

# Optimal Development Policies with Financial Frictions\*

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## Abstract

Motivated by the observation that many emerging economies pursue active development and industrial policies, we study optimal dynamic Ramsey policies in a standard growth model with financial frictions. We first study a one-sector economy, and then generalize the framework to multiple sectors in order to consider sectoral and exchange rate policies. In the one-sector economy, the optimal policy intervention initially increases labor supply and lowers wages, resulting in higher entrepreneurial profits and faster wealth accumulation. This in turn relaxes borrowing constraints in the future, leading to higher labor productivity and wages. The use of additional policy instruments, such as subsidized credit, is desirable as well. In the long run, the optimal policy reverses sign. In a multi-sector economy, optimal policy subsidizes sectors with a latent comparative advantage. Furthermore, if tradables sectors are undercapitalized relative to non-tradables, optimal policy compresses wages thereby improving competitiveness, but this does not necessarily imply a depreciated real exchange rate.

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# 1 Introduction

Is there a role for governments in underdeveloped countries to accelerate economic development by intervening in product and factor markets? Should they use taxes and subsidies? If so, which ones? To answer these questions, we study optimal policy intervention in a standard growth model with financial frictions. In our framework, forward-looking heterogeneous producers face borrowing (collateral) constraints which result in a misallocation of capital and depressed productivity. This framework is, therefore, similar to the one commonly adopted in the macro-development literature to study the relationship between financial development and aggregate productivity (see e.g. [Banerjee and Duflo, 2005](#); [Song, Storesletten, and Zilibotti, 2011](#); [Buera and Shin, 2013](#)). Our paper is the first to study the optimal Ramsey policies in such an environment, along with their implications for a country’s development dynamics.

From a more positive perspective, we are motivated by the observation that many emerging economies pursue active development and industrial policies ([Harrison and Rodriguez-Clare, 2010](#)). Such policies can broadly be divided into two categories: *uniform* policies that affect the economy as a whole and *targeted* policies that target particular sectors or firms. [Table 1](#) provides a summary of historical accounts of the policies observed in seven fast-growing East Asian countries.<sup>1</sup> Typical examples of uniform policies include the economy-wide suppression of factor prices, in particular wages. For example, South Korea in the 1970s imposed an official upper limit on the growth of real wages.<sup>2</sup> Examples of targeted policies include subsidies to presumed *comparative advantage* sectors and subsidized credit to particular firms. From a neoclassical perspective such policies are, of course, unambiguously detrimental. In this paper we argue that, under particular circumstances, some of these policies may instead be beneficial.

We first consider *uniform* policies in the context of a one-sector economy, and then generalize our framework to multiple sectors to study *targeted sectoral* policies. In both cases, we work with a tractable workhorse macro-development model, which allows us to obtain sharp analytical characterizations. Our one-sector economy is populated by two types of agents: a continuum of heterogeneous entrepreneurs and a continuum of homogeneous workers. Entrepreneurs differ in their wealth and their productivity, and borrowing constraints

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<sup>1</sup>We discuss these historical accounts in more detail at the end of this introduction.

<sup>2</sup>South Korea’s Economic Planning Board directed firms to keep nominal wage growth below 80 percent of the sum of inflation and aggregate productivity growth, which resulted in real-wage growth lagging behind productivity growth (e.g., see [Kim and Topel, 1995](#); [Kim and Leipziger, 1997](#), and [Online Appendix B](#)).

Table 1: Summary of historical accounts of development policies

	Rough period	Suppressed wages	Subsidized credit	Subsidized intermediates	Subsidies to export sector
Japan	1950–1970		✓	✓	✓
Taiwan	1950–1980	✓	✓	✓	
Korea	1960–1980	✓	✓	✓	✓
Malaysia	1960–1990	✓	✓	✓	✓
Singapore	1960–1990	✓			✓
Thailand	1960–1990	✓	✓	✓	✓
China	1980–present		✓		

Note: For a detailed discussion and underlying sources, see the end of this introduction and [Online Appendix B](#). The checkmarks in the table indicate instances when we found explicit evidence of the policies used, and hence it is not an exhaustive account of all policies that were in place in these countries.

limit the extent to which capital can reallocate from wealthy to productive individuals. In the presence of financial frictions, productive entrepreneurs make positive profits; they then optimally choose how much of these to consume and how much to retain for wealth accumulation. Workers decide how much labor to supply to the market and how much to save. Section 2 lays out the structure of the economy and characterizes the decentralized laissez-faire equilibrium. As a result of financial frictions, marginal products of capital are not equalized in equilibrium, and constrained entrepreneurs obtain a higher rate of return than that available to workers. It is this differential in rates of return that is exploited by the policy interventions we consider.

In Section 3, we introduce various tax instruments into this economy and study the optimal Ramsey policies given the available set of instruments. We consider the problem of a benevolent planner subject to the same financial frictions present in the decentralized economy. We first consider the case with a subset of tax instruments, which effectively allow the planner to manipulate worker savings and labor supply decisions, and then show how the results generalize to cases with a much greater number of instruments, including a credit subsidy to the firms (entrepreneurs).

In a one-sector economy, our main result is that the optimal policy intervention involves distorting labor supply of workers, yet the direction of this distortion is different for developing and developed countries, defined in terms of the level of their financial wealth relative to the steady state.<sup>3</sup> In particular, it is optimal to *increase* labor supply in the initial phase

<sup>3</sup>In the simple version of the model, countries only differ in their financial wealth (the only state variable), while other factors affecting the steady state are held constant (including exogenous parameters of the collateral constraint). In this special case the level of financial wealth and overall economic development (income per capita) are one-to-one. In a richer environment with cross-country steady state (or balanced growth path) differences, the precise determinant of the optimal policy is the distance of financial wealth from its steady state level, which nonetheless is likely positively correlated with income per capita.

of transition, when entrepreneurs are undercapitalized, and *reduce* labor supply once the economy comes close enough to the steady state, where entrepreneurs are well capitalized. Hence, optimal policy is *stage-dependent*. Greater labor supply reduces equilibrium wages paid by entrepreneurs, increasing their profits and accelerating wealth accumulation. This, in turn, makes future financial constraints less binding, resulting in greater labor productivity and higher wages.<sup>4</sup> We show in Section 3.3 that it is optimal to distort labor supply in this fashion, even when credit or capital subsidies are available, which are arguably more direct instruments for targeting the underlying inefficiency.

While our benchmark analysis focuses on a labor supply subsidy for concreteness, there are of course many equivalent ways of implementing the optimal allocation, including non-tax market regulation which is widespread in practice (as we discuss in Section 3 and [Online Appendix B](#)). The common feature of such policies is that, in the short-run, they make workers work hard even though wages paid by firms are low. We show that such *pro-business* development policies are optimal even when the planner puts zero weight on the welfare of entrepreneurs. Indeed, the planner finds it optimal to hurt workers in the short-run so as to reward them with high wages and shorter work hours in the long-run. An alternative way of thinking about this result is that the labor supply decision of workers involves a dynamic *pecuniary externality* (see [Greenwald and Stiglitz, 1986](#)): workers do not internalize the fact that working more leads to faster wealth accumulation by entrepreneurs and higher wages in the future. The planner corrects this using a Pigouvian subsidy.<sup>5</sup>

After having analyzed optimal policies in a one-sector economy, we take advantage of the tractability of our framework and extend the model to multiple tradable and non-tradable sectors in Section 4. We explore the implications of financial frictions for optimal sectoral and exchange rate policies and obtain two main insights. First, we show that financial frictions create a wedge between the short-run and long-run (*latent*) comparative advantage of a country, and the optimal policy tilts the allocation of resources towards the latent comparative advantage sectors, thereby speeding up the transition.

Second, we study the implications of optimal development policies for the real exchange

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<sup>4</sup>We show that the reduced labor supply in the long-run is reminiscent of a labor union allocation, and it allows the planner to redistribute the gains from entrepreneurs towards workers. This insight provides an efficiency rationale for the difference in the relative bargaining power of labor and business observed in Asian and European countries without relying on differences in preferences or political systems.

<sup>5</sup>We show that a reduced form of our model is mathematically equivalent to a setup in which production is subject to a learning-by-doing externality, whereby working more today increases future productivity, as in [Krugman \(1987\)](#) and [Young \(1991\)](#). While reduced forms are similar, the economies are structurally different: the dynamic externality in our framework is a pecuniary one, stemming from the presence of financial frictions and operating via misallocation of resources, rather than a technological one.

rate and wages in the tradable sector, a measure of a country’s competitiveness. If the tradable sector is relatively undercapitalized, it is optimal to compress wages in order to allow tradables producers to accumulate wealth faster by enhancing their international competitiveness. Whether a decrease in wages paid by tradables producers goes hand in hand with a real devaluation is a more subtle issue that depends on the policy instruments available to the planner. We show that *only if* the planner cannot differentially tax tradable and non-tradable sectors, she recurs to third-best intertemporal distortions, which result in a depreciated real exchange rate, and otherwise the real exchange rate actually appreciates. Therefore, while compressing wages is a robust feature of optimal policy, we find that the real exchange rate is not a particularly useful guide for the policymakers.

The objective of developing a tractable multi-sector model for the analysis of optimal development policies under financial frictions motivates a number of modeling choices, which we discuss in detail in Section 5. Our framework builds on Moll (2014), and in particular makes use of the insight that heterogeneous agent economies remain tractable if individual production functions feature constant returns to scale. By additionally assuming that productivities are drawn from sector-specific Pareto distributions, we are able to considerably generalize this framework to multiple tradable and non-tradable sectors while still maintaining tractability, which is essential for the optimal policy analysis.

Our paper is related to the large theoretical literature studying the role of financial market imperfections in economic development, and in particular the more recent literature relating financial frictions to aggregate productivity. We contribute to this literature by studying optimal Ramsey policies and the resulting implications for a country’s transition dynamics in both a one-sector and a multi-sector environments.<sup>6</sup> Our approach generates two main new insights. First, when entrepreneurial wealth is scarce, policies that compress wages increase welfare by facilitating wealth accumulation. Yet, eventually these policies

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<sup>6</sup>In addition to papers cited above, see the early contributions by Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997) and Piketty (1997), the more recent papers by Jeong and Townsend (2007), Erosa and Hidalgo-Cabrillana (2008), Caselli and Gennaioli (2013), Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Midrigan and Xu (2014) and the recent surveys by Matsuyama (2008) and Townsend (2010). These papers are part of a growing literature exploring the macroeconomic effects of micro distortions (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Bartelsman, Haltiwanger, and Scarpetta, 2013). The modeling of financial frictions in the paper also follows the tradition in the recently burgeoning macro-finance literature (e.g. Kiyotaki and Moore 1997; see Brunnermeier, Eisenbach, and Sannikov 2013 for a comprehensive survey) and the literature on entrepreneurship and wealth distribution (Cagetti and De Nardi, 2006; Bassetto, Cagetti, and Nardi, 2013; Quadrini, 2011). A few papers in this literature evaluate the effects of various policies, including Banerjee and Newman (2003), Buera, Moll, and Shin (2013), Buera and Nicolini (2013) and Buera, Kaboski, and Shin (2012), but none study Ramsey-optimal policies. There is an even larger *empirical* literature showing the importance of finance for development (see Levine, 2005, for a survey).

need to be reversed, as the economy approaches its long-run steady state. Second, extending this logic to a multi-sector economy, optimal policy subsidizes sectors with a long-run comparative advantage, and supports the competitiveness of tradables sectors if those sectors are undercapitalized relative to non-tradables.

In related work, [Caballero and Lorenzoni \(2014\)](#) analyze the Ramsey-optimal response to a preference shock in a two-sector small open economy with financial frictions in the tradable sector. In both papers financial frictions give rise to a pecuniary externality, which justifies a policy intervention that distorts the allocation of resources across sectors. But the focus of the two papers is different: ours studies long-run development policies whereas theirs studies cyclical policies.<sup>7</sup> Our work is also related to [Bacchetta, Benhima, and Kalantzis \(2014\)](#) who study optimal exchange rate policies, but in a framework with liquidity constraints on the household side.

Another closely related paper is [Song, Storesletten, and Zilibotti \(2014\)](#) who study the effects of capital controls, and policies regulating interest rates and the exchange rate. Their positive analysis shows that, in China, such policy interventions may have compressed wages and increased the wealth of entrepreneurs, relaxing the borrowing constraints of private firms, just like in our framework. Relative to their paper, our normative analysis shows that policies leading to compressed wages not only foster productivity growth but may, in fact, be *optimal* in the sense of maximizing welfare. Furthermore, we argue that the optimal use of such policy tools may be stage-dependent, requiring a policy reversal along the transition path.

In terms of methodology, we follow the dynamic public finance literature and study a Ramsey problem (see e.g. [Barro, 1979](#); [Lucas and Stokey, 1983](#)). The environment we study is similar to [Judd \(1985\)](#) and [Straub and Werning \(2014\)](#) in that it features a distributional conflict between capitalists and workers, but with the difference that capitalists are heterogeneous and face financial frictions and incomplete markets.<sup>8</sup> Our work differs from the classical Ramsey taxation literature in that we study optimal policy intervention in the presence of financial frictions, rather than the optimal financing of an exogenously given

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<sup>7</sup>The frameworks used by the two papers also differ. In particular, our paper is well-suited for studying transition dynamics, because we follow the tradition of the macro-development literature and depart minimally from the neoclassical growth model by introducing financial frictions. Another related paper is [Angeletos, Collard, Dellas, and Diba \(2013\)](#) who study Ramsey policies in a setup with heterogeneous producers and within-period liquidity constraints, but their focus on optimal public debt management as supply of collateral is very different from ours. A related strand of work emphasizes a different type of pecuniary externality that operates through prices in borrowing constraints, for example [Lorenzoni \(2008\)](#), [Jeanne and Korinek \(2010\)](#) and [Bianchi \(2011\)](#). Yet another type of pecuniary externality is analyzed in the earlier work on the “big push” (e.g., see [Murphy, Shleifer, and Vishny, 1989](#)).

