Non-grant Microfinance, Incentives, and Efficiency

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Abstract

This paper shows that charging interest on funds provided by donors or investors to microfinance institutions (MFIs) can increase efficiency, the total number of extended loans, and borrowers' welfare compared to grant or concessionary funding. In a setting in which MFIs supply costly effort, putting a price on, or raising the price of microcredit funds improves the MFIs' incentives to supply effort in loan administration or monitoring, to extend more loans, and/or cut down overhead costs. The result is robust to several variations of the benchmark model that allow for endogenous MFI lending interest rate, motivated MFIs, and endogenous loan overhead costs.

Keywords: non-grant microfinance, for-profits, incentives, overhead costs

JEL Classifications: O12, D21

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1 Introduction

The 2000s saw major changes in the microfinance sector. While the early varieties of microcredit used to rely almost exclusively on grant or concessionary public funds, recent years have brought the entry and expansion of non-grant and for-profit providers of microcredit funds.[1] In addition, several high-profile microfinance institutions (MFIs) that had been formerly funded through grants or near-grant 'soft' loans have gone public to be able to tap into commercial sources of capital and grow faster. A leading example is Mexico's largest microlender *Banco Compartamos* which was founded in 1990 as an NGO, then switched its operations to a for-profit company in 2000, and performed an IPO in 2007 (Rosenberg, 2007). Another example is *SKS Microfinance* in India which was listed on the Bombay Stock Exchange in 2010 and came under scrutiny in relation to a wave of suicides by heavily indebted borrowers in the state of Andhra Pradesh.[2]

These developments have led to a backlash from some prominent public figures including the 'father of microfinance' Muhammad Yunus who exclaimed in a *New York Times* article titled "Sacrificing Microcredit for Megaprofits": "...I never imagined that one day microcredit would give rise to its own breed of loan sharks" (Yunus, 2011). Yunus went on to call for stricter regulation of the sector, to prevent lenders "...take advantage of the vulnerable". For additional background on the evolution of microfinance from its roots as purely non-profit to its current state and discussion on whether the additional funds or entry by for-profits in the sector benefit or harm the poor, see Cull et al. (2009). Empirically, the notion that charging higher interest rates by for-profit microlenders is automatically bad for the poor does not find confirmation in the randomized study by Angelucci, Karlan and Zinman (2013) who report overall positive effects and little harm to borrowers from an expansion of group lending at an APR as high as 110%.

This paper contributes to the ongoing debate on the role of non-grant funding and for-profits in microfinance by making a simple theoretical point to argue that some of the objections that have been raised could be misguided. I show that under certain conditions, charging positive interest on donor-provided funds to microlenders (alternatively, investors requiring a positive return) can have an incentivizing effect on MFIs and lead to increased efficiency and total lending in the sector, leading in turn to higher welfare for microcredit borrowers. The result obtains both in the case of an exogenous ('open economy') and endogenous (e.g., 'free entry') MFI lending interest rate and is robust to variations of the baseline setting which allow for non-zero MFI profits, motivated MFIs, and endogenous loan overhead costs.

The key mechanism underlying the result is that microlenders are assumed to supply a costly action (hereafter, 'effort') during their operations. For example, the action could be effort in loan monitoring, helping borrowers choose or implement their projects, or other administrative tasks.

[1] For example, Reille, Forster and Rozas (2011) document an increase in institutional investors in microfinance, which include banks, private equity funds, pension funds, and insurance companies, from $0.5bln in 2005 to over $3.5bln (27% of the total surveyed by CGAP) in 2010.

[2] De Quidt, Fetzer and Ghatak (2012) provide an excellent review of the circumstances surrounding these events and the current state of the Indian microfinance sector in general.

Supplying a higher effort level increases the expected total repayment from the loans an MFI makes. The MFI effort is unobservable or non-contractible to the donor or investor. In this setting, I show that an increase in the price (the interest rate) of funds provided to MFIs can increase the MFIs' incentives to supply effort or reduce overhead costs. The higher interest rate increases the donor's incentives to provide larger amount of funds to the MFIs. In turn, the need to repay the funds plus interest causes MFIs to issue a larger number of loans and motivates them to supply more effort in order to increase the probability of loan repayment (borrower's project success) or expected total repayment, or to cut overhead costs. Combined, these effects (more loans and higher borrower's project success probability) lead to an increase in borrower welfare.

Most of the existing theoretical literature on microfinance has analyzed the contractual relationships and outcomes between microlenders and their borrowers (Besley and Coate, 1995; Ghatak and Guinnane, 1999; Ghatak, 2000; Ahlin, 2014 among many others). In contrast, in this paper I keep the borrowers' side of the setting as simple as possible and focus on the contractual relationship between the microcredit funds donors or investors and the microlenders (MFIs).

In a related and complementary work, Ghosh and Van Tassel (2013) derive conditions under which microfinance donors, by charging interest on provided funds, alleviate an adverse selection problem by squeezing out of the market high-cost microlenders which would otherwise operate if grant funding were given. While I also study the effect of charging interest on funds as opposed to grant funding, in contrast to Ghosh and Van Tassel (2013), the argument here is not about cost heterogeneity and adverse selection among MFIs but instead about the incentives of each individual microlender to supply costly effort in monitoring, screening, reducing overhead costs, etc. when loanable funds come at a price. Both effects are likely present in practice.

The paper also relates to other works on the role of fund providers (as opposed to microlenders) in microfinance outcomes. For example, Roy and Roy Chowdhury (2009) build a model in which a motivated NGO can help in channeling credit to the poor both in borrower selection or in project implementation. The authors show that an efficiency distortion can arise due to the fact that the NGO puts higher weight on poorer, less efficient borrowers that can benefit from on-the-job training provided by the NGO. The inefficiency can be resolved by unbundling borrower selection from project assistance. Other authors, e.g., Guha and Roy Chowdhury (2013) allow the MFIs themselves (instead of fund providers) to be partially profit- and socially-oriented by putting respective weights in the objective function and show that double-dipping (borrowers take multiple loans) can occur if MFIs are more profit-oriented.

Finally, the main premise on which the model is based, that MFIs could vary their effort, or more generally vary in their ability to perform their activities is consistent with the empirical evidence as reviewed in Cull et al. (2009) and relates to some of the theoretical literature on NGOs (for example, Aldashev and Verdier, 2010).

