, then an agent of type $\theta_0 + \varepsilon$ would strictly prefer contract $(y^*(\theta_0 - \varepsilon), m^*(\theta_0 - \varepsilon))$ to

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$(y^*(\theta_0 + \varepsilon), m^*(\theta_0 + \varepsilon))$ by continuity, which would violate (6).

The idea of the proof is to perturb the optimal contract $(y^*(\theta), m^*(\theta))_{\theta \in \Theta}$ around the point of discontinuity θ_0 and obtain a higher value of V^p, which would contradict the optimality of the initial contract. Take some $a \in (0,1)$ and define $\hat{y}_0$ by

$$\frac{\partial l^a(\theta_0, \hat{y}_0)}{\partial \theta} = a \frac{\partial l^a(\theta_0, \hat{y}_1)}{\partial \theta} + (1-a)\frac{\partial l^a(\theta_0, \hat{y}_2)}{\partial \theta}; \tag{32}$$

clearly, such $\hat{y}_0 \in (\hat{y}_1, \hat{y}_2)$ exists (and is unique) for any $a \in (0,1)$, since $\frac{\partial l^a(\theta_0, y)}{\partial y}$ is continuous and monotonic (increasing) in y. Trivially, (32) is equivalent to

$$\frac{\frac{\partial l^a(\theta_0, \hat{y}_0)}{\partial \theta} - \frac{\partial l^a(\theta_0, \hat{y}_2)}{\partial \theta}}{\frac{\partial l^a(\theta_0, \hat{y}_1)}{\partial \theta} - \frac{\partial l^a(\theta_0, \hat{y}_0)}{\partial \theta}} = \frac{a}{1-a}.$$

We now pick $\hat{m}_0$ to be such that

$$l^a(\theta_0, \hat{y}_0) + \hat{m}_0 = l^a(\theta_0, \hat{y}_1) + \hat{m}_1 = l^a(\theta_0, \hat{y}_2) + \hat{m}_2.$$

Since $l^a(\theta_0, y)$ is strictly convex in y, we have $l^a(\theta_0, \hat{y}_0) < \max(l^a(\theta_0, \hat{y}_1), l^a(\theta_0, \hat{y}_2))$, and therefore $\hat{m}_0 > \min(\hat{m}_1, \hat{m}_2) \geq 0$.

By construction, agent of type θ_0 is indifferent between $(\hat{y}_0, \hat{m}_0)$, $(\hat{y}_1, \hat{m}_1)$, and $(\hat{y}_2, \hat{m}_2)$. In contrast, agents with $\theta < \theta_0$ strictly prefer $(\hat{y}_1, \hat{m}_1)$ to $(\hat{y}_0, \hat{m}_0)$ (this immediately follows from the single-crossing condition), and prefer $(y^*(\theta), m^*(\theta))$ to $(\hat{y}_1, \hat{m}_1)$ (from (6), as $(\hat{y}_1, \hat{m}_1)$ is a limit of feasible contracts), while agents with $\theta > \theta_0$ weakly prefer $(y^*(\theta), m^*(\theta))$ to $(\hat{y}_2, \hat{m}_2)$, which they strictly prefer to $(\hat{y}_0, \hat{m}_0)$. Consider the function

$$z(\theta) = l^a(\theta, \hat{y}_0) + \hat{m}_0 - L^a(\theta),$$

which is naturally interpreted as the "gap" in utility from choosing $(y^*(\theta), m^*(\theta))$, which agent θ does, and choosing $(\hat{y}_0, \hat{m}_0)$ if he had such an option. From Claim 3 it follows that function $z(\theta)$ is continuous for $\theta \in \Theta$, it is positive and strictly decreasing for $\theta < \theta_0$, it is positive and strictly increasing for $\theta > \theta_0$, and it equals zero at $\theta = \theta_0$.

Let us take a sufficiently small $\varepsilon > 0$ and augment the set of available choices $(y^*(\theta), m^*(\theta))_{\theta \in \Theta}$ by adding $(\hat{y}_0, \hat{m}_0 - \varepsilon)$ to it. From the properties of function $z(\theta)$ it follows that players with $\theta \in (\theta_1(\varepsilon), \theta_2(\varepsilon))$ will switch to $(\hat{y}_0, \hat{m}_0 - \varepsilon)$ while the rest will not (and those with types $\theta_1(\varepsilon)$ and $\theta_2(\varepsilon)$ will be indifferent); here, $\theta_1(\varepsilon)$ and $\theta_2(\varepsilon)$ are continuous functions of θ such that $\theta_1(\varepsilon)$ is decreasing and $\theta_2(\varepsilon)$ is increasing in ε. As $\varepsilon \to 0$, $\theta_1(\varepsilon) \to \theta_0$ and $\theta_2(\varepsilon) \to \theta_0$. Let us find the limit of $\frac{\theta_0 - \theta_1(\varepsilon)}{\theta_2(\varepsilon) - \theta_0}$ (and simultaneously show that it exists and is finite). To do that, it is convenient to consider the inverse functions, $\varepsilon_1(\theta_1)$, defined for $\theta_1 \leq \theta_0$, and $\varepsilon_2(\theta_2)$, defined for $\theta_2 \geq \theta_0$.