<sup>8</sup>See [Aiyagari \(1995\)](#) and [Shin \(2006\)](#) for related analyses of Ramsey problems in environments with idiosyncratic risk and incomplete markets, but without collateral constraints.

stream of government expenditure or optimal debt management.

**Empirical relevance** There exist a large number of historical accounts that the sort of policies prescribed by our normative analysis have been used in countries with successful development experiences. [Online Appendix B](#) discusses in detail development policies in seven fast-growing East Asian countries (Japan, Korea, Taiwan, Malaysia, Singapore, Thailand and China), as summarized in [Table 1](#) above.<sup>9</sup> Typical policies include the suppression of wages and intermediate input prices. In the beginning of the introduction we already discussed wage suppression policies observed in South Korea during the 1960s and 1970s.<sup>10</sup> As stated in [Table 1](#), similar policies were also implemented in other Asian economies. Another very commonly observed policy is some form of subsidized credit to particular sectors or firms, often conditional on their export status. Many of these policies are reminiscent of the normative prescriptions in our theoretical analysis for economies in the early stages of development. In practice, such policies were frequently imposed for reasons other than the pursuit of development policy, e.g. due to political, ideological or rent-seeking considerations (see also [Harrison and Rodriguez-Clare, 2010](#)). Yet, they appear to have been effective in supporting domestic producers in the global market.

Equally commonly observed are different forms of export-promotion policies. In particular, policymakers, as well as academics, often advocate support for *comparative advantage* sectors (see e.g. [Lin, 2012](#); [Harrison and Rodriguez-Clare, 2010](#)). Such policies have even been embraced by the World Bank. For example, the World Bank’s [Growth Commission \(2008\)](#) argues that export promotion policies may be beneficial, at least as long as they are only temporary.<sup>11</sup> This view is consistent with our result in [Section 4](#) that optimal policy entails temporary, or stage-dependent, subsidies to latent (or long-run) comparative advantage sectors, provided that such sectors can indeed be identified in practice.<sup>12</sup> In the same section we evaluate another commonly advocated development policy, namely devaluation of the real exchange rate. [Rodrik \(2008\)](#) provides a recent systematic study of the effects of this policy across many developing countries (while [Woodford, 2008](#), offers a cautious inter-

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<sup>9</sup>Appendix B is available at [http://www.princeton.edu/~itskhoki/papers/FinFrictionsDevoPolicy\\_AppendixB.pdf](http://www.princeton.edu/~itskhoki/papers/FinFrictionsDevoPolicy_AppendixB.pdf).

<sup>10</sup>Labor unions were also restricted in South Korea ([Kim and Topel, 1995](#); [Schuman, 2010](#), and [Online Appendix B](#)). On the anecdotal side, president Park Chung Hee in his annual national address declared 1965 a “year to work”; twelve months later, he humorlessly named 1966 a “year of harder work” ([Schuman, 2010](#)).

<sup>11</sup>The Growth Commission report studies the growth experiences and policies of the thirteen economies that have grown at an average rate of seven percent a year or more for 25 years or longer since 1950, and argues that all of them benefited substantially from exports and many promoted exports in some way.

<sup>12</sup>Identifying sectors with latent comparative advantage in practice is a challenging task, as pointed out for example by [Stiglitz and Yusuf \(2001, p.135\)](#). [Hidalgo, Klingler, Barabási, and Hausmann \(2007\)](#) and [Lin \(2012\)](#) offer two empirical approaches to this challenge.

pretation of the evidence). The argument in favor of such intervention rests on the premise that the scale of the tradable sector is inefficiently small (in particular, due to binding borrowing constraints), and an exchange rate devaluation is aimed at reducing labor costs and increasing international competitiveness of this sector.

From a more historical perspective, [Feinstein \(1998\)](#) and [Voth \(2001\)](#) provide evidence that the rapid economic growth in 18th century Britain was in part due to reduced labor and land prices as well as long work hours. [Ventura and Voth \(2013\)](#) argue that these were caused by expanding government borrowing which crowded out unproductive agricultural investment and reduced factor demand by this declining sector. Lower factor prices, in turn, increased profits in the new industrial sectors, allowing the capitalists in these sectors to build up wealth, which in the absence of an efficient financial system was the major source of investment.<sup>13</sup>

## 2 An Economy with Financial Frictions

In this section we describe our baseline economy with financial frictions. We consider a one-sector small open economy populated by two types of agents: workers and entrepreneurs.<sup>14</sup> The economy is set in continuous time with an infinite horizon and no aggregate shocks so as to focus on the properties of transition paths. We first describe the problem of workers, followed by that of entrepreneurs. We then characterize some aggregate relationships and properties of the decentralized equilibrium in this economy. The technical details of derivations and proofs, as well as various extensions, can be found in [Online Appendix A](#).<sup>15</sup>

### 2.1 Workers and entrepreneurs

A representative worker (household) in the economy has preferences given by

$$\int_0^{\infty} e^{-\rho t} u(c(t), \ell(t)) dt, \tag{1}$$

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<sup>13</sup>In the United States, the idea of subsidies targeted at particular industries goes back at least to [Hamilton's \(1791\)](#) “Report on Manufactures,” in which he advocated “pecuniary bounties” (production subsidies) as “one of the most efficacious means of encouraging manufactures.”

<sup>14</sup>In [Appendix A.12](#) we extend our analysis to the case of a closed economy, where we show that our main qualitative insights regarding labor market policies go through, despite the presence of an additional state variable. The additional insight from the closed economy analysis is that the planner also chooses to distort the intertemporal margin to encourage a faster accumulation of capital.

<sup>15</sup>[Appendix A is available at \[http://www.princeton.edu/~itskhoki/papers/FinFrictionsDevoPolicy\\\_AppendixA.pdf\]\(http://www.princeton.edu/~itskhoki/papers/FinFrictionsDevoPolicy\_AppendixA.pdf\).](http://www.princeton.edu/~itskhoki/papers/FinFrictionsDevoPolicy_AppendixA.pdf)

where  $\rho$  is the discount rate,  $c$  is consumption, and  $\ell$  is market labor supply. We assume that  $u(\cdot)$  is increasing and concave in its first argument and decreasing and convex in its second argument, with a positive and finite Frisch elasticity of labor supply (see Appendix A.1). Where it leads to no confusion, we drop the time index  $t$ .

Households take the market wage  $w(t)$  as given, as well as the price of the consumption good, which we choose as the numeraire. They borrow and save using non-state-contingent bonds, which pay risk-free interest rate  $r(t) \equiv r^*$ , hence facing the flow budget constraint:

$$c + \dot{b} \leq w\ell + r^*b, \quad (2)$$

where  $b(t)$  is the household asset position. The solution to the household problem satisfies a standard Euler equation and a static optimality (labor supply) condition.

The economy is also populated by a unit mass of entrepreneurs that produce the homogeneous tradable good. Entrepreneurs are heterogeneous in their wealth  $a$  and productivity  $z$ , and we denote their joint distribution at time  $t$  by  $\mathcal{G}_t(a, z)$ . In each time period of length  $\Delta t$ , entrepreneurs draw a new productivity from a Pareto distribution  $G_z(z) = 1 - z^{-\eta}$  with shape parameter  $\eta > 1$ , where smaller  $\eta$  corresponds to a greater heterogeneity of the productivity draws. We consider the limit economy in which  $\Delta t \rightarrow 0$ , so we have a continuous-time setting in which productivity shocks are *iid* over time.<sup>16</sup> Finally, we assume a law of large numbers so the share of entrepreneurs experiencing any particular sequence of shocks is deterministic.

Entrepreneurs have preferences

$$\mathbb{E}_0 \int_0^\infty e^{-\delta t} \log c_e(t) dt \quad (3)$$

where  $\delta$  is their discount rate. Each entrepreneur owns a private firm which can use  $k$  units of capital and  $n$  units of labor to produce

$$A(t)(zk)^\alpha n^{1-\alpha} \quad (4)$$

units of output, where  $\alpha \in (0, 1)$  and  $A(t)$  is aggregate productivity following an exogenous path. Entrepreneurs hire labor in the competitive labor market at wage  $w(t)$  and purchase

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<sup>16</sup>In Appendix A.10 we show that our results generalize to an environment with persistent productivity process. Persistent shocks imply that we need to keep track of the wealth shares across different productivity types, but the properties of the optimal policies are qualitatively unchanged. Also, as explained in Moll (2014), an iid process in continuous time can be viewed as the limit of a mean-reverting process as the speed of mean reversion goes to infinity.

capital in a capital rental market at rental rate  $r(t) \equiv r^*$ .<sup>17</sup>

Entrepreneurs face collateral constraints:

$$k \leq \lambda a, \tag{5}$$

where  $\lambda \geq 1$  is an exogenous parameter. By placing a restriction on an entrepreneur's leverage ratio,  $k/a$ , the constraint captures the common prediction from models of limited commitment that the amount of capital available to an entrepreneur is limited by his personal assets.<sup>18</sup> At the same time, the particular formulation of the constraint is analytically convenient and allows us to derive most of our results in closed form. As shown in Moll (2014), the constraint could be generalized in a number of ways at the expense of some extra notation.<sup>19</sup> We can also allow for evolution of  $\lambda$  over time, and show below that this is isomorphic to changes in the exogenous aggregate productivity,  $A$ . Finally, note that by varying  $\lambda \in [1, \infty)$ , we can trace out different degrees of the efficiency of capital markets from autarky to perfect markets, so that  $\lambda$  captures the degree of financial development.

An entrepreneur's wealth evolves according to

$$\dot{a} = \pi(a, z) + r^* a - c_e, \tag{6}$$

where  $\pi(a, z)$  are her profits

$$\pi(a, z) \equiv \max_{\substack{n \geq 0, \\ 0 \leq k \leq \lambda a}} \left\{ A(zk)^\alpha n^{1-\alpha} - wn - r^* k \right\}.$$

Maximizing out the choice of labor,  $n$ , profits are linear in capital,  $k$ . It follows that the

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<sup>17</sup>The setup with a rental market is chosen solely for simplicity. As shown by Moll (2014), it is equivalent to a setup in which entrepreneurs own and accumulate capital  $k$  and can trade in a risk-free bond, provided the entrepreneurs know their productivity one period in advance (see also Buera and Moll, 2012).

<sup>18</sup>The constraint can be derived from the following limited commitment problem. Consider an entrepreneur with wealth  $a$  who rents  $k$  units of capital. The entrepreneur can steal a fraction  $1/\lambda$  of rented capital. As a punishment, he would lose his wealth. In equilibrium, the financial intermediary will rent capital up to the point where individuals would have an incentive to steal the rented capital, implying a collateral constraint  $k/\lambda \leq a$ , or  $k \leq \lambda a$ . See Banerjee and Newman (2003) and Buera and Shin (2013) for a similar motivation of the same form of constraint. Note that this constraint is essentially static as it rules out long-term contracts (as, for example, in Kehoe and Levine, 2001). On the other hand, as Banerjee and Newman put it, "there is no reason to believe that more complex contracts will eliminate the imperfection altogether, nor diminish the importance of current wealth in limiting investment." La Porta, de Silanes, Shleifer, and Vishny (1997, 1998) show empirically that legal creditor rights are important determinants of private credit.

<sup>19</sup>For example, we could allow the maximum leverage ratio  $\lambda$  to be an arbitrary function of productivity so that (5) becomes  $k \leq \lambda(z)a$ . The maximum leverage ratio may also depend on the interest rate, wages and other aggregate variables. What *is* crucial is that the collateral constraint is linear in wealth.

optimal capital choice is at a corner: it is zero for entrepreneurs with low productivity, and the maximal amount allowed by the collateral constraints,  $\lambda a$ , for those with high enough productivity. Throughout the paper we assume that along all transition paths considered, there always exist entrepreneurs with low enough productivity that choose to be inactive. In this case, we have the following characterization (see Appendix A.2):

**Lemma 1** *Factor demands and profits are linear in wealth and can be written as:*

$$k(a, z) = \lambda a \cdot \mathbf{1}_{\{z \geq \underline{z}\}}, \quad (7)$$

$$n(a, z) = [(1 - \alpha)A/w]^{1/\alpha} z k(a, z), \quad (8)$$

$$\pi(a, z) = \left[ \frac{z}{\underline{z}} - 1 \right] r^* k(a, z), \quad (9)$$

where the productivity cutoff  $\underline{z}$  satisfies:

$$\alpha \left( \frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}} A^{1/\alpha} \underline{z} = r^*. \quad (10)$$

Marginal entrepreneurs with productivity  $\underline{z}$  break even and make zero profits, while entrepreneurs with productivity  $z > \underline{z}$  receive Ricardian rents given by (9), which depend on both their productivity edge and the scale of operation determined by their wealth through the collateral constraint. Entrepreneurs' labor demand depends on both their productivity and their capital choice, with marginal products of labor equalized across active entrepreneurs. At the same time, the choice of capital among active entrepreneurs is shaped by the collateral constraint, which depends only on their assets and not on their productivity. Therefore, entrepreneurs with higher productivity  $z$  have a higher marginal product of capital, reflecting the misallocation of resources in the economy.