2 Model

2.1 Basic setting

A donor or investor (for example, an NGO, a government, an investment fund) funds small business projects operated by borrowers with zero wealth via a competitive microfinance sector composed of a continuum of mass one of identical microlenders (MFIs). Each project requires a loan of size 1 and yields a gross return $\theta > 1$ if successful or zero otherwise. The project returns are i.i.d. across projects / borrowers. Each MFI finances a large number of projects. The donor/investor's opportunity cost of funds is normalized to one. MFIs charge gross interest rate R, where $\theta > R > 1$. One can think of R as the going market rate on microfinance loans or as exogenously given – this would not be essential for the main results. Holding R fixed can be also interpreted as shutting down the pass-through from cost of funds to MFIs to final borrower interest rates. The case of an endogenous MFI lending rate R (opening the pass-through between MFI cost of funds and borrower interest rates) is studied in Section 2.4.

The donor or investor charges a gross interest rate $r \in [0, R]$, on funds provided to MFIs. If $r = 0$, the funds can be thought of as 'pure grant' (no repayment is required). If $r = 1$, the donor wants the funds repaid but charges zero interest rate (a 'concessionary loan'). If $r > 1$, the lender requires to be repaid more than his opportunity cost of funds (a 'for-profit' investor).

The donor / investor decides on the amount of funding x (which equals the number of loans MFIs make) to maximize the following objective function

$$V(x) \equiv B(x) + rx - x \tag{1}$$

where $B(x)$ is strictly concave with $B'(0) > 1$ and $B'(\infty) < 1 - r$.

The interpretation of the function $B(x)$ is the *net benefit* to the donor/investor of providing funds x *in addition to* the interest income net of opportunity cost of funds, $rx - x$. For example, $B(x)$ could equal $-c(x)$ where $c(x)$ is a convex cost function of supplying funds x. Alternatively, $B(x)$ could equal $\beta(x)$ with $\beta(.)$ concave, corresponding to the net, possibly non-monetary, (e.g., outreach) utility which the donor obtains from providing the funds.[3] The assumption $B'(0) > 1$ ensures that a positive amount x would be provided even at $r = 0$ (pure grant funding). Assuming $B'(\infty) < 1 - r$ ensures the boundedness of $V(x)$ and a finite optimal x.

MFIs perform a costly action, $z \in Z \equiv [\underline{z}, \bar{z}] \in \mathbb{R}_+$ which affects the probability, $p(z)$ with which an MFI loan is repaid, that is, the probability that a project financed by the loan is successful. One can think of z as 'effort' supplied by MFIs in borrower monitoring, loan administration, financial advice, etc. The MFI effort level z is not observed by the donor/investor or is non-contractible. If a loan is not repaid, which assuming limited liability by the borrower, happens when the project fails, then the MFI obtains zero return on that loan. The function $p(z)$ is strictly increasing and

[3] This is similar to the assumption of McIntosh and Wydick (2005) who define the objective of non-profit MFIs to be maximizing outreach.

strictly concave and satisfies $p(z) \in [0,1)$, $\forall z \in Z$ and $p'(\underline{z}) = +\infty$. Alternatively, one can think of $p(z)$ as the fraction of the x MFI loans which are repaid. Both interpretations imply that the expected total repayment received by the MFIs is $p(z)Rx$.[4]

Suppose that it costs $c(z,x) = zf(x)$ to supply MFI effort level z when financing x projects.[5] The function $f(x)$ is increasing and concave ($f' \geq 0$ and $f'' \leq 0$) with $f(0) > 0$. Alternatively, all results go through for f increasing and strictly concave with $f(0) \geq 0$. The role of $f(x)$ is to allow for scaling-up of the total costs of MFI effort (monitoring, loan administration, etc.), as the number of loans x increases. For example, monitoring borrowers to lower average default probabilities costs more the more borrowers there are. The assumed concavity of f implies that, for any given z, there are economies of scale to making more loans – that is, total costs increase in x but at a (weakly) decreasing rate. It is reasonable to think that monitoring, loan administration, etc. actions have this feature. The results below would not hold if the action z were such that MFIs total effort costs are instead increasing in the number of loans x at an accelerating rate. Note that the special case of effort costs constant in x, for example, $f(x) = k$ for some constant $k > 0$ is also allowed.

Assuming that microfinance borrowers are risk-neutral and maximize expected net income implies that the borrowers' total expected payoff given total funding (number of loans made), x and MFI action level, z equals:

$$W(x,z) = xp(z)(\theta - R) \quad \text{(BP)}$$

while the expected payoff (net income) per borrower is $w(z) = p(z)(\theta - R)$. Clearly, $W(x,z)$ is strictly increasing in x and z and $w(z)$ is strictly increasing in z. Beyond keeping track of these expected payoffs, the borrowers' side of the model is kept as simple as possible on purpose, to focus on the interaction between donors/investors and the MFI lenders.

2.2 Optimal MFI funds and effort

The donor/investor chooses the amount of funds, x to maximize his objective, (1) subject to two constraints. First, due to the effort non-contractibility, the MFIs' individual incentives to supply costly effort must be taken into account. Given a required per-loan repayment (gross interest rate) r, an MFI would choose its effort level z to maximize its own expected payoff, which equals the expected revenue, $p(z)Rx$ minus total costs (repaying the donor plus effort costs), $rx + zf(x)$. Call this the 'incentive-compatibility constraint' (ICC),

$$z = \arg\max_{\tilde{z}} \; p(\tilde{z})Rx - rx - \tilde{z}f(x) \quad \text{(ICC)}$$

[4] Suppose there are N loans made to investment projects with i.i.d. outcomes. Each project repays R if it is successful and zero otherwise. For simplicity, call $p \equiv p(z)$ the probability of success and define Y to be the random variable "number of successes among N draws from the binomial distribution with parameter p". It is well known that $E(Y) = Np$. Since each success yields R, the expected total repayment then equals $E(Y)R = NpR$.

[5] The linearity in z is without loss of generality. One could alternatively assume a convex function $\phi(z)$ but, since only the ratio $\frac{p'(z)}{\phi'(z)}$ matters for the optimal effort decision, assuming constant marginal effort cost (given x) is without loss of generality.

In Section 2.3 I also discuss the possibility of "motivated" MFIs as in Besley and Ghatak (2005), that is MFIs who also derive non-pecuniary utility from supplying the action z.