By construction, $\varepsilon_1 (\theta_1)$ satisfies

$$L^a (\theta_1) = l^a (\theta_1, \hat{y}_0) + \hat{m}_0 - \varepsilon_1 (\theta_1).$$

Hence,

$$
\begin{aligned}
\varepsilon_1 (\theta_1) & = & l^a (\theta_1, \hat{y}_0) + \hat{m}_0 - L^a (\theta_1) \\
& = & l^a (\theta_1, \hat{y}_0) - l^a (\theta_0, \hat{y}_0) + L^a (\theta_0) - L^a (\theta_1).
\end{aligned}
$$

Therefore Claim 3 implies that $\varepsilon_1 (\theta_1)$ has a left derivative at $\theta_1 = \theta_0$:

$$\frac{d^l \varepsilon_1 (\theta_1)}{d\theta_1} = \frac{\partial l^a (\theta_0, \hat{y}_0)}{\partial \theta} - \frac{\partial l^a (\theta_0, \hat{y}_1)}{\partial \theta}.$$

Similarly,

$$\frac{d^r \varepsilon_2 (\theta_1)}{d\theta_1} = \frac{\partial l^a (\theta_0, \hat{y}_0)}{\partial \theta} - \frac{\partial l^a (\theta_0, \hat{y}_2)}{\partial \theta}.$$

We then have

$$
\begin{aligned}
\frac{\partial l^a (\theta_0, \hat{y}_0)}{\partial \theta} - \frac{\partial l^a (\theta_0, \hat{y}_1)}{\partial \theta} & = & \lim_{\theta_1 \to \theta_0-} \frac{\varepsilon_1 (\theta_0) - \varepsilon_1 (\theta_1)}{\theta_0 - \theta_1} \\
& = & \lim_{\varepsilon \to 0+} \frac{-\varepsilon}{\theta_0 - \theta_1 (\varepsilon)}, \\
\frac{\partial l^a (\theta_0, \hat{y}_0)}{\partial \theta} - \frac{\partial l^a (\theta_0, \hat{y}_2)}{\partial \theta} & = & \lim_{\theta_1 \to \theta_0+} \frac{\varepsilon_2 (\theta_2) - \varepsilon_2 (\theta_0)}{\theta_2 - \theta_0} \\
& = & \lim_{\varepsilon \to 0+} \frac{\varepsilon}{\theta_2 (\varepsilon) - \theta_0}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\lim_{\varepsilon \to 0+} \frac{\theta_0 - \theta_1 (\varepsilon)}{\theta_2 (\varepsilon) - \theta_0} & = & \frac{\lim_{\varepsilon \to 0+} \frac{\varepsilon}{\theta_2 (\varepsilon) - \theta_0}}{-\lim_{\varepsilon \to 0+} \frac{-\varepsilon}{\theta_0 - \theta_1 (\varepsilon)}} \\
& = & \frac{\frac{\partial l^a (\theta_0, \hat{y}_0)}{\partial \theta} - \frac{\partial l^a (\theta_0, \hat{y}_2)}{\partial \theta}}{\frac{\partial l^a (\theta_0, \hat{y}_1)}{\partial \theta} - \frac{\partial l^a (\theta_0, \hat{y}_0)}{\partial \theta}} = \frac{a}{1 - a}.
\end{aligned}
\tag{33}
$$

We are now ready to estimate the welfare effect of this perturbation. The agent of any type is weakly better off, and for some types the agent is strictly better off: for $\theta \in (\theta_1 (\varepsilon), \theta_2 (\varepsilon))$ switched to $(\hat{y}_0, \hat{m}_0 - \varepsilon)$ which he strictly prefers to $(y^* (\theta), m^* (\theta))$ which he was choosing before, and the rest have not changed their contract. We therefore only need to compute the change in principal's payoff. This change equals

$$\int_{\theta_1(\varepsilon)}^{\theta_2(\varepsilon)} (l^p (\theta, y (\theta)) - l^p (\theta, \hat{y}_0)) f (\theta) d\theta = \int_{\theta_1(\varepsilon)}^{\theta_2(\varepsilon)} \int_{\hat{y}_0}^{y(\theta)} \frac{\partial l^p (\theta, y)}{\partial y} dy f (\theta) d\theta \tag{34}$$

$$= \int_{\theta_0}^{\theta_2(\varepsilon)} (l^p (\theta, y (\theta)) - l^p (\theta, \hat{y}_0)) f (\theta) d\theta - \int_{\theta_1(\varepsilon)}^{\theta_0} (l^p (\theta, \hat{y}_0) - l^p (\theta, y (\theta))) f (\theta) d\theta.$$