The corner solution for the choice of capital in (7) is the consequence of the constant returns to scale assumption, which yields analytical tractability to our framework by easing the aggregation. While there is a sharp discontinuity in the tractability once one departs from constant returns to scale in production, there is no reason to expect such discontinuity in optimal policies, as the equilibrium allocation itself is continuous in returns to scale (see discussion in Section 5).

Finally, entrepreneurs choose consumption and savings to maximize (3) subject to (6) and (9). Under our assumption of log utility, combined with the linearity of profits in wealth, there exists an analytic solution to their consumption policy function,  $c_e = \delta a$ , and therefore

the evolution of wealth satisfies (see Appendix A.3):

$$\dot{a} = \pi(a, z) + (r^* - \delta)a. \quad (11)$$

This completes our description of workers' and entrepreneurs' individual behavior.

## 2.2 Aggregation and equilibrium

We first provide a number of useful equilibrium relationships. Aggregating (7) and (8) over all entrepreneurs, we obtain the aggregate capital and labor demand:<sup>20</sup>

$$\kappa = \lambda x \underline{z}^{-\eta}, \quad (12)$$

$$\ell = \frac{\eta}{\eta - 1} [(1 - \alpha)A/w]^{1/\alpha} \lambda x \underline{z}^{1-\eta}, \quad (13)$$

where  $x(t) \equiv \int a d\mathcal{G}_t(a, z)$  is aggregate (or average) entrepreneurial wealth. Note that we have made use of the assumption that productivity shocks are iid over time which implies that, at each point in time, wealth  $a$  and productivity  $z$  are independent in the cross-section of entrepreneurs. Intuitively, the aggregate demand for capital in (12) equals the aggregate leveraged wealth of the entrepreneurs,  $\lambda x$ , times the fraction of active entrepreneurs,  $\mathbb{P}\{z \geq \underline{z}\} = \underline{z}^{-\eta}$ , given Pareto productivity distribution.

Aggregate output in the economy can be characterized by a production function:

$$y = Z \kappa^\alpha \ell^{1-\alpha} \quad \text{with} \quad Z \equiv A \left( \frac{\eta}{\eta-1} \underline{z} \right)^\alpha, \quad (14)$$

where  $Z$  is aggregate total factor productivity (TFP) which is the product of aggregate technology  $A$  and the average productivity of active entrepreneurs,  $\mathbb{E}\{z|z \geq \underline{z}\} = \frac{\eta}{\eta-1}\underline{z}$ . Using (12)–(14) together with the productivity cutoff condition (10), we characterize the equilibrium relationship between average wealth  $x$ , labor supply  $\ell$  and aggregate output  $y$ , and express other equilibrium objects as functions of these three variables (see Appendix A.2):

**Lemma 2** (a) *Equilibrium aggregate output satisfies:*

$$y = y(x, \ell) \equiv \Theta x^\gamma \ell^{1-\gamma}, \quad (15)$$

<sup>20</sup>Specifically,  $\kappa(t) = \int k_t(a, z) d\mathcal{G}_t(a, z)$  and  $\ell(t) = \int n_t(a, z) d\mathcal{G}_t(a, z)$ . Below, aggregate output in (14) equals  $y(t) = \int A(t)(zk_t(a, z))^\alpha n_t(a, z)^{1-\alpha} d\mathcal{G}_t(a, z)$ , integrating individual outputs in (4), and expressing it in terms of aggregate capital and labor,  $\kappa$  and  $\ell$  in (12) and (13). See derivations in Appendix A.2.

where

$$\Theta \equiv \frac{r^*}{\alpha} \left[ \frac{\eta\lambda}{\eta-1} \left( \frac{\alpha A}{r^*} \right)^{\eta/\alpha} \right]^\gamma \quad \text{and} \quad \gamma \equiv \frac{\alpha/\eta}{\alpha/\eta + (1-\alpha)}.$$

(b) The productivity cutoff  $\underline{z}$  and the division of income in the economy can be expressed as:

$$\underline{z}^\eta = \frac{\eta\lambda}{\eta-1} \frac{r^*}{\alpha} \frac{x}{y}, \quad (16)$$

$$w\ell = (1-\alpha)y, \quad (17)$$

$$r^*\kappa = \alpha \frac{\eta-1}{\eta} y, \quad (18)$$

$$\Pi = \frac{\alpha}{\eta} y, \quad (19)$$

where  $\Pi(t) \equiv \int \pi_t(a, z) d\mathcal{G}_t(a, z)$  are aggregate profits of the entrepreneurs.

Equations (17)–(19) characterize the split of aggregate income in the economy with financial frictions. The share of labor still equals  $(1-\alpha)$ , as in the frictionless world, since the choice of labor is unconstrained. However, the presence of financial frictions results in active entrepreneurs making positive profits,  $\Pi > 0$ , in contrast with the neoclassical limit. Therefore, a fraction of national income is received by entrepreneurs at the expense of rentiers, whose share of income,  $r^*\kappa/y$ , falls below  $\alpha$ . This happens due to the depressed demand for capital in a frictional environment and despite the maintained rate of return on capital  $r^*$ .

Lemma 2 further expresses equilibrium aggregates as functions of the state variable  $x$  and labor supply  $\ell$ . Note that given (15), both the equilibrium wage rate,  $w = (1-\alpha)y/\ell$ , and the productivity cutoff,  $\underline{z}$ , are increasing functions of  $x/\ell$ . High entrepreneurial wealth,  $x$ , increases capital demand and allows a given labor supply to be absorbed by a smaller subset of more productive entrepreneurs, raising both the average productivity of active entrepreneurs and aggregate labor productivity (hence wages). If labor supply,  $\ell$ , increases, less productive entrepreneurs become active to absorb it, which in turn reduces average productivity and wages.<sup>21</sup> Nonetheless, both higher  $x$  and higher  $\ell$  lead to an increase in aggregate output and aggregate incomes of all groups in the economy—workers, entrepreneurs and rentiers. Lastly, parameter  $\gamma$  in (15) increases in capital intensity ( $\alpha$ ) and in productivity heterogeneity (i.e., decreases in  $\eta$ ). It is a measure of the severity of financial frictions,<sup>22</sup> and therefore plays an important role in the analysis of optimal policies in Section 3.

<sup>21</sup>Note that this effect of increased labor supply on the marginal product of labor through declining productivity,  $\underline{z}$ , is *partly* offset by the expansion in demand for capital,  $\kappa$ , as can be seen from (12).

<sup>22</sup>Indeed,  $\gamma = \Pi/(w\ell + \Pi)$  is the share of profits in the total income of imperfectly-mobile factors (i.e., labor and entrepreneurial wealth), and hence is a measure of distance from the frictionless neoclassical limit.

Finally, integrating (11) across all entrepreneurs, aggregate entrepreneurial wealth evolves according to:

$$\dot{x} = \frac{\alpha}{\eta} y(x, \ell) + (r^* - \delta)x, \quad (20)$$

where from Lemma 2 the first term on the right-hand side equals aggregate entrepreneurial profits  $\Pi$ . Therefore, greater labor supply increases output, which raises entrepreneurial profits and speeds up wealth accumulation.

A *competitive equilibrium* in this small open economy is defined in the usual way. Workers and entrepreneurs solve their respective problems taking prices as given, while the path of wages clears the labor market at each point in time and capital is in perfectly-elastic supply at interest rate  $r^*$ . Equilibrium can be summarized as a time path for  $\{c, \ell, b, y, x, w, z\}_{t \geq 0}$  satisfying (2), standard household optimality, and (15)–(20), given an initial household asset position  $b_0$ , initial entrepreneurial wealth  $x_0$ , and a path of exogenous productivity  $A$ . Actions of individual entrepreneurs can then be recovered from (7)–(9) and (11).

### 2.3 The excess return of entrepreneurs

The key to understanding the rationale for policy intervention in our economy is that entrepreneurs earn an excess return relative to workers. Indeed, the workers face a rate of  $r^*$ , while an entrepreneur with productivity  $z$  generates a return  $R(z) \equiv r^* (1 + \lambda [\frac{z}{\underline{z}} - 1]^+)$   $\geq r^*$ , with  $R(z) > r^*$  for  $z > \underline{z}$ . Because of the collateral constraint, an entrepreneur with productivity  $z > \underline{z}$  cannot expand his capital to drive down his return towards  $r^*$ . Similarly, not only individual entrepreneurs but also entrepreneurs *as a group* earn an excess return. In particular, the average rate of return across entrepreneurs is given by:

$$\mathbb{E}_z R(z) = r^* \left( 1 + \frac{\lambda \underline{z}^{-\eta}}{\eta - 1} \right) = r^* + \frac{\alpha y}{\eta x} > r^*, \quad (21)$$

where the first equality integrates  $R(z)$  using the Pareto distribution  $G_z(z)$  and the second equality uses the equilibrium cutoff expression (16).

Given that workers and entrepreneurs face different rates of return, which fail to equalize due to the financial friction, a Pareto improvement can be achieved by a wealth transfer from workers to all entrepreneurs (independently of their productivity) combined with a reverse transfer at a later point in time.<sup>23</sup> This perturbation illustrates sharply the nature

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<sup>23</sup>More precisely, we show in Appendix A.4 that any transfer of resources from workers to entrepreneurs at  $t = 0$  and a reverse transfer at  $t' > 0$  with interest accumulated at a rate  $r_\omega = r^* + \omega \int_0^{t'} \frac{\alpha y(t)}{\eta x(t)} dt > r^*$  for

of inefficiency in the model, and provides a natural benchmark for thinking about various other policy interventions. Yet, such transfers may not be a realistic policy option for a number of reasons, which we discuss in detail Appendix A.6.<sup>24</sup> Furthermore, the type of transfer policy discussed here effectively allows the government to get around the specific financial constraint we have adopted, and hence it is not particularly surprising that it results in a Pareto improvement. Such a transfer policy may also not prove robust to alternative formulations of the financial friction. It is for these reasons that the main focus of the paper is on Ramsey-optimal taxation with a given set of simple policy tools. While also having the capacity to Pareto-improve upon the laissez-faire allocation, the policy tools we study in the next section constitute a more realistic and, we think, more robust alternative to transfers.

### 3 Optimal Policy in a One-Sector Economy

In this section we study optimal Ramsey interventions with a given set of policy tools. We start our analysis with two tax instruments, a labor income tax and a savings tax, operating directly on the decision margins of the households. In Section 3.3, we then extend our analysis to include additional tax instruments directly affecting the decisions of entrepreneurs. In particular, these instruments include a credit subsidy and a subsidy to the cost of capital of the entrepreneurs, which are arguably more direct instruments to address the financial friction. Nonetheless, we show that our main insights, in particular that the optimal policy involves a labor subsidy when entrepreneurial wealth is low, are robust to allowing for credit and/or capital subsidies. With the additional tax instruments, we give the planner a set of tools that operate on all endogenous choice margins in the model, yet do not allow the planner to target individual entrepreneurs (e.g., based on their productivity) or implement direct transfers of wealth between workers and entrepreneurs.<sup>25</sup>

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some  $\omega \in (0, 1)$  would necessarily lead to a strict Pareto improvement for all workers and entrepreneurs.

<sup>24</sup>For example, large transfers to entrepreneurs may be infeasible for budgetary reasons, for distributional and political economy reasons, or due to the associated informational frictions and informational requirements for administering them.

<sup>25</sup>In other words, we rule out any direct redistribution of wealth *among* entrepreneurs (which would clearly be desirable given the inefficient allocation of capital) and *between* entrepreneurs and workers. In Appendix A.5, we relax the latter restriction and allow for direct transfers between workers and entrepreneurs. Here again our conclusion regarding the optimality of a labor subsidy when entrepreneurial wealth is low remains intact, as long as the feasible transfers are finite. Put differently, the only case in which there is no benefit from increasing labor supply in the initial transition phase, is when an unbounded transfer from workers to entrepreneurs is available, which allows the planner to immediately jump the economy to its steady state.

### 3.1 Economy with taxes

In the presence of labor income and savings taxes on workers,  $\tau_\ell(t)$  and  $\tau_b(t)$ , the budget constraint of the households changes from (2) to:

$$c + \dot{b} \leq (1 - \tau_\ell)w\ell + (r^* - \tau_b)b + T, \quad (22)$$

where  $T$  are the lump-sum transfers from the government (lump-sum taxes if negative). In our framework, Ricardian equivalence applies, so that only the combined wealth of households and the government matter. Therefore, without loss of generality, we assume that the government budget constraint is balanced period by period:

$$T = \tau_\ell w\ell + \tau_b b. \quad (23)$$

More generally, if the government can run a budget deficit and issue debt, we can guarantee implementation of the Ramsey policies without lump-sum taxes.<sup>26</sup> In the presence of taxes, the optimality conditions of households become:

$$\frac{\dot{u}_c}{u_c} = \rho - r^* + \tau_b, \quad (24)$$

$$-\frac{u_\ell}{u_c} = (1 - \tau_\ell)w, \quad (25)$$

while the wages still satisfy the labor demand relationship (17).