Second, MFIs must break even – call this the 'participation constraint' (PC),[6]

$$p(z)Rx - rx - zf(x) \geq 0 \text{ (PC)}$$

The optimal amount of funds (loans), x^* and MFI effort, z^* are chosen to maximize the objective, (1) subject to the constraints (ICC) and (PC). Since $p(z)$ is strictly increasing and strictly concave with $p'(\underline{z}) = +\infty$, (ICC) implies that, for any given $r \in [0, R]$ and $x > 0$, the optimal effort level chosen by an MFI either satisfies the first-order condition,

$$p'(z)R\frac{x}{f(x)} = 1 \text{ (ICC')}$$

at an interior $z^* \in (\underline{z}, \bar{z})$ or, if $p'(\bar{z})R\frac{x}{f(x)} > 1$, then $z^* = \bar{z}$ (see Example 1 with functional forms below). Given the interest rate r, the problem of the donor/investor in the interior solution case is:

$$\max_{x,z} \; B(x) + rx - x \tag{DP}$$
$$\text{subject to (ICC') and (PC)}$$

I next analyze the effect of a local increase in the MFI funds rate r (for instance, moving from pure grant funding, $r = 0$ to concessionary loan funding, $r > 0$) on the total amount of funds x provided (loans made) and MFI effort level z solving problem (DP). Suppose that,

Assumption A1:

$$r < p(\bar{z})R \tag{2}$$

Together with the assumptions on the derivative of $B(x)$ stated earlier, Assumption A1 guarantees the existence of non-trivial solution to problem (DP).[7] Clearly, if the MFI funds rate r is too large relative to the MFI lending rate R, there is no way to satisfy the MFI participation constraint even at maximum effort. Under Assumption A1 and the assumptions on $B(x)$ and $f(x)$ the solution (x^*, z^*) to (DP) satisfies $x^* > 0$.

Depending on parameter values, there are two possible scenarios that can occur when solving problem (DP), either

case (i) constraint (PC) does not bind (MFIs break even or make profits) at the optimal (x^*, z^*) where x^* is determined from the lender's FOC, $B'(x) + r - 1 = 0$ and z^* solves (ICC) evaluated at x^*, or

[6] Alternatively, zero expected MFI profits could also arise as a result of competition (free entry) in the MFI sector among profit-maximizing microlenders. See Section 2.3 for more discussion.

[7] If $p(\bar{z})R = r$ and $\underline{z} = 0$ the problem has the trivial corner solution $z^* = x^* = 0$. Otherwise, if $p(\bar{z})R < r$, or if $p(\bar{z})R = r$ and $\underline{z} > 0$, it is clear from (PC) that no feasible x and z exist.

case (ii) MFIs make a loss at the unconstrained solution from case (i), which implies that constraint (PC) must bind at optimum and hence the optimal x and z are determined by solving (ICC') and (PC) taken at equality.

In both cases (i) and (ii) we obtain,

Proposition 1.

Suppose Assumption A1 holds. Then,

(a) the MFI effort level, z^* *and total number of loans made,* x^* *solving problem (DP) are both increasing (strictly at an interior solution) in the donor/investor funds rate,* r*.*

(b) the borrowers' total expected payoff, $W(x^*, z^*)$ *is strictly increasing in* r *and the borrowers' individual expected payoffs,* $w(z^*)$ *are increasing (strictly at an interior solution) in* r*.*

Proof (see Appendix)

The intuition for Proposition 1 is as follows. In case (i), when the MFI participation constraint (PC) does not bind, the MFI funds provider chooses his preferred funding amount x^* that solves $B'(x) = 1 - r$. A local increase in r makes providing funds more beneficial to the donor since he receives additional revenue. Consequently, the donor provides more funds – x^* increases. In addition, the MFI's marginal benefit of supplying effort (see condition ICC') is increasing in the funds received (loans made) and hence its effort choice z^* goes up as well (strictly for an interior z^*).

In case (ii), with a binding MFI participation constraint (PC), note that the higher is the funds rate r, the larger is the amount a microlender needs to pay back to the funds provider in order to break even. But the more an MFI needs to repay, the higher the effort it needs to supply. Supplying more effort increases the expected marginal return on microcredit loans by raising the probability of success $p(z)$ for all financed projects, or equivalently, raising the fraction of financed projects that succeed, which in turn increases the MFI's incentive to supply effort. Hence, putting a price (if $r = 0$ initially) or raising the price on MFI funds can have an incentivizing effect and lead to more loans made and higher project success rate. Combined, these factors increase the expected total and individual payoffs of microcredit borrowers.

The classic 'debt-overhang' problem in credit markets, whereby a higher required repayment reduces a borrower's incentives to supply costly effort (e.g., see Ghosh at al., 2000) does not apply here since this argument relies on limited liability – the borrower repays only if her project succeeds. This possibility does not arise between the donor/investor (the lender) and the MFI (the borrower) in the current setting because of the large number of MFI loans issued which finance projects with i.i.d. returns (MFI's income is non-stochastic).

Example 1

Let $0 \leq r < 1$ *and suppose* $B(x) = \sqrt{x}$, $p(z) = \sqrt{z}$, $f(x) = \frac{\sqrt{x}}{2}$ *and* $Z = [0, \bar{z})$ *with* $\bar{z} < 1$. *It is easy to check that all assumptions on* $B(x)$, $p(z)$ *and* $f(x)$ *are satisfied. Assumption A1 requires also* $r < R\sqrt{\bar{z}}$.