To check that this expression is positive, it is sufficient, given the continuity of $f(\theta)$ at θ_0 and existence of limits $\lim_{\theta \to \theta_0-} \frac{\partial l^p(\theta,y)}{\partial y} = \frac{\partial l^p(\theta_0,y)}{\partial y}$ and $\lim_{\theta \to \theta_0+} \frac{\partial l^p(\theta,y)}{\partial y} = \frac{\partial l^p(\theta_0,y)}{\partial y}$, to prove that

$$\lim_{\varepsilon \to 0} \left((\theta_2(\varepsilon) - \theta_0)(l^p(\theta,\hat{y}_2) - l^p(\theta,\hat{y}_0)) - (\theta_0 - \theta_1(\varepsilon))(l^p(\theta,\hat{y}_0) - l^p(\theta,\hat{y}_1)) \right) > 0.$$

In light of (33), it suffices to prove that

$$(1 - a)(l^p(\theta,\hat{y}_2) - l^p(\theta,\hat{y}_0)) > a(l^p(\theta,\hat{y}_0) - l^p(\theta,\hat{y}_1)). \tag{35}$$

By Part 1, $l^p(\theta,\hat{y}_2) > l^p(\theta,\hat{y}_0)$ and $l^p(\theta,\hat{y}_0) > l^p(\theta,\hat{y}_1)$, and (35) is equivalent to

$$\frac{l^p(\theta_0,\hat{y}_2) - l^p(\theta_0,\hat{y}_0)}{l^p(\theta_0,\hat{y}_0) - l^p(\theta_0,\hat{y}_1)} > \frac{\frac{\partial l^a(\theta_0,\hat{y}_0)}{\partial \theta} - \frac{\partial l^a(\theta_0,\hat{y}_2)}{\partial \theta}}{\frac{\partial l^a(\theta_0,\hat{y}_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,\hat{y}_0)}{\partial \theta}}.$$

By adding 1 to both sides, we find this is equivalent to

$$\frac{l^p(\theta_0,\hat{y}_2) - l^p(\theta_0,\hat{y}_1)}{l^p(\theta_0,\hat{y}_0) - l^p(\theta_0,\hat{y}_1)} > \frac{\frac{\partial l^a(\theta_0,\hat{y}_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,\hat{y}_2)}{\partial \theta}}{\frac{\partial l^a(\theta_0,\hat{y}_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,\hat{y}_0)}{\partial \theta}}.$$

Now, rearranging (this is safe since the denominators are positive) and changing the sign, we get

$$\frac{\frac{\partial l^a(\theta_0,\hat{y}_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,\hat{y}_0)}{\partial \theta}}{l^p(\theta_0,\hat{y}_0) - l^p(\theta_0,\hat{y}_1)} > \frac{\frac{\partial l^a(\theta_0,\hat{y}_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,\hat{y}_2)}{\partial \theta}}{l^p(\theta_0,\hat{y}_2) - l^p(\theta_0,\hat{y}_1)}. \tag{36}$$

Claim 1 and part 1 of the current claim imply that $\theta \leq \hat{y}_1 < \hat{y}_0 < \hat{y}_2$, hence the assumption of the lemma implies that (36) holds. This implies that the proposed deviation is profitable, contradicting y^* is discontinuous at θ_0.

To finish the proof of this result, notice that for any $\theta_0 \leq y_1 < y_0 < y_2$ the following holds:

$$\frac{\frac{\frac{\partial l^a(\theta_0,y_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,y_0)}{\partial \theta}}{y_0 - y_1}}{\frac{l^p(\theta_0,y_0) - l^p(\theta_0,y_1)}{y_0 - y_1}} > \frac{\frac{\frac{\partial l^a(\theta_0,y_0)}{\partial \theta} - \frac{\partial l^a(\theta_0,y_2)}{\partial \theta}}{y_2 - y_0}}{\frac{l^p(\theta_0,y_2) - l^p(\theta_0,y_0)}{y_2 - y_0}}.$$

This is equivalent to $\frac{\frac{\partial l^a(\theta_0,y_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,y_0)}{\partial \theta}}{l^p(\theta_0,y_0) - l^p(\theta_0,y_1)} > \frac{\frac{\partial l^a(\theta_0,y_1)}{\partial \theta} - \frac{\partial l^a(\theta_0,y_2)}{\partial \theta}}{l^p(\theta_0,y_2) - l^p(\theta_0,y_1)}$, there, the argument above implies the claim in the theorem.