The following result simplifies considerably the analysis of the optimal policies:

**Lemma 3** *Any aggregate allocation  $\{c, \ell, b, x\}_{t \geq 0}$  satisfying*

$$c + \dot{b} = (1 - \alpha)y(x, \ell) + r^*b, \quad (26)$$

$$\dot{x} = \frac{\alpha}{\eta}y(x, \ell) + (r^* - \delta)x, \quad (27)$$

where  $y(x, \ell)$  is defined in (15), can be supported as a competitive equilibrium under appropriately chosen policies  $\{\tau_\ell, \tau_b, T\}_{t \geq 0}$ , and the equilibrium characterization in Lemma 2 applies.

Intuitively, equations (26) and (27) are the respective aggregate budget constraints of workers and entrepreneurs, where we have substituted the government budget constraint (23)

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<sup>26</sup>As we show below, in the long run  $\tau_b = 0$  and  $\tau_\ell > 0$ , so that the government can roll forward the debt it has accumulated in the short run without violating the intertemporal budget constraint. This can be achieved either without any lump-sum taxes or transfers  $T = 0$ , or with lump-sum transfers to households  $T > 0$  in case the government happens to run a gross surplus in the long run.

and the expressions for the aggregate wage bill and profits as a function of aggregate income (output) from Lemma 2, which still applies in this environment.<sup>27</sup> The additional two constraints on the equilibrium allocation are the optimality conditions of workers, (24) and (25), but they can always be ensured to hold by an appropriate choice of labor and savings subsidies for workers. Finally, given a dynamic path of  $\ell$  and  $x$ , we can recover all remaining aggregate variables supporting the allocation from Lemma 2.

Similarly to the *primal approach* in the Ramsey taxation literature (e.g. Lucas and Stokey, 1983), Lemma 3 allows us to replace the problem of choosing a time path of the policy instruments subject to a corresponding dynamic equilibrium outcome by a simpler problem of choosing a dynamic aggregate allocation satisfying the *implementability constraints* (26) and (27). These two constraints differ somewhat from those one would obtain following the standard procedure of the primal approach because we exploit the special structure of our model (summarized in Lemma 2) to derive more tractable conditions.

### 3.2 Optimal Ramsey policies

We assume for now that the planner maximizes the welfare of households and puts zero weight on the welfare of entrepreneurs.<sup>28</sup> As will become clear, this is the most conservative benchmark for our results. The Ramsey problem in this case is to choose policies  $\{\tau_\ell, \tau_b, T\}_{t \geq 0}$  to maximize household utility (1) subject to the resulting allocation being a competitive equilibrium. From Lemma 3, this problem is equivalent to maximizing (1) with respect to the aggregate allocation  $\{c, \ell, b, x\}_{t \geq 0}$  subject to (26)–(27), which we reproduce as:

$$\begin{aligned} & \max_{\{c, \ell, b, x\}_{t \geq 0}} \int_0^\infty e^{-\rho t} u(c, \ell) dt \\ \text{subject to} \quad & c + \dot{b} = (1 - \alpha)y(x, \ell) + r^*b, \\ & \dot{x} = \frac{\alpha}{\eta}y(x, \ell) + (r^* - \delta)x, \end{aligned} \tag{P1}$$

and given initial conditions  $b_0$  and  $x_0$ . (P1) is a standard optimal control problem with controls  $(c, \ell)$  and states  $(b, x)$ , and we denote the corresponding co-state vector by  $(\mu, \mu\nu)$ . To ease the analysis and ensure the existence of a finite steady state, in what follows we assume  $\delta > \rho = r^*$ . However, neither the first inequality nor the second equality are essential

<sup>27</sup>Formally, Lemma 2 holds as the introduced policy instruments do not directly affect the policy functions of entrepreneurs given their wealth  $a$ , productivity  $z$ , and wage rate  $w$ .

<sup>28</sup>We relax this assumption in the end of this section, where we also show that the optimal policy not only achieves a redistribution of welfare from entrepreneurs to workers, but also can ensure a Pareto improvement.

for the pattern of optimal policies along the transition path, which is our focus.

Before characterizing the solution to (P1), we provide a brief discussion of the nature of this planner's problem. Apart from the Ramsey interpretation that we adopt as the main one, this planner's problem admits two additional interpretations. First, it corresponds to the planner's problem adopted in Caballero and Lorenzoni (2014), which rules out any transfers or direct interventions into the decisions of agents, and only allows for aggregate market interventions which affect agent behavior by moving equilibrium prices (wages in our case). Second, the solution to this planner's problem is a constrained efficient allocation under the definition developed in Dávila, Hong, Krusell, and Ríos-Rull (2012) for economies with exogenously incomplete markets and borrowing limits, as ours, where standard notions of constrained efficiency are hard to apply. Under this definition, the planner can choose policy functions for all agents respecting, however, their budget sets and exogenous borrowing constraints. Indeed, in our case the planner does not want to change the policy functions of entrepreneurs, but chooses to manipulate the policy functions of households exactly in the way prescribed by the solution to (P1). The implication is that the planner in this case does not need to identify who are the entrepreneurs in the economy, relaxing the informational requirement of the Ramsey policy. As we show in later sections, the baseline structure of the planner's problem (P1) is maintained in a number of extensions we consider.

The optimality conditions for the planner's problem (P1) are given by (see Appendix A.7):

$$\frac{\dot{u}_c}{u_c} = \rho - r^* = 0, \quad (28)$$

$$-\frac{u_\ell}{u_c} = (1 - \gamma + \gamma\nu) \cdot (1 - \alpha) \frac{y}{\ell}, \quad (29)$$

$$\dot{\nu} = \delta\nu - (1 - \gamma + \gamma\nu) \frac{\alpha y}{\eta x} \quad (30)$$

An immediate implication of (28) is that the planner does not distort the intertemporal margin,  $\tau_b \equiv 0$ , as implied by (24). There is no need to distort the workers' saving decision since, holding labor supply constant, consumption does not have a direct effect on output  $y(x, \ell)$ , and hence wealth accumulation in (27).

In contrast, the laissez-faire allocation of labor, satisfying (25) with  $\tau_\ell \equiv 0$ , is in general suboptimal. Indeed, combining planner's optimality (29) with (17) and (25), the *labor wedge* (tax) can be expressed in terms of the co-state  $\nu$  as:

$$\tau_\ell = \gamma(1 - \nu), \quad (31)$$

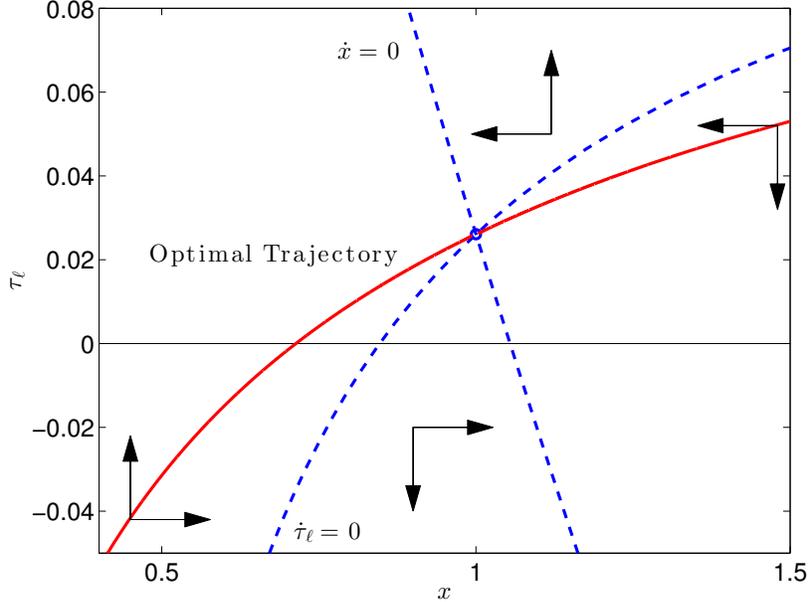


Figure 1: Planner’s allocation: phase diagram for transition dynamics

and whether labor supply is subsidized or taxed depends on whether  $\nu$  is greater than one.

Indeed, the planner has two reasons to distort the choice of labor supply,  $\ell$ . First, the workers take wages as given and do not internalize that  $w = (1 - \alpha)y/\ell$  (see Lemma 2), that is, by restricting labor supply the workers can increase their wages. As the planner only cares about the welfare of workers, this *monopoly effect* induces the planner to reduce labor supply. The statically optimal monopolistic labor tax equals  $\gamma$  in our model, and corresponds to the first term after opening brackets in (31).

Second, the workers do not internalize the positive effect of their labor supply on entrepreneurial profits and wealth accumulation, which affects future output and wages. This *dynamic productivity effect* through wealth accumulation forces the planner to increase labor supply, and it is reflected in the term  $\gamma\nu > 0$  in (31). When entrepreneurial wealth is scarce, its shadow value for the planner is high ( $\nu > 1$ ), and the planner subsidizes labor.<sup>29</sup> Otherwise, the static consideration dominates, and the planner taxes labor. Finally, recall that  $\gamma$  is a measure of the distortion arising from the financial frictions, and in the frictionless limit with  $\gamma = 0$ , the planner does not need to distort any margin.

We rewrite the optimality conditions (29) and (30), replacing the co-state  $\nu$  with the

<sup>29</sup>Note that (30) can be solved forward to express  $\nu$  as a net present value of future marginal products of wealth,  $\partial y/\partial x = \gamma y/x$ , which are monotonically decreasing in  $x$  with  $\lim_{x \rightarrow 0} \partial y/\partial x = \infty$  (see Appendix A.8).

labor tax  $\tau_\ell = \gamma(1 - \nu)$ :

$$-\frac{u_\ell}{u_c} = (1 - \tau_\ell) \cdot (1 - \alpha) \frac{y(x, \ell)}{\ell}, \quad (32)$$

$$\dot{\tau}_\ell = \delta(\tau_\ell - \gamma) + \gamma(1 - \tau_\ell) \frac{\alpha y(x, \ell)}{\eta x} \quad (33)$$

The planner's allocation  $\{c, \ell, b, x\}_{t \geq 0}$  solving (P1), satisfies (26)–(28) and (32)–(33). With  $r^* = \rho$ , the marginal utility of consumption is constant over time,  $u_c(t) \equiv \bar{\mu}$ , and the system separates in a convenient way. Given a level of  $\bar{\mu}$ , which can be pinned down from the intertemporal budget constraint, the optimal labor wedge can be characterized by means of two ODEs in  $(\tau_\ell, x)$ , (27) and (33), together with the static optimality condition (32).<sup>30</sup> These can be analyzed by means of a phase diagram (Figure 1) and other standard tools (see Appendix A.8) to yield:

**Proposition 1** *The solution to the planner's problem (P1) corresponds to the globally stable saddle path of the ODE system (27) and (33), as summarized in Figure 1. In particular, starting from  $x_0 < \bar{x}$ , both  $x(t)$  and  $\tau_\ell(t)$  increases over time towards the unique positive steady state  $(\bar{\tau}_\ell, \bar{x})$ , with labor supply taxed in steady state:*

$$\bar{\tau}_\ell = \frac{\gamma}{\gamma + (1 - \gamma)(\delta/\rho)} > 0. \quad (34)$$

*Labor supply is subsidized,  $\tau_\ell(t) < 0$ , when entrepreneurial wealth,  $x(t)$ , is low enough. The planner does not distort the workers' intertemporal margin,  $\tau_b(t) \equiv 0$ .*

The optimal steady state labor wedge is strictly positive, meaning that in the long-run the planner suppresses labor supply rather than subsidizing it. This tax is however smaller than the optimal *monopoly tax* equal to  $\gamma$  (i.e.,  $0 < \bar{\tau}_\ell < \gamma$ ), because with  $\delta > r^*$  the entrepreneurial wealth accumulation is bounded and the financial friction is never resolved (i.e., even in steady state the shadow value of entrepreneurial wealth is positive,  $\bar{\nu} > 0$ ). Nonetheless, in steady state the redistribution force necessarily dominates the dynamic productivity considerations.<sup>31</sup> This, however, is not the case along the entire transition path

<sup>30</sup>Interestingly, the system of ODEs (27) and (33) depends on the form of the utility function (and, in particular, on preference parameters such as the Frisch elasticity of labor supply) only indirectly through labor supply  $\ell(x, \tau_\ell)$  defined by (32), as we discuss further in Appendix A.8. Furthermore, these parameters do not affect the value of the steady state tax.

<sup>31</sup>In face of uninsured idiosyncratic risk, entrepreneurs accumulate a lot of wealth along the transition to steady state, making it optimal for the planner to eventually start taxing them (i.e.,  $\nu < 1$  in the long run). Of course, this result relies on the assumption that the planner puts no weight on the welfare of entrepreneurs, which we relax below.