Consider first case (i) in which (PC) does not bind at optimum (conditions are provided below). The donor's first-order condition implies $x^* = \frac{1}{4(1-r)^2}$ *and so, using (ICC') assuming interior solution,* $z^* = \frac{R^2}{4(1-r)^2}$ *– hence both* x^* *and* z^* *are increasing in* r. *We also have,* $W(x^*, z^*) = \frac{R}{8(1-r)^3}(\theta - R)$ *and* $w(z^*) = \frac{R}{2(1-r)}(\theta - R)$, *both of which strictly increase in* r. *For example, at* $r = 0$ *we have:* $x^* = \frac{1}{4}$, $z^* = \frac{R^2}{4}$ *(assume* $\frac{R^2}{4} < \bar{z}$*), and MFI profits are* $(p(z^*)R - r)x^* - z^* f(x^*) = \frac{R^2}{16} > 0$ *– (PC) does not bind indeed. By a standard continuity argument, a local increase in the funds rate* r *starting at* $r = 0$ *(a switch from pure grant to concessionary funding) thus strictly increases the number of loans provided,* x^* *and MFI effort,* z^* *and results in higher borrower welfare.*

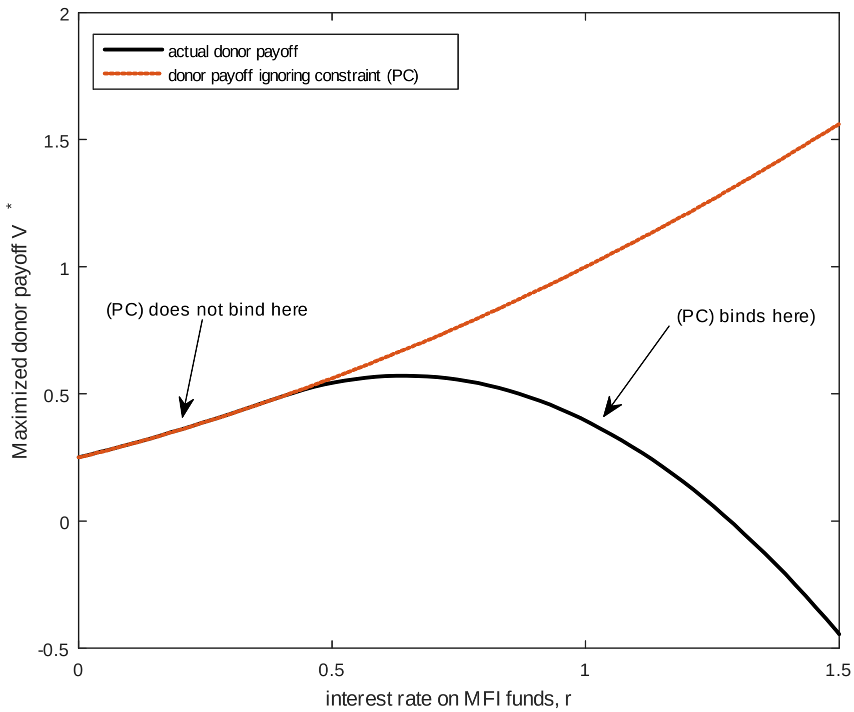
Next, consider case (ii) in which (PC) binds at optimum. That is, the MFI profits at the case (i) unconstrained solution, $x^* = \frac{1}{4(1-r)^2}$ *and* $z^* = \frac{R^2}{4(1-r)^2}$ *are negative,* $\frac{R^2}{16(1-r)^3} - \frac{r}{4(1-r)^2} < 0$, *or* $R < 2\sqrt{1-r}\sqrt{r}$. *At an interior solution for effort, (ICC') implies* $z = R^2 x$. *Substituting into (PC) yields* $R\sqrt{x}Rx - rx - R^2 x\frac{\sqrt{x}}{2} = 0$ *and hence the optimum is* $x^c = \frac{4r^2}{R^4}$ *and* $z^c = \frac{4r^2}{R^2}$, *both of which increase in* r. *We also obtain* $W(x^c, z^c) = \frac{8r^3}{R^5}(\theta - R)$ *and* $w(z^c) = \frac{2r}{R}(\theta - R)$ *which are also strictly increasing in* r. *Interior effort obtains for* $r < \frac{R\sqrt{\bar{z}}}{2}$ *(see the proof of Proposition 1). If, instead,* $r \in [\frac{R\sqrt{\bar{z}}}{2}, R\sqrt{\bar{z}})$ *then the MFIs supply the maximum effort* $\bar{z}$ *and a local increase in* r *does not raise MFI effort and the individual payoff* $w(\bar{z})$ *but still increases the total number of loans* x^c *and the borrowers' total payoff,* $W(x^c, \bar{z})$.

2.3 Discussion

1. The analysis above took the funding interest rate r as given and considered a local increase. If constraint (PC) does not bind at the initial r (for example, suppose MFIs make positive profit at the $r = 0$ optimum, x^* and z^*) then, by the Envelope theorem, the donor payoff $V(x^*)$ is strictly increasing in r since $\frac{\partial V(x^*)}{\partial r} = x^*$. In addition, by Proposition 1, total lending, the MFIs effort, and borrower welfare all locally increase in r.

As r increases further, however, the MFI participation constraint (PC) can eventually bind and then (see case (ii) above) the funding amount x^c and the MFI effort z^c which solve (DP) are found from (ICC) and setting (PC) at equality. In this case there are two countervailing effects on the donor payoff $V(x)$ from increasing the funds rate r. On the one hand, the term rx goes up which raises $V(x)$ but, on the other hand the funds amount x^c no longer satisfies the donor's FOC, which lowers $V(x)$. Since in case (ii) MFIs make a loss at the x^* which solves the donor's FOC $B'(x) = 1 - r$, the latter effect means that the donor must increase the funding amount x^c above x^* to enable MFIs to break even (and microcredit to be provided), at the cost of reducing his own payoff. Therefore, in the case of binding (PC), an optimal funds rate r can exist, balancing these two effects. The exact outcome can be worked out numerically and in general depends on

Figure 1: Payoff maximizing funds rate, r



the assumed functional forms and parameters.

For example, suppose $B(x) = 2x - x^2$, $p(z) = \sqrt{z}$, $f(x) = 1$, *and* $Z = [0, \bar{z})$ *with* $\bar{z} < 1$. *Figure 1 below illustrates the maximized donor payoff* $V(x^*(r))$ *plotted as function of* r *over the interval* $[0, 1.5]$ *at* $R = 1.5$.[8] *In this example the donor payoff-maximizing rate* r *is about 0.65.*

In either case, the interest rate r on funds provided to MFIs is bounded in the model by the largest r for which a solution to problem (DP) exists, which is $Rp(\bar{z})$; otherwise (PC) cannot be satisfied. In reality, r could be also capped by competition among donors, donors' mission, or by the possibility of strategic default by MFIs. These issues remain beyond the scope of the paper. Everything needed for the conclusion that an increase in the interest rate r can increase funding, efficiency, and borrower welfare is that the Proposition 1 result hold locally in a neighborhood to the right of $r = 0$ for grant donors or to the right of $r = 1$ for investors (see also Section 3).