Part 3. We first prove the following auxiliary result. Suppose $(y(\theta), m(\theta))$ satisfies (6) and $y(\theta)$ is continuous on Θ. Then for any $\theta_1, \theta_2 \in \Theta$, we have

$$m(\theta_2) - m(\theta_1) = l^a(\theta_1, y(\theta_1)) - l^a(\theta_2, y(\theta_2)) + \int_{\theta_1}^{\theta_2} \left(\frac{\partial l^a(\theta, y(\theta))}{\partial \theta} \right) d\theta. \tag{37}$$

Indeed, from (9), we have

$$
\begin{aligned}
\int_{\theta_1}^{\theta_2} \left(\frac{\partial l^a(\theta, y(\theta))}{\partial \theta} \right) d\theta &= L^a(\theta_2) - L^a(\theta_1) \\
&= l^a(\theta_2, y(\theta_2)) + m(\theta_2) - l^a(\theta_1, y(\theta_1)) - m(\theta_1).
\end{aligned}
$$

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Rearranging, we obtain (37).

Now, by Part 2, $y^*(\theta)$ is a continuous function. Suppose, to obtain a contradiction, that there exists $\theta_0 \in \Theta$ such that $y(\theta_0) > \theta_0 + b(\theta_0)$. Because $y^*(\cdot)$ is continuous, without loss of generality we may assume that $0 < \theta_0 < 1$. There are two possibilities: either for all $\theta < \theta_0$, $y^*(\theta) \geq \theta + b(\theta)$, or there exists $\theta' < \theta_0$ such that $y^*(\theta') < \theta' + b(\theta')$. We start with the first possibility.

Suppose $y^*(\theta) \geq \theta + b(\theta)$ for all $\theta < \theta_0$. Let $\theta_1 = \inf\{\theta : y^*(\theta) < \theta + b(\theta)\}$ if such θ exists; otherwise, let $\theta_1 = 1$. Define function $y(\theta)$ by

$$y(\theta) = \begin{cases} y^*(\theta) & \text{if } \theta > \theta_1, \\ \theta + b & \text{if } \theta \leq \theta_1; \end{cases}$$

Note that by continuity, $y^*(\theta_1) = \theta_1 + b(\theta_1)$, hence the above function is continuous. Furthermore, let

$$m(\theta) = \begin{cases} m^*(\theta) & \text{if } \theta > \theta_1, \\ m^*(\theta_1) & \text{if } \theta \leq \theta_1. \end{cases}$$

Given that scheme $y^*(), m^*()$ satisfies (6) and (7), it is straightforward to verify that scheme $(y(), m())$ also satisfies (6) and (7). In the modified scheme, the utility of the agent at $\theta \geq \theta_0$ is unchanged. If $\theta < \theta_0$, then, by (9)

$$
\begin{aligned}
L^a(\theta, y(\theta), m(\theta)) &= L^a(\theta_1, y(\theta_1), m(\theta_1)) - \int_\theta^{\theta_1} \frac{\partial l^a(\xi, y(\xi))}{\partial \theta} d\xi \\
&< L^a(\theta_1, y^*(\theta_1), m^*(\theta_1)) - \int_\theta^{\theta_1} \frac{\partial l^a(\xi, y^*(\xi))}{\partial \theta} d\xi = L^a(\theta, y^*(\theta), m^*(\theta));
\end{aligned}
$$

this holds because at θ_1 the contract is unchanged, and

$$\int_\theta^{\theta_1} \frac{\partial l^a(\xi, y^*(\xi))}{\partial \theta} d\xi - \int_\theta^{\theta_1} \frac{\partial l^a(\xi, y(\xi))}{\partial \theta} d\xi = \int_\theta^{\theta_1} \int_{y(\xi)}^{y^*(\xi)} \frac{\partial^2 l^a(\xi, y)}{\partial \theta \partial y} d\xi < 0,$$

since $y(\xi) < y^*(\xi)$ whenever $\xi < \theta_1$ is close to θ_0. Consequently, all types of agent are at least weakly better off. The principal, is obviously better off, since for some θ, $y(\theta)$ became closer to θ than $y^*(\theta)$. This contradicts that contract $(y^*(\theta), m^*(\theta))$ solves the problem (4).

Now suppose that there exists $\theta' < \theta_0$ such that $y^*(\theta') < \theta' + b(\theta')$. Let $\theta_1 = \min\{\theta \in [\theta', \theta_0] : y^*(\theta) = \theta + b(\theta)\}$, $\theta_2 = \inf\{\theta \in [\theta_1, \theta_0] : y^*(\theta) > \theta + b(\theta)\}$; by continuity, $\hat{\theta}_1$ and $\hat{\theta}_2$ are well-defined and they may or may not coincide. By construction, if $\theta \in [\theta_1, \theta_2]$, then $y^*(\theta) = \theta + b(\theta)$; moreover, for sufficiently small $\varepsilon > 0$ we have $y^*(\theta_1 - \varepsilon) < \theta_1 - \varepsilon + b(\theta_1 - \varepsilon)$ and $y^*(\theta_2 + \varepsilon) > \theta_1 + \varepsilon + b(\theta_1 + \varepsilon)$. This implies, in particular, that $m^*(\theta)$ is bounded away from 0 on $[\theta_1, \theta_2]$ (this is a trivial corollary of the result from the beginning of the proof, since $m^*(\theta_1 - \varepsilon)$ is non-negative).