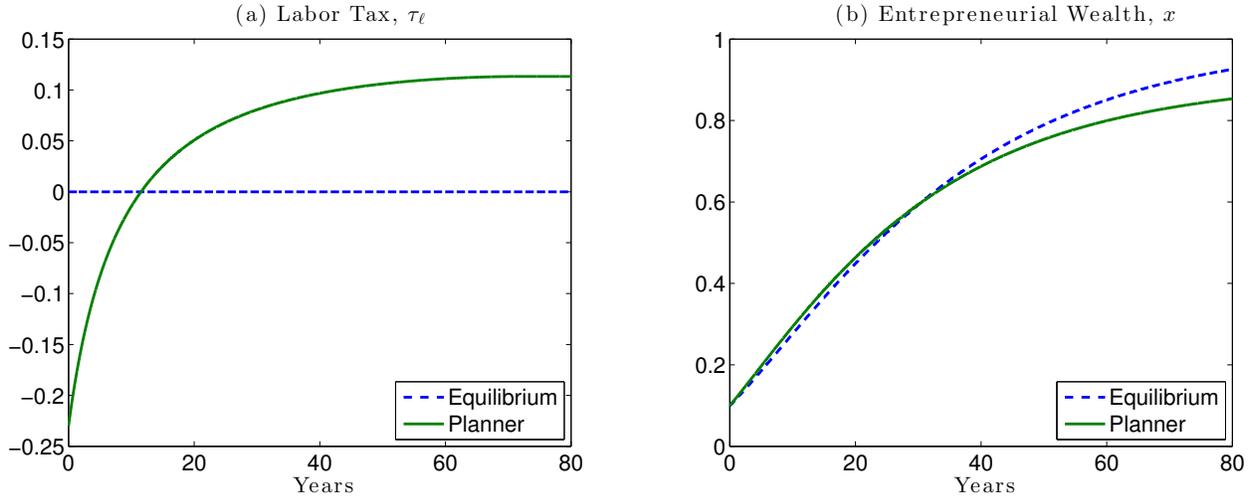


Figure 2: Planner's allocation: labor tax  $\tau_\ell(t)$  and entrepreneurial wealth  $x(t)$

Note: steady state entrepreneurial wealth in the laissez-faire equilibrium is normalized to 1 in panel (b).

to steady state, as we prove in Proposition 1 and as can be seen from the phase diagram in Figure 1. Consider a country that starts out with entrepreneurial wealth considerably below its steady-state level, i.e. in which entrepreneurs are initially severely undercapitalized. Such a country finds it optimal to subsidize labor supply during the initial transition phase, until entrepreneurial wealth reaches a high enough level.

Figures 2 and 3 illustrate the transition dynamics for key variables, comparing the allocation chosen by the planner to the one that would obtain in a laissez-faire equilibrium.<sup>32</sup> The left panel of Figure 2 plots the optimal labor tax, which is negative in the early phase of transition (i.e., a labor supply subsidy), and then switches to being positive in the long run. This is reflected in the initially increased and eventually depressed labor supply in the planner's allocation in Figure 3a. The purpose of the labor supply subsidy is to speed up entrepreneurial wealth accumulation (Figures 2b and 3b), which in turn translates into higher productivity and wages in the medium run, at the cost of their reduction in the short run (Figures 3c and 3d). The labor tax and suppressed labor supply in the long run are used to redistribute the welfare gains from entrepreneurs towards workers through the resulting increase in wages.<sup>33</sup>

<sup>32</sup>Our numerical examples use the following benchmark parameter values:  $\alpha = 1/3$ ,  $\delta = 0.1$ ,  $\rho = 0.03$ ,  $\eta = 1.06$  and  $\lambda = 2$ , as well as balanced growth preferences with a constant Frisch elasticity  $1/\phi$ :  $u(c, \ell) = \log c - \psi \ell^{1+\phi}/(1+\phi)$ , with  $\psi = 1$  and  $\phi = 1$ . The initial condition  $x_0$  is 10% of the steady state level in the laissez-faire equilibrium, and the initial wealth of workers is  $b_0 = 0$ .

<sup>33</sup>Interestingly, even if the reversal of the labor subsidy were ruled out (by imposing a restriction  $\tau_\ell \leq 0$ ), the planner still wants to subsidize labor during the early transition, emphasizing that the purpose of this policy is not merely a reverse redistribution at a later date. We have also worked out an alternative setup in

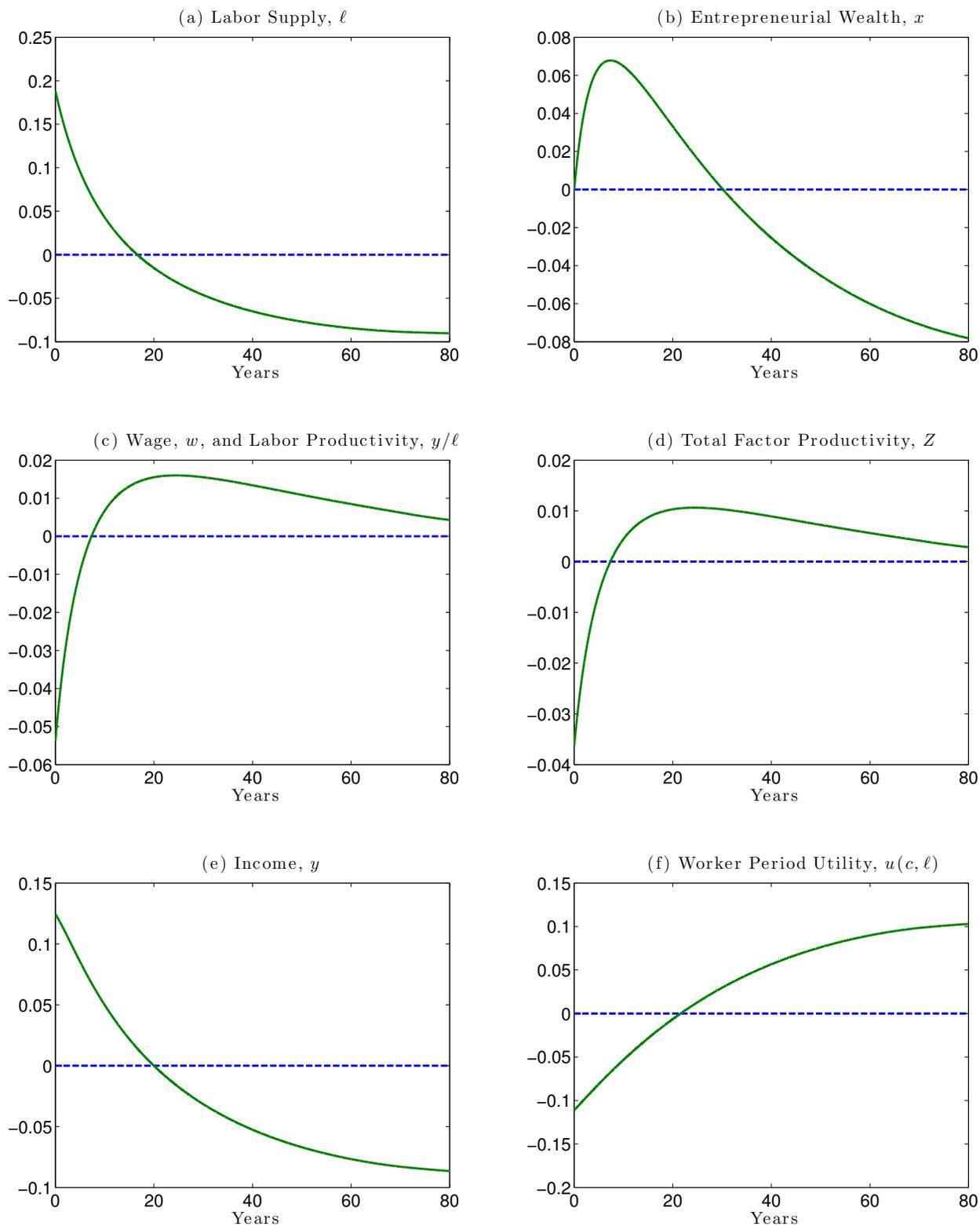


Figure 3: Planner's allocation: proportional deviations from the laissez-faire equilibrium

Note: in (d), the deviations in TFP are the same as deviation in  $z^\alpha$ , as follows from (14); in (e), income deviations characterize simultaneously the deviations in output ( $y$ ), wage bill ( $w\ell$ ), profits ( $\Pi$ ), capital income ( $r^*\kappa$ ), and hence capital ( $\kappa$ ), as follows from Lemma 2.

Figure 3e shows that during the initial phase of the transition, the optimal policy increases GDP, as well as the incomes of all groups of agents—workers, active entrepreneurs and rentiers (inactive entrepreneurs)—according to Lemma 2. Output  $y$  is higher both due to a higher labor supply  $\ell$  and increased capital demand  $\kappa$  (the capital-output ratio,  $\kappa/y$ , remains constant according to (18)). This increase in demand is met by an inflow of capital, which is in perfectly elastic supply in a small open economy. The effect of the increase in inputs  $\ell$  and  $\kappa$  is partly offset by a reduction in the TFP due to a lower productivity cutoff  $\underline{z}$ .

Although our numerical example is primarily illustrative, it can be seen that the transition dynamics in this economy may take a very long time, and the quantitative effects of the Ramsey policies may be quite pronounced.<sup>34</sup> In particular, in our example the Ramsey policy increases labor supply by up to 18% and GDP by up to 12% during the initial phase of the transition, which lasts around 20 years. This is supported by an initial labor supply subsidy of over 20%, which switches to a 12% labor tax in the long run. Despite the increased labor income, workers initially suffer in flow utility terms (Figure 3f) due to increased labor supply. Workers are compensated with a higher utility in the future, reaping the benefits of both higher wages and lower labor supply, and gain on net intertemporally.<sup>35</sup>

**Implementation** The Ramsey-optimal allocation can be implemented in a number of different ways. For concreteness, we focus here on the early phase of transition, when the planner wants to increase labor supply. The way we set up the problem, the optimal allocation during this initial phase is implemented with a labor supply subsidy,  $\varsigma_\ell(t) \equiv -\tau_\ell(t) > 0$ , financed by a lump-sum tax on workers (or government debt accumulation). In this case, workers’ gross labor income including subsidy is  $(1 + \varsigma_\ell)(1 - \alpha)y$ , while their net income subtracting the lump-sum tax is still given by  $(1 - \alpha)y$ , hence resulting in no direct change in their budget set. Note that increasing labor supply unambiguously increases net labor in-

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which firms are collectively owned by workers but subject to a financial friction that makes a direct transfer between the two groups difficult. In this alternative environment, there is no distributional conflict between workers and entrepreneurs and therefore no need for the planner to use a monopoly tax to extract resources from entrepreneurs. The main result of this section, that the planner subsidizes labor supply early on in the transition if initial entrepreneurial wealth is low enough, is unchanged in this alternative setup.

<sup>34</sup>Slow transition dynamics are a generic feature of models in which heterogeneous producers face collateral constraints. Such models therefore have the potential to explain observed growth episodes such as the post-war miracle economies. This is in contrast to transitions in the neoclassical growth model which are characterized by very fast convergence. See Buera and Shin (2013) who make this argument by means of a quantitative theory of endogenous TFP dynamics in the presence of financial frictions.

<sup>35</sup>While we found the patterns in Figures 3a–e to be robust to alternative setups, the specific pattern of the flow-utility changes in Figure 3f is more sensitive to the assumptions, and in particular changes substantially when workers are assumed to be hand-to-mouth (as well as in the closed economy), in which case the utility gains come in significantly sooner due to the increasing path of consumption tied to that of output.

come ( $w\ell$ ), but decreases the net wage rate ( $w$ ) paid by the firms. This is why we sometimes refer to this policy as *wage suppression*.

An equivalent implementation is to give a wage bill subsidy to firms financed by a lump-sum tax on workers. In this case, the equilibrium wage rate increases, but the firms pay only a fraction of the wage bill, and the resulting allocation is the same. There are of course alternative implementations that rely on directly controlling the *quantity* of labor supplied, rather than its *price*; for example, *forced labor*—a forced increase in the hours worked relative to the competitive equilibrium. Such a non-market implementation pushes workers off their labor supply schedule and the wage is determined by moving along the labor demand schedule of the business sector. Our theory is silent on the relative desirability of one form of intervention over another. See [Weitzman \(1974\)](#) for a discussion. Furthermore, desirable allocations may be achieved without any tax interventions by means of market regulation, e.g. by shifting the bargaining power from workers to firms in the labor market, as is often the case in practice (see [Online Appendix B](#) and [Appendix A.6](#) for further discussion).

The general feature of all these implementation strategies is that they make workers work hard even though wages paid by firms are low. Put differently, the common feature of all policies is their *pro-business* tilt in the sense that they reduce the effective labor costs to firms, allowing them to expand production and generate higher profits, in order to facilitate the accumulation of wealth in the absence of direct transfers to entrepreneurs.

**Learning-by-doing analogy** One alternative way of looking at the planner’s problem (P1) is to note that from (15), GDP depends on current labor supply  $\ell(t)$  and entrepreneurial wealth  $x(t)$ . From (20), entrepreneurial wealth accumulates as a function of past profits, which are a constant fraction of past aggregate incomes, or outputs. Therefore, current output depends on the entire history of past labor supplies,  $\{\ell\}_{t \geq 0}$ , and the initial level of wealth,  $x_0$ . Importantly, in the competitive equilibrium workers do not take into account the effect of their labor supply decisions on the accumulation of this state variable. In contrast, the planner internalizes this. This setup, hence, is isomorphic at the aggregate to a model of a small open economy with a learning-by-doing externality in production (see, for example, [Krugman, 1987](#); [Matsuyama, 1992](#)). Entrepreneurial wealth in our setup plays the same role as physical productivity in theories with learning-by-doing. As a result, some of our policy implications have a lot in common with those that emerge in economies with learning-by-doing externalities, as we discuss in [Section 4](#). The detailed micro-structure of our environment not only provides discipline for the aggregate planning problem, but also

differs in qualitative ways from an environment with learning-by-doing. For example, as explained above (and in more detail in Appendix A.5) transfers between entrepreneurs and workers would be a powerful tool in our environment, but have no bite in an economy with learning by doing.