2. Proposition 1 does not require that MFIs profits be bounded from below by zero. Suppose

[8] The maximized donor payoff is computed using either the solution at which (PC) does not bind ($x^* = \frac{1+r}{2}$ and $z^* = \frac{R^2(1+r)^2}{16}$) or the solution at which (PC) binds ($x^c = \frac{4r}{R^2}$ and $z^c = \frac{4r^2}{R^2}$), as is appropriate for any given r (see cases i and ii in Section 2.2).

that, instead of (PC), MFIs must satisfy the alternative participation constraint,

$$p(z)Rx - rx - zf(x) \geq \pi \text{ (IPC)}$$

for some positive or negative value π. This could correspond to MFIs with market power ($\pi > 0$) or MFIs with a 'soft' budget constraint ($\pi < 0$). Going over the steps in the proof of Proposition 1, it is easy to see that all results obtain, as long as π is not too large or small.

3. The results on borrower welfare in Proposition 1 hinge on the implicit assumption that, as the MFI action level z goes up, its borrowers do not incur any additional costs that could reduce their welfare. For example, if MFIs put more effort to raise the repayment rate of their borrowers, this could come from more intensive monitoring and may cause the borrowers to exert additional costly effort themselves. Not accounting for this possibility would overstate the gains to the borrowers from the increase in MFI's effort z caused by the increase in r. On the other hand, if the MFI effort z is in the form of the loan administration quality or financial advice to borrowers, then the borrowers are unlikely to incur extra costs as z goes up.

4. In modeling the MFI's effort choice, I align with the classical principal-agent literature by treating the 'agent' (the MFI) as an expected income maximizer. If, in contrast, e.g., as in Besley and Ghatak (2005), MFIs were "motivated", that is, derive additional non-pecuniary utility $\psi > 0$ per unit of supplied effort, then the incentive-constraint (ICC') becomes $p'(z)Rx + \psi = f(x)$ – the marginal cost of MFI effort is effectively decreased.[9] It is then easy to show, following the steps in the proof of Proposition 1, that, for ψ positive and sufficiently small, by continuity, the proposition results remain intact – the number of loans, MFI's effort level, and the borrowers' payoffs all increase in r.[10]

2.4 Endogenous MFI lending rate, R

Thus far and in Proposition 1 the MFI lending interest rate R was treated as exogenously given. As shown below, this assumption is not critical for the main results, with the possible exception of the effect on borrower welfare. Specifically, consider the following variant of the model in which, for example because of free entry in the MFI sector or because of the non-profit mandate of MFIs, the microcredit interest rate R adjusts so that MFIs always make zero expected profits given the funding terms (r, x).

[9] Guha and Roy Chowdhury (2013) model 'motivated' MFIs differently – they assume that their objective function is a weighted sum of the borrowers' total payoff and the MFI's own payoff.

[10] In the context of Example 1, in the case of binding (PC) and interior effort choice, we have $z = \frac{R^2x^2}{4(1-\psi)^2}$ from the new (ICC) which, substituting into (PC) and solving for x yields, $x^{**} = \frac{4r(1-\psi)^2}{R^2(1-2\psi)}$ and thus $z^{**} = \frac{4r^2(1-\psi)^2}{R^2(1-2\psi)^2}$. Therefore, for $\psi \in (0, 1/2)$ both x^* and z^*, and hence $W(x^*, z^*)$ and $w(z^*)$, are still strictly increasing in r. In the case of non-binding (PC), it is imediate to see that x^* solving $B'(x) = 1 - r$ and z^* solving the new (ICC') $p'(z)Rx^* + \psi = f(x)$, both locally increase in r.

The donor/investor maximizes his payoff, $V(x) = B(x) + rx - x$ by choice of x and z subject to the MFIs' incentive constraint. The MFIs choose z taking the market lending rate R as given,

$$z = \arg\max_{\tilde{z}}\ p(\tilde{z})Rx - rx - \tilde{z}f(x) \quad \text{(ICC)}$$

The equilibrium lending rate R must be such that,

$$p(z)Rx - rx - zf(x) = 0 \quad \text{or,}$$
$$R = \frac{rx + zf(x)}{p(z)x} \quad \text{(ER)}$$

Condition (ER) ensures zero expected profits for the MFIs in equilibrium. The equilibrium R^* will satisfy this condition evaluated at the optimal x and z. The donor/investor takes into account the effect of his choice of x and z (subject to ICC) on the equilibrium MFI lending rate R^*. Suppose also that the parameters are such that $R = \frac{rx+zf(x)}{p(z)x} < \theta$ at the optimal x and z – the project return exceeds the required repayment; otherwise no one would like to borrow from MFIs (see Example 2 for functional forms and parameters that satisfy these assumptions).

The funding amount x is chosen by the donor/investor to maximize his objective $V(x)$ subject to the incentive constraint (ICC) and the equilibrium condition (ER). Consider first the case of interior z satisfying the MFI's first-order condition (ICC'), $p'(z)R\frac{x}{f(x)} = 1$ derived as in Section 2.2. Substituting R from (ER) into (ICC'), we obtain

$$\frac{p'(z)}{p(z)}\frac{rx}{f(x)} + \frac{p'(z)z}{p(z)} = 1 \quad \text{(ICC")}$$

The donor maximizes $V(x)$ subject to (ICC"). This implies that $x^*(r)$ solves

$$B'(x) = 1 - r \quad \text{(FOC)}$$

and so $x^*(r) = B'^{-1}(1 - r)$ which is strictly increasing in r. The optimal MFI effort solves (ICC") evaluated at $x^*(r)$,

$$\frac{p'(z)}{p(z)}\frac{rx^*(r)}{f(x^*(r))} + \frac{p'(z)z}{p(z)} = 1 \qquad \text{(eq3)}$$

We show in Proposition 2 below that the solution to (eq3), $z^*(r)$ is strictly increasing in r. If, instead, $\frac{p'(\bar{z})}{p(\bar{z})}\frac{rx^*(r)}{f(x^*(r))} + \frac{p'(\bar{z})\bar{z}}{p(\bar{z})} > 1$, then there is a corner solution for effort, $z^* = \bar{z}$ which is non-decreasing in r.

Proposition 2 (endogenous MFI lending rate)

With endogenous MFI lending rate R,

(a) the optimal MFI effort, z^* and total number of loans made, x^* are both increasing (strictly at an interior solution) in the interest rate on donor/investor funds, r.