Let us construct an alternative $y(\theta)$ as follows. We take ε_1 and ε_2 to be such small positive numbers such that

$$\int_{\theta_1-\varepsilon_1}^{\theta_1}\int_{y^*(\theta)}^{\theta+b(\theta)}\left(-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}\right)dyd\theta = \int_{\theta_2}^{\theta_2+\varepsilon_2}\int_{\theta+b(\theta)}^{y^*(\theta)}\left(-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}\right)dyd\theta, \qquad (38)$$

and pick a small $\varepsilon_0 > 0$. We require that

$$y(\theta) = \begin{cases} y^*(\theta) \text{ if } \theta \le \theta_1 - \varepsilon_1 - \varepsilon_0, \\ \in (y^*(\theta),\theta+b(\theta)) \text{ if } \theta \in (\theta_1-\varepsilon_1-\varepsilon_0,\theta_1-\varepsilon_1), \\ \theta+b(\theta) \text{ if } \theta \in [\theta_1-\varepsilon_1,\theta_2+\varepsilon_2], \\ \in (\theta+b(\theta),y^*(\theta)) \text{ if } \theta \in (\theta_2-\varepsilon_2,\theta_2+\varepsilon_2+\varepsilon_0), \\ y^*(\theta) \text{ if } \theta \ge \theta_2+\varepsilon_2+\varepsilon_0, \end{cases}$$

and that

$$\int_{\theta_1-\varepsilon_1-\varepsilon_0}^{\theta_1}\int_{y^*(\theta)}^{y(\theta)}\left(-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}\right)dyd\theta = \int_{\theta_2}^{\theta_2+\varepsilon_2+\varepsilon_0}\int_{y(\theta)}^{y^*(\theta)}\left(-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}\right)dyd\theta. \qquad (39)$$

Now, if we define $m(\theta)$ to be such that the agent's loss function $L^a(\theta,y(\theta),m(\theta))$ satisfies (9) and coincides with $L^a(\theta,y^*(\theta),m^*(\theta))$ for $\theta \notin (\theta_1-\varepsilon_1-\varepsilon_0,\theta_2+\varepsilon_2+\varepsilon_0)$, we would get a contract $(y(\theta),m(\theta))$ that satisfies (6) and (7), as well as (8).

Under the new contract $(y(\theta),m(\theta))$, all agents with type $\theta \in (\theta_1-\varepsilon_1-\varepsilon_0,\theta_2+\varepsilon_2+\varepsilon_0)$ are better off; moreover, the agents with types $\theta \in [\theta_1,\theta_2]$ are better off by at least (38). The change in the principal's utility is given by

$$\int_{\theta_2}^{\theta_2+\varepsilon_2+\varepsilon_0}\left(l^p(\theta,y^*(\theta))-l^p(\theta,y(\theta))\right)f(\theta)d\theta - \int_{\theta_1-\varepsilon_1-\varepsilon_0}^{\theta_1}\left(l^p(\theta,y(\theta))-l^p(\theta,y^*(\theta))\right)f(\theta)d\theta$$

$$= \int_{\theta_2}^{\theta_2+\varepsilon_2+\varepsilon_0}\int_{y^*(\theta)}^{y(\theta)}\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)dyd\theta - \int_{\theta_1-\varepsilon_1-\varepsilon_0}^{\theta_1}\int_{y^*(\theta)}^{y(\theta)}\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)dyd\theta.$$

It suffices to show that

$$\int_{\theta_2}^{\theta_2+\varepsilon_2+\varepsilon_0}\int_{y^*(\theta)}^{y(\theta)}\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)dyd\theta > \int_{\theta_1-\varepsilon_1-\varepsilon_0}^{\theta_1}\int_{y^*(\theta)}^{y(\theta)}\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)dyd\theta.$$

Dividing this by (39), we are to prove

$$\frac{\int_{\theta_2}^{\theta_2+\varepsilon_2+\varepsilon_0}\int_{y^*(\theta)}^{y(\theta)}\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)dyd\theta}{\int_{\theta_2}^{\theta_2+\varepsilon_2+\varepsilon_0}\int_{y^*(\theta)}^{y(\theta)}\left(-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}\right)dyd\theta} > \frac{\int_{\theta_1-\varepsilon_1-\varepsilon_0}^{\theta_1}\int_{y^*(\theta)}^{y(\theta)}\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)dyd\theta}{\int_{\theta_1-\varepsilon_1-\varepsilon_0}^{\theta_1}\int_{y^*(\theta)}^{y(\theta)}\left(-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}\right)dyd\theta}.$$