**Pareto weight on entrepreneurs** Our analysis generalizes in a natural way to the case where the planner puts an arbitrary non-zero Pareto weight on the welfare of the entrepreneurs. In Appendix A.3, we derive the expected present value of an entrepreneur with assets  $a_0$  at time  $t = 0$ , denoted  $V_0(a_0)$ . In Appendix A.11 we extend the baseline planner’s problem (P1) to allow for an arbitrary Pareto weight,  $\theta \geq 0$ , on the utilitarian welfare function for all entrepreneurs,  $\mathbb{V}_0 = \int V_0(a)dG_{a,0}(a)$ . We show that the resulting optimal policy parallels that characterized in our main Proposition 1, with the optimal labor tax now given by:

$$\tau_\ell^\theta(t) = \gamma(1 - \nu(t)) - e^{(\rho-\delta)t} \frac{\theta}{\delta\bar{\mu}} \frac{\gamma}{x(t)}. \quad (35)$$

Therefore, the optimal tax schedule simply shifts down (for a given value of  $\nu$ ) in response to a greater weight on the entrepreneurs in the social objective. That is, the transition is associated with a larger subsidy to labor supply initially and a smaller tax on labor later on.<sup>36</sup> In this sense, we view our results above as a conservative benchmark, since even when the planner does not care about entrepreneurs, he still chooses a pro-business policy tilt during the early transition.

### 3.3 Additional tax instruments

In order to evaluate the robustness of our conclusions, we now briefly consider the case with additional tax instruments which directly affect the decisions of entrepreneurs. In particular we introduce a capital subsidy  $\varsigma_k$ , which in our environment is also equivalent to a credit subsidy. The key result of this section is that, even though this much more direct policy instrument is available, it is nevertheless optimal to distort workers’ labor supply decisions by suppressing wages early on during the transition and increasing them in the long-run. In Appendix A.9, we characterize a more general case, which additionally allows for a revenue (sales) subsidy, a profit subsidy, and an asset subsidy to entrepreneurs.

Specifically, we now consider the profit maximization of an entrepreneur that faces a

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<sup>36</sup>Interestingly, the long-run optimal tax rate is the same for all  $\theta \geq 0$ , as a consequence of our assumption that entrepreneurs are more impatient than workers,  $\delta > \rho$ . When  $\delta = \rho$ , the long-run tax depends on  $\theta$  and can be negative for  $\theta$  large enough.

wagebill subsidy  $\varsigma_w$  and a cost of capital subsidy  $\varsigma_k$ .<sup>37</sup>

$$\pi(a, z) = \max_{\substack{n \geq 0, \\ 0 \leq k \leq \lambda a}} \left\{ A(zk)^\alpha n^{1-\alpha} - (1 - \varsigma_w)wn - (1 - \varsigma_k)r^*k \right\}. \quad (36)$$

Credit (capital) subsidies are, arguably, a natural tax instrument to address the financial friction, and they have been an important element of real world industrial policies (see Table 1 as well as McKinnon, 1981; Diaz-Alejandro, 1985; Leipziger, 1997).

In the presence of the additional subsidies to entrepreneurs, the equilibrium characterization in Lemma 2 no longer applies and needs to be generalized, as we do in Appendix A.9. In particular, we show that the aggregate output function now generalizes (15) and is given by:

$$y(x, \ell) = (1 - \varsigma_k)^{-\gamma(\eta-1)} \Theta x^\gamma \ell^{1-\gamma},$$

with  $\gamma$  and  $\Theta$  defined as before. Furthermore, the planner's problem has a similar structure to (P1), with the added optimization over the choice of the additional subsidies. This allows us to prove the following:

**Proposition 2** *When the only planner's policy tools are a wagebill subsidy and a capital subsidy to entrepreneurs, the optimal Ramsey policy is to use both of them in tandem, and set them according to:*

$$\frac{\varsigma_w}{1 - \varsigma_w} = \frac{\varsigma_k}{1 - \varsigma_k} = \frac{\alpha}{\eta}(\nu - 1), \quad (37)$$

where  $\nu$  is the shadow value of entrepreneurial wealth, which evolution follows the same pattern as described in Section 3.2.

The key implication of Proposition 2 is that even when a credit (capital) subsidy  $\varsigma_k$  is available, the planner still finds it optimal to use the labor (wagebill) subsidy  $\varsigma_w$  alongside. This is because credit subsidies introduce distortions of their own by affecting the extensive margin of selection into entrepreneurship.<sup>38</sup> As a result, the planner prefers to combine both

<sup>37</sup>Note that the capital subsidy  $\varsigma_k$  is similar to a credit subsidy  $\varsigma_r$ , i.e. a subsidy on  $(k - a)$  rather than  $k$ . In fact, in our framework, where all active entrepreneurs have the same leverage ratio  $\lambda$ , the two instruments have identical effects (by operating via a reduction in the cost of capital) when  $\varsigma_k = (\lambda - 1)\varsigma_r/\lambda$ . Furthermore, the wagebill subsidy  $\sigma_w$  to the entrepreneurs is a perfect substitute (as a policy tool) for the labor income tax  $\tau_\ell$  on the households, as both of these instruments operate through a labor wedge. Specifically, by replacing  $\tau_\ell$  with  $\sigma_w = -\tau_\ell/(1 + \tau_\ell)$ , the planner can replicate the allocation obtained using the labor income tax.

<sup>38</sup>Note that the most direct way to address the financial friction is to relax the collateral constraint (5) by increasing  $\lambda$ , which would lead in equilibrium to reallocation of capital from less to more productive entrepreneurs and exit of the marginal ones. In contrast, a capital subsidy leads to additional entry on the margin, resulting in greater production inefficiency and lower TFP.

instruments in order to minimize the amount of created deadweight loss. Furthermore, note that the two subsidies are perfectly coordinated, leaving undistorted the capital-labor ratio chosen by the entrepreneurs. Lastly, observe that the shadow value of entrepreneurial wealth  $\nu$  is, as before, the sufficient state variable for the stance of the optimal policy, given the parameters of the model.<sup>39</sup> When  $\nu > 1$ , the entrepreneurial wealth is scarce, and the planner subsidizes both entrepreneurial production margins. As wealth accumulates,  $\nu$  declines and eventually becomes less than 1, a point at which the planner starts taxing both margins, just like before in Proposition 1. As a general principle, whenever entrepreneurial wealth is scarce, the planner utilizes all available policy instruments in a pro-business manner.

## 4 Optimal Policy in a Multi-Sector Economy

We now consider an extension of our economy to feature multiple sectors in order to study optimal sectoral policies. The output of some sectors is tradable internationally, while it is not in some other sectors. This extension allows us to relate to a number of popular policy discussions, for example: How should sectoral policies depend on a country's pattern of comparative advantage? And under what circumstances do successful development policies lead to a depreciated real exchange rate? We start our analysis with three types of tax instruments: a savings tax, sector-specific consumption taxes, and sector-specific labor income taxes. These taxes directly distort the actions of the households, while they have only an indirect effect on the entrepreneurs through market prices, namely sector-specific wage rates and output prices. At the end of Section 4.2, we then discuss the extension of our analysis to allow for production, credit and export subsidies.

### 4.1 Setup of the multi-sector economy

**Households** The households have general preferences over  $N+1$  goods,  $u = u(c_0, c_1, \dots, c_N)$ , where good  $i = 0$  is traded internationally, and we choose it as numeraire, normalizing  $p_0 = 1$ . Goods  $i = 1, \dots, N$  may be tradable or non-tradable, and we assume for concreteness that goods  $i = 1, \dots, k$  are tradable and goods  $i = k + 1, \dots, N$  are not tradable. The households maximize the intertemporal utility given by  $\int_0^\infty e^{-\rho t} u(t) dt$ , and supply inelastically a total

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<sup>39</sup>Contrast (37) with (31) and note that in both cases the optimal subsidies are proportional to  $\frac{\alpha}{\eta}(\nu - 1)$ , given the definition of  $\gamma$  in (15). Also note that the same allocation as in Proposition 2 can be achieved by replacing the wage subsidy  $\varsigma_w$  with a labor income tax  $\tau_\ell = -\frac{\varsigma_w}{1-\varsigma_w} = \frac{\alpha}{\eta}(1 - \nu)$ .

of  $L$  units of labor, which is split between the sectors:<sup>40</sup>

$$\sum_{i=0}^N \ell_i = L. \quad (38)$$

The after-tax wage across all sectors must be equalized in order for the households to supply labor to every sector:

$$(1 - \tau_i^\ell)w_i = w, \quad i = 0, 1, \dots, N, \quad (39)$$

where  $w$  is the common after-tax wage,  $w_i$  is the wage paid by the firms in sector  $i$  and  $\tau_i^\ell$  is the tax on labor income earned in sector  $i$ .<sup>41</sup>

The households have access to a risk-free instantaneous bond which pays out in the units of the numeraire good  $i = 0$ , and face the following budget constraint:

$$\sum_{i=0}^N (1 + \tau_i^c)p_i c_i + \dot{b} \leq (r - \tau^b)b + wL + T, \quad (40)$$

where  $p_i$  is producer price of and  $\tau_i^c$  is the consumption tax on good  $i$ ,  $b$  is the asset position of the households and  $\tau^b$  is a savings tax, and  $T$  is the lump sum transfer from the government. The solution to the household problem is given by the following optimality conditions:

$$\begin{cases} \frac{\dot{u}_0}{u_0} = \rho + \tau^b + \frac{\dot{\tau}_0^c}{1 + \tau_0^c} - r, \\ \frac{u_i}{u_0} = \frac{1 + \tau_i^c}{1 + \tau_0^c} p_i, \quad i = 1, \dots, N, \end{cases} \quad (41)$$

where  $u_i \equiv \partial u / \partial c_i$  is the marginal utility from consumption of good  $i$ . The first condition is the Euler equation for the intertemporal allocation of consumption. The second set of conditions is the optimal intratemporal consumption choice across sectors. It is easy to see that one of the taxes is redundant, and we normalize  $\tau_0^c \equiv 0$  in what follows.

**Production** The production in each sector is carried out by heterogeneous entrepreneurs, as in Section 2. Entrepreneurs within each sector face sector-specific collateral constraints parameterized by  $\lambda_i$ , operate sector-specific Cobb-Douglas technologies with productivity  $A_i$  and capital-intensity  $\alpha_i$ , and draw their idiosyncratic productivities from sector-specific

<sup>40</sup>Note that the assumption of inelastic labor supply is without loss of generality since we can always choose sector  $i = N$  to be non-tradable (so that  $c_N = y_N$ ) with competitive production according to  $y_N = \ell_N$ . Such sector is equivalent to either home production or leisure, generalizing the setup of Section 2 of the paper.

<sup>41</sup>One of the labor taxes is redundant, and we can normalize  $\tau_0^\ell \equiv 0$  (or alternatively  $\tau_N^\ell = 0$ , if the labor allocated to this sector is interpreted as leisure), but we find it more convenient to keep this extra degree of freedom in characterizing the optimal wedges.

Pareto distributions with tail parameter  $\eta_i$ . As a result, and analogous to Lemma 2, the aggregate production function in sector  $i$  is (see Appendix A.14):

$$y_i(x_i, \ell_i; p_i) = p_i^{\gamma_i(\eta_i-1)} \Theta_i x_i^{\gamma_i} \ell_i^{1-\gamma_i}, \quad (42)$$

where

$$\gamma_i = \frac{\alpha_i/\eta_i}{1 - \alpha_i + \alpha_i/\eta_i} \quad \text{and} \quad \Theta_i = \frac{r}{\alpha_i} \left[ \frac{\eta_i \lambda_i}{\eta_i - 1} \left( \frac{\alpha_i A_i}{r} \right)^{\eta_i/\alpha_i} \right]^{\gamma_i}.$$

Note that the producer price enters the reduced-form output function. A higher sectoral price allows a greater number of entrepreneurs to profitably produce, affecting both the production cutoff  $\underline{z}_i$  and the amount of capital  $\kappa_i$  used in the sector, which enter the sectoral production function corresponding to (14).

Sectoral entrepreneurial wealth  $x_i$  is in the units of the numeraire good  $i = 0$ , which we think of as the capital good in the economy. Further, we assume that entrepreneurs consume only the capital good  $i = 0$ , so that with logarithmic utility aggregate consumption of sector  $i$  entrepreneurs is  $\delta x_i$ , where  $\delta$  is the entrepreneurial discount rate.<sup>42</sup> As a result, the evolution of sectoral entrepreneurial wealth satisfies:

$$\dot{x}_i = \frac{\alpha_i}{\eta_i} p_i y_i(x_i, \ell_i; p_i) + (r - \delta)x_i, \quad (43)$$

where as before  $\alpha_i/\eta_i$  is the share of profits in the sectoral revenues, and  $(1 - \alpha_i)$  is the share of labor income:

$$w_i \ell_i = (1 - \alpha_i) p_i y_i(x_i, \ell_i; p_i). \quad (44)$$

**Government** The government chooses the tax policy  $(\tau^b, \{\tau_i^c, \tau_i^\ell\}, T)_{t \geq 0}$  and runs a balanced budget:

$$T = \tau^b b + \sum_{i=0}^N (\tau_i^c p_i c_i + \tau_i^\ell w_i \ell_i). \quad (45)$$

Note that we rule out direct sectoral transfers which would allow the planner to effectively sidestep the financial constraints.<sup>43</sup>

We consider here a small open economy which takes the price of capital  $r^* = \rho$  as given,

<sup>42</sup>We make this assumption for tractability, but the analysis extends to more general utility functions of entrepreneurs.