(b) the equilibrium MFI lending rate $R^* \equiv \frac{rx^*+z^*f(x^*)}{p(z^*)x^*}$ *is decreasing in* r*, the borrowers' total expected payoff,* $W(x^*, z^*)$ *is strictly increasing in* r*, and the borrowers' individual expected payoff,* $w(z^*)$ *is increasing (strictly at an interior solution) in* r*, if the following condition is satisfied:*

$$\frac{dR^*}{dr} \leq 0 \quad \textit{(I1)}$$

at the optimal x^*, z^* *solving (FOC) and (eq3).*[11]

(c) it is possible that the individual and total borrowers' expected payoffs, $W(x^*, z^*)$ *and* $w(z^*)$ *are increasing in* r *even when the sufficient condition (I1) is not satisfied (that is, when* $\frac{dR^*}{dr} > 0$*).*

Proof (see Appendix)

The intuition for part (a) follows directly from the fact that MFI's optimal effort, z^* increases in the total funds (number of loans) x as the marginal product of effort goes up while the total amount of funds x^* increases in r because the donor's marginal benefit of lending increases. Regarding part (b), note first that the effect of a higher r on the MFI lending rate R^* is ambiguous and in general depends on the functional forms or parameter values. A higher interest rate r directly increases R^* in order to satisfy the equilibrium condition (ER) but, on the other hand, the higher induced effort level z^* works in the opposite direction – since projects succeed more often (or more projects succeed on average) a lower lending rate is required to obtain the same expected revenue $p(z^*)R$ per dollar lent. Consequently, there are also two countervailing effects from raising the donor interest rate r on borrowers' payoffs: the increase in z^* triggered by the increase in r raises the borrowers' expected payoff as it increases $p(z^*)$, whereas the possible increase in the interest rate R^* that borrowers must pay can act in the opposite direction.

Proposition 2(b) shows that the equilibrium MFI lending rate R^* being weakly decreasing in r is a sufficient condition for the borrowers' total and individual welfare to both increase in r, since z^* and x^* increase in r. Intuitively, the higher effort level z^* implies that borrowers' projects succeed more often, which together with the (weakly) lower interest R^* makes individual borrowers better off in expectation. The borrowers' total expected payoff, $W(x, z^*)$ goes up for three reasons: the larger number of loans made, the lower interest rate, and the higher fraction of loans succeeding.

For part (c), note that condition (I1) is just a sufficient condition – borrower welfare, $w(z^*)$ could still increase in r even if R^* is also increasing in r (condition I1 did not hold), provided that the effect of the increase in MFI effort z^* on $w(z^*) = p(z^*)(\theta - R^*)$ outweighs the effect of the higher interest rate R^*. Similarly, total borrower welfare $W(x^*, z^*)$ can locally increase in r even when $\frac{dR^*}{dr} > 0$ *and* $\frac{dw(z^*)}{dr} \leq 0$, as long as the effect of the increase in x^* dominates. The formal conditions under which these scenarios occur are described in the proof of Proposition 2. See also

[11] See Example 2 for functional forms and parameters for which condition (I1) involving endogenous variables is satisfied. Unfortunately, for general functional forms, $p(z)$, $B(x)$ and $f(x)$ it is impossible to supply a sufficient condition involving only exogenous objects.

numerical Example 2 below for more details and for functional forms and parameters satisfying condition (I1) and all of the assumptions.

To conclude, Proposition 2 shows that assuming an exogenous MFI lending rate R, as in Proposition 1, is not crucial for the main results of the paper. Indeed, Proposition 2 demonstrates that a local increase in the donor/investor rate r could, under the appropriate conditions, lead to *all* of the following: (i) more loans given; (ii) higher MFI effort; (iii) higher borrower welfare (individual and/or total); and (iv) lower MFI lending rate R^*.

Example 2

This example illustrates Proposition 2 and shows that condition (I1) involving endogenous variables is easy to satisfy. Suppose $B(x) = \sqrt{x}$, $p(z) = \sqrt{z}$, $f(x) = 1$ *and* $Z = [0, \bar{z})$ *with* $\bar{z} < 1$. *Let also* $0 < r < 1$.

From (FOC) we obtain $x^* = \frac{1}{4(1-r)^2}$, *which is strictly increasing in* r. *At interior effort (interiority requires*[12] $rx^* < \bar{z}$*), (eq3) implies* $z^* = rx^*$ *which is strictly increasing in* r. *The equilibrium MFI lending rate is* $R^*(r) = \frac{rx^* + z^* f(x^*)}{p(z^*)x^*} = 2\sqrt{\frac{r}{x^*}} = 2(1-r)\sqrt{r}$ *which is decreasing in* r *for* $r \geq 1/3$. *Condition (I1) is thus satisfied for any* $r \geq 1/3$. *Then, to ensure* $R^* < \theta$ *(so that people want to borrow) it is sufficient to assume* $\theta > R^*(1/3) = 0.77$. *Turning to borrower welfare,* $w(z^*) = p(z^*)\theta - r = \frac{\theta\sqrt{r}}{2(1-r)} - r$ *and so* $\frac{dw(z^*)}{dr} > 0 \iff \theta > \frac{4\sqrt{r}(1-r)^2}{1+r}$. *For* θ *large enough (*$\theta > 1$ *is sufficient) the latter inequality holds for any* $r \in (0,1)$ *and so, for such* θ *both the individual and total borrower expected payoffs locally increase in* r.

3 Endogenous overhead costs

This section considers a variation of the model explicitly modeling MFI effort as an action affecting their overhead costs per loan. Specifically, suppose it costs an MFI $d(z) > 1$ in gross to administer each loan it makes, where, as before, $z \in Z$ is the level of effort supplied. For example, $d(z) = 1.2$ would mean that, if a loan of size 1 is disbursed, 0.2 is spent on overhead costs administering it. The overhead costs are incurred *before* the loans are repaid. Suppose that the function $d(z)$ is strictly decreasing in z with $d(z) > 1$, $\forall z \in Z$. The interpretation is that an MFI spending more effort (e.g., on a more efficient management model, accounting practices, etc.) reduces the overhead costs per dollar lent.