This would be true if we prove that for any (θ_L,y_L) and (θ_H,y_H) such that $\theta_1-\varepsilon_1-\varepsilon_0 < \theta_L < \theta_1$, $y^*(\theta_L) < y_L < y(\theta_L)$, $\theta_2 < \theta_H < \theta_2+\varepsilon_2+\varepsilon_0$, $y(\theta_H) < y_H < y^*(\theta_H)$,

$$\frac{\frac{\partial l^p(\theta_H,y_H)}{\partial y}f(\theta_H)}{-\frac{\partial^2 l^a(\theta_H,y_H)}{\partial\theta\partial y}} > \frac{\frac{\partial l^p(\theta_L,y_L)}{\partial y}f(\theta_L)}{-\frac{\partial^2 l^a(\theta_L,y_L)}{\partial\theta\partial y}}.$$

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Since $\dfrac{\frac{\partial l^p(\theta,y)}{\partial y}f(\theta)}{-\frac{\partial^2 l^a(\theta,y)}{\partial\theta\partial y}}$ is strictly increasing in y for any fixed θ, and $y_L < \theta_L + b\left(\theta_L\right)$, $y_H > \theta_H + b\left(\theta_H\right)$, it suffices to prove that

$$\frac{\frac{\partial l^p(\theta_H,\theta_H+b(\theta_H))}{\partial y}f\left(\theta_H\right)}{-\frac{\partial^2 l^a(\theta_H,\theta_H+b(\theta_H))}{\partial\theta\partial y}} \geq \frac{\frac{\partial l^p(\theta_L,\theta_L+b(\theta_L))}{\partial y}f\left(\theta_L\right)}{-\frac{\partial^2 l^a(\theta_L,\theta_L+b(\theta_L))}{\partial\theta\partial y}}.$$

However, this follows from the assumption. This completes the proof. ■

Proof of Theorem 7. Existence follows from Theorem 5. Uniqueness follows from the strict concavity of the function and linearity of constraints. ■

Proof of Claim 8. Suppose, to obtain a contradiction, that this does not hold. Then there is θ_0 such that $y\left(0\right) < y\left(\theta_0\right) < y\left(1\right)$ (which means, in particular, that $0 < \theta_0 < 1$) and $y\left(\theta_0\right) \neq \min\left\{z\left(\theta_0\right),\theta_0 + b\right\}$. First, consider the case where $z\left(\theta\right)$ is increasing or constant (note that it is a linear function of θ). Suppose $y\left(\theta_0\right) < \min\left\{z\left(\theta_0\right),\theta_0 + b\right\}$. Then, by continuity of $y\left(\theta\right)$ (a constraint in the optimization problem) and the assumption that $y\left(\theta_0\right) < y\left(1\right)$, there exists $\theta' > \theta_0$ such that $y\left(\theta'\right) < \min\left\{z\left(\theta'\right),\theta' + b\right\}$ and $y\left(\theta_0\right) < y\left(\theta'\right)$. But then slightly increasing $y\left(\theta\right)$ for $\theta \in \left(\theta_0,\theta'\right)$ while preserving $y\left(\theta_0\right)$ and $y\left(\theta'\right)$ would decrease the value function without violating the constraints. Now suppose $y\left(\theta_0\right) > \min\left\{z\left(\theta_0\right),\theta_0 + b\right\}$; since $y\left(\theta_0\right) \leq \theta_0 + b$ (a constraint in the optimization problem), we must have $z\left(\theta_0\right) < y\left(\theta_0\right) \leq \theta_0 + b$. Since $z\left(\theta\right)$ is increasing or constant and $y\left(\cdot\right)$ is continuous, we can choose $\theta' < \theta_0$ such that $z\left(\theta'\right) < y\left(\theta'\right) < y\left(\theta_0\right)$. Then if we slightly decrease $y\left(\theta\right)$ for $\theta \in \left(\theta',\theta_0\right)$ while preserving $y\left(\theta'\right)$ and $y\left(\theta_0\right)$ would decrease the value function without violating the constraints. So, if $z\left(\theta\right)$ is not decreasing, we get to a contradiction.