<sup>43</sup>In the presence of unbounded transfers, the planner instantaneously jumps every sector to its optimal steady state level of financial wealth  $\bar{x}_i^*$ , while no other policy instruments is used, just as in the one-sector economy in Appendix A.5.

as well as the international prices of the tradable goods  $(p_0, p_1, \dots, p_k)$  for  $k \leq N$ . The prices of the non-tradables  $p_i$  for  $i = k + 1, \dots, N$  are determined to clear the respective markets:

$$c_i = y_i(x_i, \ell_i; p_i), \quad i = k + 1, \dots, N. \quad (46)$$

## 4.2 Optimal policy

We now analyze optimal policy in this framework. With the structure above, we can prove the following *primal approach* lemma, which generalizes the earlier Lemma 3:

**Lemma 4** *Given initial condition  $b(0)$  and  $\{x_i(0)\}$ , for any allocation  $(\{c_i, \ell_i, x_i\}_{i=0}^N, b)_{t \geq 0}$  that satisfies the following dynamic system:*

$$\begin{cases} \dot{b} &= r^*b + \sum_{i=0}^N [(1 - \alpha_i)p_i y_i(x_i, \ell_i; p_i) - p_i c_i], \\ \dot{x}_i &= \frac{\alpha_i}{\eta_i} p_i y_i(x_i, \ell_i; p_i) + (r^* - \delta)x_i, \quad i = 0, 1, \dots, N, \\ c_i &= y_i(x_i, \ell_i; p_i), \quad i = k + 1, \dots, N, \\ L &= \sum_{i=0}^N \ell_i, \end{cases} \quad (47)$$

*there exists a path of taxes  $(\tau^b, \{\tau_i^c, \tau_i^\ell\}, T)_{t \geq 0}$  that decentralize this allocation as an equilibrium in the multi-sector economy, where  $r^*$  and  $(p_0, \dots, p_k)$  are international prices and  $(p_{k+1}, \dots, p_N)$  can be chosen by the planner along with the rest of the allocation.*

Therefore, we can consider a planner who maximizes household utility with respect to  $(b, \{c_i, \ell_i, x_i\}_{i=0}^N, \{p_i\}_{i=k+1}^N)_{t \geq 0}$  and subject to the set of constraint in (47) with corresponding Lagrange multipliers denoted by  $\mu \cdot (1, \{\nu_i\}_{i=0}^N, \{\psi_i\}_{i=k+1}^N, \omega)$ . Given a dynamic allocation, we recover the corresponding paths of taxes  $\tau^b$  and  $\{\tau_i^c, \tau_i^\ell\}$  from household optimality (41) together with sectoral labor demand which satisfies (44). Lastly, note that the availability of consumption taxes allows the planner to create a wedge between the sectoral production and consumption prices, and in the non-tradable sectors this allows the planner to manipulate equilibrium producer prices.

We now outline and discuss some general results, and in the following sections consider two illustrative special cases to explore in more detail the implications for comparative advantage and the real exchange rate. After considering the problem with the full set of instruments, we consecutively limit the set of taxes available to the planner. First, we

additionally rule out the sectoral labor taxes ( $\tau_i^\ell \equiv 0$ ) by imposing

$$(1 - \alpha_i)p_i y(x_i, \ell_i; p_i) = w \ell_i, \quad i = 0, 1, \dots, N, \quad (48)$$

in addition to constraints in (47), and we denote the corresponding Lagrange multipliers by  $\mu \xi_i$  for  $i = 0, \dots, N$ .<sup>44</sup> Second, we also rule out static consumption taxes ( $\tau_i^c \equiv 0$ ), leaving the planner with only the intertemporal tax  $\tau^b$ . In this case, we additionally impose

$$u_i = p_i u_0, \quad i = 1, \dots, N, \quad (49)$$

and denote by  $\mu \chi_i$  the corresponding Lagrange multipliers.

We prove the following result which applies to both the case with the full set of instruments and the cases with limited instruments:

**Lemma 5** (a) *The planner never uses consumption taxes on tradable goods ( $\tau_i^c \equiv 0$  for  $i = 0, \dots, k$ );* (b) *The planner does not use the intertemporal tax ( $\tau^b \equiv 0$ ) as long as static sectoral taxes (labor and/or consumption) are available.*

The planner never uses consumption taxes on tradable goods because they only distort consumption and have no effect on producers, who face unchanged international prices.<sup>45</sup> As in the analysis of the one-sector economy of Section 3.2, the planner does not distort the intertemporal margin (the Euler equation of households) as long as she has access to some static sectoral instruments, either consumption or labor income taxes. Indeed, such instruments are more direct, operating immediately over the sectoral allocation of resources, which is affected only indirectly by the intertemporal allocation of consumption through income effects. This implies that the wide-spread policies of ‘financial repression’ and government reserve accumulation can only be third-best in a small-open economy with financial frictions, and would only be used in the absence of static sectoral instruments (as we discuss in more detail in Section 4.4, along with the implications for the real exchange rate).

Next consider the case where the planner has at her disposal both sectoral labor and consumption taxes. We show:

**Proposition 3** *The optimal consumption and labor taxes in the multi-sector economy are*

<sup>44</sup>Note that in this case the common wage rate  $w$  becomes a variable of planner’s optimization.

<sup>45</sup>The situation is different if the planner has access to production or export taxes for tradable goods, which we discuss below.

given by:

$$\tau_i^c = \begin{cases} 0, & i = 0, 1, \dots, k, \\ \frac{1}{\eta_i - 1}(1 - \nu_i), & i = k + 1, \dots, N, \end{cases} \quad \text{and} \quad \tau_i^\ell = \begin{cases} \gamma_i(1 - \nu_i), & i = 0, 1, \dots, k, \\ -\tau_i^c, & i = k + 1, \dots, N, \end{cases}$$

where  $\nu_i$  is the shadow value of entrepreneurial wealth in sector  $i$ .

The planner does not tax consumption of tradables (as was already pointed out in Lemma 5), but does tax the consumption of non-tradables in proportion with  $(1 - \nu_i)$ . In other words, the planner subsidizes the consumption of non-tradables in sectors that have  $\nu_i > 1$ , meaning that they are financially constrained, and this subsidy is larger the more fat-tailed is the distribution of sectoral productivities (the smaller is  $\eta_i$ ). When tradable sectors are financially constrained,  $\nu_i > 1$ , the planner instead subsidizes labor supply to these sectors,  $\tau_i^\ell < 0$ , generalizing the result in a one-sector economy in (31). In contrast, the labor tax for the non-tradable sectors perfectly offsets the labor wedge introduced by the consumption subsidy,  $\tau_i^\ell = -\tau_i^c$ .

To understand the overall effect of these various tax instruments, it is useful to define the *overall labor wedge* for sector  $i$  as:

$$1 + \tau_i \equiv \frac{(1 - \alpha_i) \frac{u_i y_i}{\ell_i}}{(1 - \alpha_0) \frac{u_0 y_0}{\ell_0}} = (1 - \tau_0^\ell) \frac{1 + \tau_i^c}{1 - \tau_i^\ell}.$$

In words, the overall labor wedge is the combination of the product-market wedge  $1 + \tau_i^c$ , capturing deviations of consumers' marginal rate of substitution from relative sectoral prices, and the labor-market wedge  $(1 - \tau_0^\ell)/(1 - \tau_i^\ell)$  capturing deviations of the economy's marginal rate of transformation from relative prices. When the overall labor wedge is positive, the planner diverts the allocation of labor away from sector  $i$  (relative to the numeraire sector), and vice versa. Using Proposition 3, for tradable sectors we have

$$\tau_i = \frac{\tau_i^\ell - \tau_0^\ell}{1 - \tau_i^\ell} = \frac{\gamma_0(\nu_0 - 1) - \gamma_i(\nu_i - 1)}{1 + \gamma_i(\nu_i - 1)}, \quad i = 1, \dots, k \quad (50)$$

and for non-tradable sectors

$$\tau_i = -\tau_0^\ell = \gamma_0(\nu_0 - 1), \quad i = k + 1, \dots, N. \quad (51)$$

Consider first the overall labor wedge for non-tradables in (51). Somewhat counterintuitively, it is shaped exclusively by the need to subsidize the *tradable* sectors (in particular, the

numeraire sector, which we chose as the base, since the wedges are relative by definition). This is because the need for financing in the non-tradable sector is addressed with respective consumption taxes,  $\tau_i^c$ . In other words, the presence of both consumption and production taxes for non-tradable sectors allows the planner to subsidize the non-tradable producers via an increase in producer price  $p_i$  (due to  $\tau_i^c < 0$ ) without distorting the labor supply to these sectors. This option is unavailable in the tradable sectors which face exogenous international producer prices. Consider next the overall labor wedge for tradables in (50). The allocation of labor is distorted in favor of the tradable sector  $i$  (relative to numeraire sector 0), that is  $\tau_i < 0$ , whenever  $\gamma_i(\nu_i - 1) > \gamma_0(\nu_0 - 1)$ , and vice versa. In the following subsections 4.3 and 4.4 we consider special cases in which we can further characterize the conditions under which certain sectors are subsidized or taxed.

The results here generalize to the case with a larger set of policy instruments. Specifically, when credit and/or output (export) subsidies are available, the planner optimally combines them with the labor subsidies to the constrained sectors according to the values of  $\nu_i$ . The planner wants to use all of these instruments in tandem to achieve the best outcome with minimal distortions, as we showed in Section 3.3 and Appendix A.9 in the context of a one-sector economy. The advantage of output (export) subsidies over consumption subsidies in the tradable sectors is that they directly change effective producer prices even when the international prices are taken as given.

### 4.3 Comparative advantage and industrial policies

Proposition 3 characterizes policy in a general multi-sector economy in terms of planner's shadow values  $\nu_i$ , which represent the tightness of sectoral financial constraints. To make further progress in characterizing the policy in terms of the primitives of the economy, we consider in turn a few illuminating special cases. In this subsection we focus on the economy with tradable sectors only. For simplicity, we focus on two tradable sectors  $i = 0, 1$ , but the results extend straightforwardly to an economy with any number  $k \geq 2$  of tradable sectors.

First, we consider the case in which sectors are symmetric in everything except in what we call their *latent*, or long-run, comparative advantage. In particular, we assume that  $\eta_i \equiv \eta$  and  $\alpha_i \equiv \alpha$  for both sectors, and as a result  $\gamma_i \equiv \gamma$ . In this case, from (42), sectoral revenues which also determine wages and profits are given by  $p_i y_i = p_i^\zeta \Theta_i x_i^\gamma \ell_i^{1-\gamma}$  where  $\zeta \equiv 1 + \gamma(\eta - 1)$ . We define a sector's latent comparative advantage to be the effective revenue productivity term  $p_i^\zeta \Theta_i$ . As reflected in its definition,  $\Theta_i$  may differ across sectors due

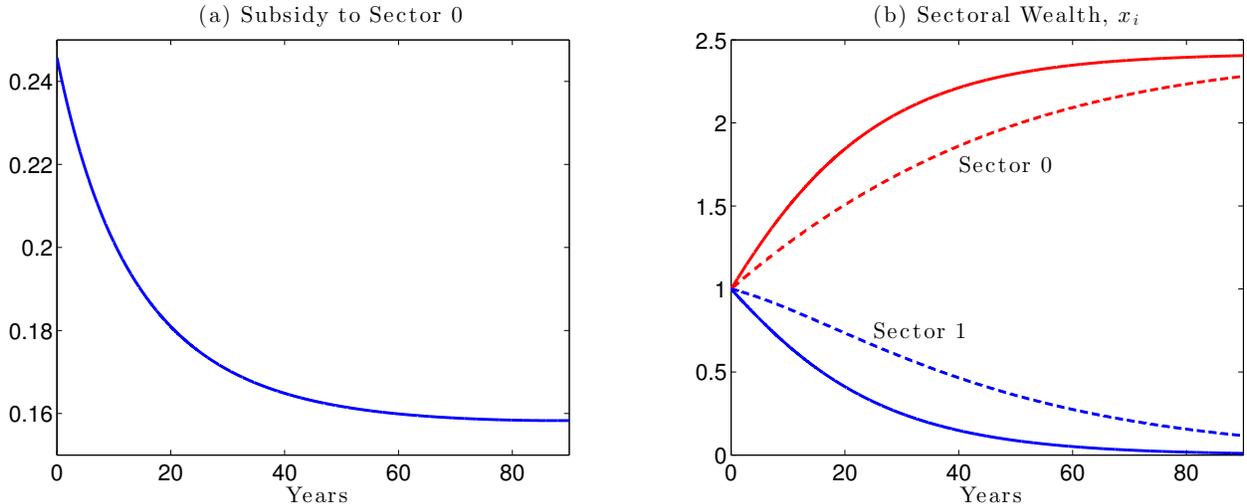


Figure 4: Planner's allocation in an economy with two tradable sectors

Note: The sectors are symmetric in everything (including the initial entrepreneurial wealth), but their latent comparative advantage:  $p_0^\zeta \Theta_0 > p_1^\zeta \Theta_1$ , i.e. sector 0 has the latent comparative advantage. Panel (a) plots the labor supply subsidy to the comparative advantage sector 0; the long-run level of the subsidy is not consequential, as the comparative disadvantage sector 1 shrinks to zero. Panel (b) plots the evolution of the sectoral entrepreneurial wealth under the decentralized allocation (dashed lines) and under the planner's allocation (solid lines).

to either physical productivity  $A_i$  or financial constraints  $\lambda_i$ , which for example depend on the pledgeability of sectoral assets. Importantly, a sector's *actual*, or short-run, comparative advantage may differ from this latent comparative advantage: in particular, it is also shaped by the allocation of sectoral entrepreneurial wealth  $x_i$  and is given by  $p_i^\zeta \Theta_i x_i^\gamma$ . In the short run, the country may specialize against its latent comparative advantage, if entrepreneurs in that sector are poorly capitalized (as was pointed out in [Wynne, 2005](#)). In the long-run, the latent comparative advantage forces dominate, and entrepreneurial wealth relocates towards the sector with the highest  $p_i^\zeta \Theta_i$ .