As in the Section 2, assume there is a large number of MFIs charging gross interest R per loan of size 1. The repayment R is now obtained with certainty – there is no risk in the project outcome, which is always $\theta > 0$. Define the total borrowers' payoff as $W(x) = x(\theta - R)$ and individual payoff as $w = \theta - R$.

Suppose MFIs are provided $F > 0$ in funds (this amount could be determined by maximizing a donor objective function as in Section 2) and charged a gross interest rate $r \geq 0$ by the donor/investor. The MFIs choose the number of loans x to disburse and the effort level z. Two

[12] For example, if $\bar{z} = 1 - \varepsilon$ for a very small and positive ε, we need $\frac{r}{4(1-r)^2} < 1$ which is satisfied for $r < 0.61$.

constraints must be satisfied. First, the available funds F must suffice to cover the disbursed loans x and the associated overhead costs,

$$xd(z) = F \quad \text{(C1)}$$

Constraint (C1) is an ex-ante feasibility or budget constraint, simply reflecting the assumption that the MFI's only source of funds is the donor/investor. Second, suppose that either because of free entry or their mandate, MFIs make zero profits – their gross revenue Rx is used to pay what is owed to the donor, rF plus the effort costs $zf(x)$,

$$Rx - rF - zf(x) = 0 \quad \text{(C2)}$$

where the function $f(x)$ has the same properties and interpretation as in Section 2.

Proposition 3 (endogenous overhead costs)

Suppose the inequality,

$$\frac{d'(z^*)}{d(z^*)^2}[z^* f'(x^*) - R] - \frac{f(x^*)}{F} > 0 \qquad \text{(C3)}$$

holds at z^* and x^* solving (C1) and (C2), at least locally to the right of r. Then, the MFIs' effort level, z^*, the total number of loans made, x^*, and the borrowers' total payoff $W(x^*)$ are all increasing in the funds interest rate r.

Proof (see Appendix)

Proposition 3 provides a condition under which a local increase in the interest rate on MFI funds, r results in increased effort by MFIs. Consequently, per loan overhead costs, $d(z^*) - 1$ are reduced and more loans are given (more projects are financed). This increases efficiency. The intuition is similar to that given after Proposition 1 – the increase in the funds rate r raises the amount MFIs need to pay back, which induces them to supply higher effort to reduce their overhead costs and be able to break even. The higher effort level and lower overhead costs $d(z)$ in turn allow more loans to be given out of the same funding F, which increases the total payoff to borrowers.

Example 3

This example shows that it is easy to satisfy condition (C3) in Proposition 3. Suppose $RF = 50$, $f(x) = 1$ and $d(z) = 100 - 4z$ with $z \in (0, 99/4)$. Let initially $r = 0$ (pure grant). Then equation (7) in the proof of Proposition 3 implies $z^* \simeq 24.5$, taking the larger root of the quadratic equation. Therefore, $d(z^*) \simeq 2$ and, since $f(x) = 1$, condition (C3) becomes $-\frac{Rd'(z^*)}{d(z^*)^2} - \frac{1}{F} > 0$ which is equivalent to $\frac{200}{d(z^*)^2} > 1$ and is clearly satisfied at z^*. Hence, a local increase in the interest rate on MFI funds starting from $r = 0$ would strictly increase the MFIs' optimal effort z^*, the total number of loans x^*, and the borrowers' total payoff, $W(x^*)$ while reducing overhead costs per loan.

4 Conclusions

I show that charging interest on the funds provided to microlenders may induce them to supply more effort, for instance in monitoring or assistance with borrower projects (increasing the expected repayment rate) or make MFIs reduce per-loan overhead costs. This paper contributes to the debate on the role of non-grant funding or for-profit actors in microfinance by showing that, under certain conditions, their presence may have a positive effect on the number of loans made, the loans' expected profitability, and borrowers' expected income (welfare).

The analysis has been silent on the use of the proceeds from charging interest on funds provided to microlenders but presumably these proceeds could be used to make further loans or in other productive ways. It was also assumed that all investment projects and all MFIs are identical and that the borrowers' composition does not change when the interest rate charged to (and by) MFIs is varied. Extending the analysis to incorporate (possibly unobservable to lenders) borrower or MFI heterogeneity in productivity or costs could bring additional insights. Finally, MFIs were assumed unable to default strategically and always fully repay the funds provider. Relaxing this assumption, e.g., in a limited commitment story, could weaken the incentivizing effect of the price of funds on MFIs.

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5 Appendix

Proof of Proposition 1

(a) As explained in the main text, there are two cases: either (i) constraint (PC) does not bind at the optimal (x^*, z^*) where x^* is determined from the FOC of the lender's objective function and z^* solves (ICC) evaluated at x^* or (ii) the MFI makes negative profits at the solution from (i) and so (PC) must bind and the optimal (x^*, z^*) are determined from (ICC') and (PC) taken at equality.

(i) Non-binding participation constraint (PC) at r

The optimal number of loans, $x^* > 0$ solves:

$$B'(x) = 1 - r, \tag{3}$$

that is, calling $\phi(.) = B'^{-1}(.)$, we have $x^*(r) = \phi(1-r)$. Consider first the case of interior effort level at which (ICC') holds and call

$$\gamma(x) \equiv \frac{x}{f(x)}.$$

From the assumed properties of $f(x)$, it is easy to show that the function $\gamma(x)$ is *strictly increasing* for $x > 0$.[13]

MFI effort z then solves, using (ICC'),

$$p'(z)R\gamma(x^*) = 1$$

Since $B(x)$ is strictly concave, B'^{-1} is strictly decreasing, and so an increase in the funds rate r results in an increase in the funds amount x^* solving (3). From (ICC'), since $p(z)$ is strictly concave, z increases in $\gamma(x)$ and hence in x, and thus $\hat{z}(r)$ solving (ICC') increases in r too. Interior solution for effort occurs when $p'(\bar{z})R\gamma(\phi(1-r)) < 1$. The remaining possibility is a corner solution in effort, $z = \bar{z}$ which is non-decreasing in r. This implies, $z^*(r) = \min\{\hat{z}(r), \bar{z}\}$. For the participation constraint (PC) to be satisfied at r, x^* and z^* we must have

$$\phi(1-r)[p(z^*(r))R - r] - z^*(r)f(\phi(1-r)) \geq 0$$

For example, at $r = 0$ we need $\phi(1)p(z^*(0))R - z^*(0)f(\phi(1)) \geq 0$ where $z^*(0)$ solves $p'(z) = \frac{1}{R\gamma(\phi(1))}$. See also numerical Example 1.