Now suppose that $z\left(\theta\right)$ is strictly decreasing. Let us first suppose that $y\left(\theta_0\right) > \min\left\{z\left(\theta_0\right),\theta_0 + b\right\}$, i.e., $z\left(\theta_0\right) < y\left(\theta_0\right) \leq \theta_0 + b$. Then $z\left(1\right) < y\left(1\right) \leq 1 + b$, so we could slightly decrease $y\left(\theta\right)$ for $\theta \in \left(\theta_0,1\right]$ while preserving $y\left(\theta_0\right)$ and thereby make $y\left(\theta\right)$ closer to $z\left(\theta\right)$ on $\left(\theta_0,1\right]$. This means, in particular that in this case, if for some θ', $y\left(\theta'\right) = z\left(\theta'\right)$ then $y\left(\theta'\right) = y\left(1\right)$: indeed, this is trivially true if $\theta' = 1$, while if $\theta' < 1$ and $y\left(\theta'\right) \neq y\left(1\right)$ then there exists $\theta > \theta'$ such that $z\left(\theta\right) < y\left(\theta\right) < y\left(1\right)$, which is, as we just proved, impossible. Now consider the remaining case, $y\left(\theta_0\right) < \min\left\{z\left(\theta_0\right),\theta_0 + b\right\}$. There are two possibilities. If $z\left(1\right) \geq y\left(1\right)$, then, as before, there exists $\theta' > \theta_0$ such that $y\left(\theta'\right) < \min\left\{z\left(\theta'\right),\theta' + b\right\}$ and $y\left(\theta_0\right) < y\left(\theta'\right)$, and slightly increasing $y\left(\theta\right)$ for $\theta \in \left(\theta_0,\theta'\right)$ while preserving $y\left(\theta_0\right)$ and $y\left(\theta'\right)$ would decrease the value function. If, however, $z\left(1\right) < y\left(1\right)$, then there is some $\theta' \in \left(\theta_0,\theta_H\right)$ for which $y\left(\theta'\right) = z\left(\theta'\right)$. But then, as we argued above, $y\left(\theta'\right) = y\left(\theta_H\right) > y\left(\theta_0\right)$. Hence, if we slightly increase $y\left(\theta\right)$ for $\theta \in \left(\theta_0,\theta'\right)$ while preserving $y\left(\theta_0\right)$ and $y\left(\theta'\right)$, we would decrease the

value function, again contradicting that the contract given by $y(\cdot)$ is optimal. This contradiction completes the proof of Claim 8. ∎

Proof of Theorem 9. Since λ is endogenous, consider the following procedure. Start with $\lambda = 0$ and take the optimal action schedule $y(\theta)$ for $\lambda = 0$. If for $T = \tilde{w}$, condition (15) holds, then this $y(\theta)$ and $T = \tilde{w}$ constitute the optimal contract. If not, we increase λ gradually from $\lambda = 0$ to $\lambda = 1$, take $y(\theta)$ for this λ, and $T = \tilde{w}$; then the left-hand side of (15) is strictly increasing in λ, and if for some λ it holds with equality, we have found the optimal contract. Finally, if even for $\lambda = 1$ (15) holds as a strict inequality for $T = \tilde{w}$, this means that the minimal transfer constraint (18) is not binding, and the optimal contract is found by taking $\lambda = 1$ and T such that (15) holds (in this last case, the participation constraint will bind and T will be a function of $\tilde{u}$). Naturally, λ will be increasing in $\tilde{u}$ and decreasing in $\tilde{w}$, but also increasing in A. Our immediate goal is to show that this procedure delivers a unique λ and the unique optimal contract.

An optimal contract minimizes the Lagrangian (19) for its λ and $\mu = 1 - \lambda$ subject to (20)–(21) and in it the complementarity slackness constraints hold. If we prove that this is true only for one λ, we would get that any contract that satisfies these properties is the optimal contract. To do so, for any λ, consider L_λ^a, the value of the agent's loss function at this λ (without taking the transfer T into account), i.e., $L_\lambda^a = \int_0^1 L^a(\theta) d\theta$ for the contract that minimizes (19) for this λ. We denote $L^a[y(\cdot)]$ this value calculated at an arbitrary $y(\cdot)$, and define $L^p[y(\cdot)]$ in a similar way. Let us show that L_λ^a is strictly decreasing in λ. Indeed, suppose $\lambda' > \lambda$ and $y_\lambda(\cdot)$ and $y_{\lambda'}(\cdot)$ are the contracts that minimize (19) subject to (20)–(21). Suppose, to obtain a contradiction, that L_λ^a does not decrease. Then

$$L^a[y_\lambda(\cdot)] \leq L^a[y_{\lambda'}(\cdot)]. \tag{40}$$

However, we have

$$L^p[y_\lambda(\cdot)] + \lambda L^a[y_\lambda(\cdot)] < L^p[y_{\lambda'}(\cdot)] + \lambda L^a[y_{\lambda'}(\cdot)], \tag{41}$$

because $y_\lambda(\cdot)$ is the unique contract that minimizes (19), and the two contracts $y_\lambda(\cdot)$ and $y_{\lambda'}(\cdot)$ are different (as can be easily seen, say, from the explicit characterization in the Supplementary Appendix). Multiplying (40) by $\lambda' - \lambda > 0$ and adding to (41), we get

$$L^p[y_\lambda(\cdot)] + \lambda' L^a[y_\lambda(\cdot)] < L^p[y_{\lambda'}(\cdot)] + \lambda' L^a[y_{\lambda'}(\cdot)].$$

However, this contradicts that $y_{\lambda'}(\cdot)$ minimizes (19) for λ', which proves that L_λ^a is monotonically decreasing in λ.