(ii) Binding participation constraint at r

Now suppose that at $x^* = \phi(1-r)$ from (3) and the corresponding $z^*(r)$ from case (i) above, the MFI cannot break even, that is

$$\phi(1-r)[p(z^*(r))R - r] - z^*(r)f(\phi(1-r)) < 0.$$

This implies that (PC) must bind at optimum. So, the solution, call it (x^c, z^c), to problem (DP) in this case is obtained from (ICC) and setting (PC) at equality. Assume interior effort level first (ICC' holds). Using that $x > 0$ at the optimum as argued earlier, re-write (PC) at equality as:

$$p(z)R - \frac{zf(x)}{x} = r$$

or, substituting for $\frac{f(x)}{x}$ from (ICC'),

$$p(z)R - zp'(z)R = r \tag{4}$$

which is solved by the optimal effort z^c. The l.h.s. of (4) is strictly increasing in z since its first

[13]The concavity of $f(x)$ implies that, for any $x > 0$, we have $f(x) - f(0) \geq f'(x)x$. That is $f(x) > f'(x)x$ since $f(0) > 0$. This implies that the derivative of $\frac{x}{f(x)}$ at $x > 0$ is strictly positive. If f is strictly concave only $f(0) \geq 0$ is needed.

derivative with respect to z is

$$p'(z)R - p'(z)R - zp''(z)R > 0$$

by the strict concavity of $p(z)$. This implies that the optimal effort $z^c(r)$ solving (4) is strictly increasing in r (the right-hand side). In turn, this implies that $x^c(r)$, which solves (ICC'), $p'(z^c(r))R\gamma(x) = 1$, is also strictly increasing in r by the concavity of $p(z)$ and since $\gamma(x)$ is strictly increasing. Going back to (4), in order to have an interior effort level we need,

$$p(\bar{z})R - \bar{z}p'(\bar{z})R > r. \tag{5}$$

The remaining possibility is $z^c = \bar{z}$, which is non-decreasing in r. Then, using (PC) at equality, we have $\gamma(x^c(r)) = \frac{\bar{z}}{p(\bar{z})R-r}$ which, since $\gamma(x)$ is strictly increasing, again implies that $x^c(r)$ is strictly increasing. For this case to arise we need (using ICC): $p'(\bar{z})R\frac{\bar{z}}{p(\bar{z})R-r} \geq 1$ which, together with Assumption A1, implies the condition

$$p(\bar{z})R > r \geq p(\bar{z})R - \bar{z}p'(\bar{z})R.$$

We conclude that in both cases (i) and (ii), the optimal amount of funds is strictly increasing in r and the optimal MFI effort level is non-decreasing (increasing at an interior solution) in r.

(b) The results about the expected payoffs $W(x, z)$ and $w(z)$ follow directly from part (a) since both functions are strictly increasing in x and z.

Proof of Proposition 2

(a) We showed that $x^*(r)$ is increasing in the main text. We also showed that, in the interior effort level case, $z^*(r)$ solves (eq3) which is equivalent to:

$$r\gamma(x^*(r)) = \frac{p(z)}{p'(z)} - z \tag{6}$$

where $\gamma(x) \equiv \frac{x}{f(x)}$ as defined in the proof of Proposition 1. The function $\frac{p(z)}{p'(z)} - z$ on the r.h.s. is *strictly increasing* by the strict concavity of $p(z)$.[14] This implies that an increase in the donor/investor interest rate r, which increases $r\gamma(x^*(r))$, also raises the optimal MFI effort $z^*(r)$.

(b) Condition (I1), $\frac{dR^*}{dr} \leq 0$ implies the equilibrium MFI lending rate is decreasing in r. The borrower's individual expected payoff is $w(z^*) = p(z^*)(\theta - R^*)$ which, if (I1) holds, increases in r since z^* increases in r (see part a) and R^* (weakly) decreases in r. A similar argument applies to $W(x^*, z^*)$ since x^* also increases in r.

(c) We have $w(z^*) = p(z^*)(\theta - R^*)$. Thus, $\frac{dw(z^*)}{dr} = p'(z^*)\frac{dz^*}{dr}(\theta - R^*) - p(z^*)\frac{dR^*}{dr}$ which is

[14] Its derivative is $\frac{(p'(z))^2 - p''(z)p(z)}{(p'(z))^2} - 1 = -\frac{p''(z)p(z)}{(p'(z))^2} > 0$.

positive if and only if

$$\frac{p'(z^*)}{p(z^*)}\frac{dz^*}{dr}(\theta - R^*) > \frac{dR^*}{dr} \quad \text{(I2)}$$

Since we assume $R^* < \theta$ (otherwise no one would borrow), the l.h.s. is positive and so condition (I2) is implied by and easier to satisfy than condition (I1) in the Proposition 2 statement. Hence, we could have $\frac{dw(z^*)}{dr} > 0$ even when $\frac{dR^*}{dr} > 0$, as long as the welfare effect from the increase in z^* dominates. Similarly, $\frac{dW(x^*,z^*)}{dr} = \frac{dx^*}{dr}w(z^*) + x^*\frac{dw(z^*)}{dr}$ and hence total borrower welfare, $W(x^*, z^*)$ can locally increase in r even if $\frac{dR^*}{dr} > 0$ *and* $\frac{dw(z^*)}{dr} \leq 0$, as long as the positive effect from the increase in x^* and z^* in the first term outweighs the negative effect of the higher loan interest R^* in the second term.

Proof of Proposition 3

From (C1), we have $x = \frac{F}{d(z)}$. Substituting into (C2),

$$\frac{R}{d(z)} - \frac{zf(\frac{F}{d(z)})}{F} = r \tag{7}$$

which is solved by the MFI's optimal effort z^*. The expression in inequality (C3) in the proposition statement is the derivative of the l.h.s. of (7) with respect to z evaluated at z^* and $x^* = \frac{F}{d(z^*)}$. As long as this derivative is positive (condition C3 holds), the MFI effort z^* is increasing in r (the right hand side of (7)). Since $x^* = \frac{F}{d(z^*)}$ and $d(z)$ is strictly decreasing, this implies that the number of loans x^* is also increasing in the funds rate r.