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If $\lambda \in (0,1)$, then both (15) and (18) must hold as equalities (so $T = \tilde{w}$), and now L_λ^a monotonically decreasing means that this can hold for at most one value of λ. Now, obviously, if at $T = \tilde{w}$ (15) holds for no $\lambda \in (0,1)$, then in equilibrium we should have $\lambda = 1$, and if (15) is slack for all $\lambda \in (0,1)$, then we should have $\lambda = 0$. It is now straightforward to see that the necessary condition suggested by Kuhn-Tucker theorem are also sufficient in this problem.

To establish the comparative statics results, observe the following. The result would follow trivially from the explicit characterization in the Supplementary Appendix if λ were fixed, but λ may change if b changes. To find how, fix other parameters and let λ_b be the Lagrangian multiplier λ at the optimum for a fixed b. Take the optimal contract $\{y_b(\cdot), T_b\}$ that solves the problem for this b. Suppose b increases to $b' > b$. Then, the contract that minimizes (19) for b' and the same $\lambda = \lambda_b$ (subject to (20)–(21)) now yields a (weakly) lower utility for the agent. Consequently, the optimal contract must be achieved for a (weakly) higher multiplier, so $\lambda_{b'} \geq \lambda_b$, and thus λ is nondecreasing in b. Therefore, if b decreases, λ does not increase and may decrease. Both effects make money-burning more likely.

Now suppose that $b < 1$. An increase in A affects the optimal contract per se, but also affects λ. By a similar reasoning, higher A makes the agent worse off for a given λ, and thus λ is nondecreasing in A. However, notice that the optimal contract only depends on A and λ through the ratio λ/A (and a higher ratio makes the agent better off). The ratio λ/A cannot increase as A increases: at λ that keeps the ratio fixed, the agent gets the same contract and the same utility, hence a further increase is not needed (but it may decrease as λ cannot become higher than 1). So, as a result of a marginal increase in A, either λ/A remains the same (if $\lambda \in (0,1)$, or λ must be fixed at 0 or 1. In all these cases, the likelihood and the amount of money-burning weakly increases.

The parameters $\tilde{u}$ and $\tilde{w}$ only affect the optimal contract through λ, which is nondecreasing in $\tilde{u}$ and nonincreasing in $\tilde{w}$. As the likelihood and amount of money-burning (weakly) decrease in λ, the result follows.

Now, take any $b < 1$ and any $\tilde{u}$, $\tilde{w}$. From the explicit characterization, the contract must feature money-burning if $A > 1$ (because $\lambda \leq 1$). Moreover, the likelihood of money-burning increases in A. Consequently, there is $A^*(\tilde{u}, \tilde{w}) < 1$ such that the contract features money-burning iff $A > A^*(\tilde{u}, \tilde{w})$. If $\tilde{u}$ increases, for a given A money-burning becomes less likely, so $A^*(\tilde{u}, \tilde{w})$ is (weakly) increasing in $\tilde{u}$; similarly, it is weakly decreasing in $\tilde{w}$.

Fix other parameters and decrease $\tilde{w}$ enough. Then, the constraint (8) would not bind for any $y(\cdot)$ constructed for A and any λ and T found from (5), hence we must have $\mu = 0$ and thus $\lambda = 1$. If, however, $\tilde{w}$ is high enough, then this constraint will be violated for all possible

$y(\cdot)$ and T found from (5), which implies that $\lambda = 0$ for large $\tilde{w}$. As λ is nonincreasing in $\tilde{w}$ and depends on it continuously, the result follows.

Finally, take some $A \in (0, 1)$, some $b < 1$, and then take $\tilde{u}$ low enough and $\tilde{w}$ high enough so that there is money-burning. The characterization above suggests that for this A, $\lambda = \lambda_A$ satisfies $\lambda_A < A$. As proven above, by increasing λ we only make money-burning more likely. Now, notice that $\lambda_A < 1$. Since the contract depends on the ratio λ_A/A only, if we decrease A, then for λ_A decreased proportionately the Kuhn-Tucker conditions would hold, and this would constitute the solution. However, then $\lambda_A < A$ would be preserved, and hence there would be money-burning. Consequently, for such b, $\tilde{u}$, $\tilde{w}$ there is money-burning for all parameter values. ∎

Proof of Theorem 10. For a fixed λ, both money-burning and contingent transfers are more likely to be used for low b, as the explicit characterization reveals. However, a higher λ makes money-burning less likely and contingent transfers more likely. As in Theorem 9, we can prove that λ increases in b. Consequently, a lower b makes money-burning more likely, while the effect on contingent transfers is ambiguous.

Now suppose that $b < 1$. The constraint is binding if and only if $\mu > 0$ ($\lambda < 1$). Now, the results follow from the explicit characterization. Finally, as in Theorem 9, λ is increasing in $\tilde{u}$ and decreasing in $\tilde{w}$. As the optimal contract depends on $\tilde{u}$ and $\tilde{w}$ through λ only, the result follows. ∎

9 References

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