We can apply the results of Proposition 3 to this case. In particular, using (50) we have:

$$\tau_1 = \frac{\gamma(\nu_0 - \nu_1)}{1 + \gamma(\nu_1 - 1)},$$

and the planner shifts labor towards sector 0 whenever  $\nu_0 > \nu_1$ . We prove in Appendix A.14 that a sufficient condition for this is that sector 0 possesses a long-run comparative advantage, i.e.  $p_0^\zeta \Theta_0 > p_1^\zeta \Theta_1$ , independently of the initial allocation of wealth  $x_0$  and  $x_1$ , and hence short-run export patterns. We illustrate the optimal policy and resulting equilibrium dynamics relative to laissez-faire in Figure 4. The planner distorts the market allocation, and instead of equalizing marginal revenue products of labor across the two sectors, tilts the labor

supply towards the latent comparative advantage sector. This is because the planner’s allocation is not only shaped by the current labor productivity, which is increasing in wealth  $x_i$ , but also takes into account the shadow value of the sectoral entrepreneurial wealth, which depends on the latent comparative advantage  $p_i^\zeta \Theta_i$ . To summarize, the planner favors the long-run comparative advantage sector and speeds up the reallocation of factors towards it, consistent with some popular policy prescriptions (see e.g. Lin, 2012, and other references in the Introduction).

Second, we briefly consider the case in which sectors are asymmetric in terms of their structural parameters  $\alpha_i$  and  $\eta_i$ . To focus attention on this asymmetry, we shut down the comparative advantage forces just analyzed, so that a laissez-faire steady state features diversification of production across sectors.<sup>46</sup> We show in Appendix A.14 that the planner in this case nonetheless chooses to “pick a winner” by subsidizing one of the two sectors and independently of the initial conditions drives the economy to long-run specialization in this sector. Furthermore, there also exist cases in which the laissez-faire economy specializes in one sector, but the planner chooses to reverse the pattern of specialization.<sup>47</sup>

#### 4.4 Non-tradables, the real exchange rate and competitiveness

We now analyze in more detail a second case with only two sectors: a tradable sector  $i = 0$  and a non-tradable sector  $i = 1$ . This special case allows us to characterize more sharply the optimal sectoral taxes and particularly the implications for the real exchange rate. We find it useful to distinguish between two different measures of the real exchange rate: first, the CPI-based real exchange rate which in our two-sector model is pinned down by the after-tax

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<sup>46</sup>The wage rate paid by sector  $i$  in the long run equals (see Appendix A.14):

$$w = \left( \frac{\gamma_i}{1 - \gamma_i} \right)^{\frac{\gamma_i}{1 - \gamma_i}} \frac{[(1 - \alpha_i)p_i^\zeta \Theta_i]^{\frac{1}{1 - \gamma_i}}}{(\delta - \rho)^{\frac{\gamma_i}{1 - \gamma_i}}}.$$

When the parameter combination on the right-hand side of this expression is equalized across sectors  $i = 0, 1$ , no sector has comparative advantage in the long run. That is, there exists a multiplicity of steady states without specialization, and the specific steady state reached (in terms of intersectoral allocation of labor) depends on the initial conditions. In the alternative case, the economy specializes in the long run in the sector for which this parameter combination is largest.

<sup>47</sup>In the long-run, somewhat counterintuitively, the planner drives the economy toward specialization in the sector with the *lower*  $\gamma_i$ . The intuition for this result can be obtained from the one-sector economy in Section 3, and in particular the formula for the steady state tax (34). As explained there, the planner taxes rather than subsidizes entrepreneurs in steady state. As can be seen from (34), the size of this tax is increasing in  $\gamma$ . This is because a higher  $\gamma$  implies a larger “monopoly tax effect”, i.e. a higher desire to redistribute from entrepreneurs to workers. This intuition carries over to the multi-sector economy studied here, and the planner puts a higher steady state tax on the sector with higher  $\gamma_i$ , thereby specializing against it in the long-run. Things may be different during the transition.

price of non-tradables,  $(1 + \tau_1^c)p_1$ ; and, second, the wage-based real exchange rate which can be viewed as a measure of the country's competitiveness.<sup>48</sup> We will show below that optimal policies have potentially different implications for the two measures of real exchange rates. In particular, what happens to the CPI-based real exchange rate depends on the instruments at the planner's disposal.

**All tax instruments** We first consider the case where the planner has at her disposal the whole set of tax instruments we started with in Section 4. In this case, Proposition 3 applies and from (51) the overall labor wedge is given by:

$$\tau_1 = \gamma_0(\nu_0 - 1),$$

which is positive whenever the tradable sector is undercapitalized, that is  $\nu_0 > 1$ . Hence labor is diverted away from non-tradables to tradables and, since production features decreasing returns to labor, wages paid by tradable producers  $w_0 = (1 - \alpha_0)y_0/\ell_0$  are compressed. The implications for the after-tax price of non-tradables and hence the CPI-based real exchange rate are more subtle. Since the consumer price of non-tradables is  $p_1(1 + \tau_1^c) = u_1/u_0$ , one needs to understand the behavior of marginal utilities relative to the competitive equilibrium. A complete characterization is possible in the limit case when capital intensity in the non-tradable sector becomes very small  $\alpha_1 \rightarrow 0$ , and hence non-tradable production is frictionless. We show in Appendix A.14 that in this case, the CPI-based real exchange rate necessarily *appreciates*. Intuitively, labor is reallocated towards tradables and hence non-tradable production decreases. Since non-tradables become more scarce, their price increases and hence the CPI-based real exchange rate appreciates. In numerical experiments (omitted for brevity), we have computed time paths for the equilibrium allocation in the case with  $\alpha_1 > 0$ , which indicate that also in this case the CPI-based real exchange rate is *appreciated* relative to the competitive equilibrium when the tradable sector is sufficiently undercapitalized.

**No sectoral labor taxes** Sector-specific labor taxes might be unavailable to the planner if it is hard to allocated jobs and occupation to specific sectors in order to administer such

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<sup>48</sup>The CPI-based real exchange rate is given by  $P/P^*$ , where  $P$  and  $P^*$  are the price indexes of the home country and the rest of the world which are functions of the consumer prices of tradable and non-tradable goods. Since we analyze a small open economy,  $P^*$  is fixed from the point of view of the home country, and we normalized  $p_0 = 1$  and  $\tau_0^c = 0$ . Therefore, the real exchange rate appreciates whenever the consumer price of non-tradables  $(1 + \tau_1^c)p_1$  increases. The wage-based real exchange rate is given by  $w/w^*$ , where  $w^*$  is the wage rate in the rest of the world and is taken as given.

taxes. We thus consider the case where the planner cannot differentially tax labor in different sectors. Since labor supply is inelastic in our multisector economy, this means that the planner cannot directly affect the allocation of labor at all. Therefore, the only instrument used by the planner is the consumption tax in the non-tradable sector,  $\tau_1^c$ , since according to Lemma 5 neither the savings subsidy, nor the tradable consumption tax are used. Indeed, we show in Appendix A.14 that the planner only uses the non-tradable consumption tax and sets it according to:

$$\tau_1^c = \frac{1}{\eta_1/\alpha_1 - 1} \left[ (1 - \nu_1) + \frac{1 - \gamma_1}{\gamma_1} \kappa \right], \quad \text{where} \quad \kappa = \frac{(\nu_0 - 1)\ell_0 - \frac{1}{\eta_1/\alpha_1 - 1}(\nu_1 - 1)\ell_1}{\ell_0 + \frac{\eta_1 - 1}{\eta_1/\alpha_1 - 1}\ell_1}.$$

The expression for the non-tradable tax depends on two terms. The first term is similar to above and the planner subsidizes non-tradables whenever the sector is undercapitalized, i.e.  $\nu_1 > 1$ . In contrast, the second term  $\kappa$  captures the fact that the planner uses the consumption tax to also affect the labor allocation. Note that  $\kappa$  increases in  $\nu_0$  and decreases in  $\nu_1$ . Intuitively, if  $\nu_0$  is large, then the only way to improve the allocation is by taxing non-tradable consumption (which is reflected in the  $\kappa$  term in  $\tau_1^c$ ), thereby shifting labor to the tradable sector. From the expression above we see that non-tradable consumption is taxed (i.e.,  $\tau_1^c > 0$ ) when:

$$\frac{\ell_0}{L}(\nu_0 - 1) > \frac{\gamma_1}{1 - \gamma_1}(\nu_1 - 1),$$

which is more likely the larger is  $\nu_0$ , the smaller is  $\nu_1$ , the larger is the size of the tradable sector 0 (in terms of labor allocated to this sector), and the smaller is  $\gamma_1$ . In particular, as  $\gamma_1 \rightarrow 0$  (for example, due to  $\alpha_1 \rightarrow 0$ , i.e. as non-tradable production stops relying on capital), non-tradable consumption is taxed whenever  $\nu_0 > 1$ . As a result, the economy-wide wage  $w = (1 - \alpha)y_0/\ell_0$  decreases and hence the *wage*-based real exchange rate  $w/w^*$  depreciates. At the same time, non-tradables become more expensive due to consumption tax, and hence the *CPI*-based real exchange rate appreciates.

**No sectoral taxes** In the absence of any sectoral instruments (labor or consumption), the planner has to recur to intertemporal distortions by means of a savings subsidy, or a policy of capital controls and reserve accumulation more commonly used in practice (see Jeanne, 2012, for the equivalence result of these policies). We provide a formal analysis of this case in Appendix A.14, and here briefly discuss the results. We show that by taxing consumption today in favor of future periods, the planner shifts resources away from the non-tradable sector and towards the tradable sector, which is desirable when  $\nu_0$  is sufficiently large. The

effect of such policy on the allocation of labor across sectors is similar to that of a consumption tax on non-tradables. However, it comes with an additional intertemporal distortion on the consumption of tradables, and as a result the intertemporal policy is strictly dominated by static sectoral policies (as follows from Lemma 5). In response to the intertemporal policy, wages, price of non-tradables, and consumption of both goods decrease, while the tradable sector expands production and exports, facing unchanged international prices.<sup>49</sup> Both CPI- and wage-based real exchange rates depreciate in response to this policy, which contrasts with the previously discussed cases. This narrative is consistent with the analysis of Song, Storesletten, and Zilibotti (2014) who argue that, in China, a combination of capital controls and other policies compressed wages and increased the wealth of entrepreneurs, thereby relaxing their borrowing constraints.

To summarize the analysis of this section, one of the goals of the planner is to shift labor towards the tradable sector when it is financially constrained ( $\nu_0 > 1$ ), which can be achieved in a variety of ways depending on the available set of instruments. One common feature of the policies is that they reduce the equilibrium wage rate paid in the tradable sector, resulting in a depreciated wage-based real exchange rate and enhanced competitiveness of the tradable-sector firms. At the same time, the effect of the policies on the consumption prices and CPI-based real exchange rate depends on the available policy instruments. In particular, the planner favors static sectoral instruments, which tax non-tradable labor or consumption and result in appreciated non-tradable prices. This contrasts with the narrative in the optimal exchange rate policy literature (see Rodrik, 2008; Korinek and Serven, 2010; Benigno and Fornaro, 2012), which tends to focus on the case where static sectoral taxes are unavailable, and the planner is limited to an intertemporal instrument. Thus, the real exchange rate implications of the optimal policy crucially depend on which instruments are available, even when the nature of inefficiency remains the same. We conclude that the (standard CPI-based) real exchange rate may not be a particularly useful guide for policymakers because there is no robust theoretical link between this variable and growth-promoting policy interventions.

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<sup>49</sup>Net exports result in net foreign asset accumulation, which is however accompanied by an inflow of productive capital to satisfy increased capital demand in the tradable sector.

## 5 Discussion of Assumptions

The goal of this paper is to develop a model of transition dynamics with financial frictions in which we can analyze optimal government interventions. This motivates a number of the assumptions we adopt, which allow for tractable aggregation of the economy and result in a simple characterization of equilibrium under various government policies (see Lemma 3). We now discuss these assumptions systematically, emphasizing which ones are made purely for tractability and which ones are necessary for our results.

**Functional forms** Our results are robust to many different functional forms for the utility function of households, as long as these feature a positive and finite Frisch elasticity of labor supply. For entrepreneurs, we assume logarithmic utility as it delivers a simple closed-form consumption policy function, but the analysis can be generalized to CRRA utility (see Moll, 2014).<sup>50</sup> The Pareto productivity distribution is useful for tractable aggregation and in order to maintain log-linearity of the equilibrium conditions, but is not essential for any of the results, and in certain applications may be conveniently replaced with other distributions. The time-invariant paths of exogenous productivity ( $A_t \equiv A$ ) and maximum leverage ratio ( $\lambda_t \equiv \lambda$ ) can be immediately generalized to arbitrary deterministic or stochastic time series processes without major consequences for the results, as they affect the planner’s problem only through the reduced-form productivity  $\Theta_t$  defined in (15).

The three functional form assumptions that are essential for tractability are the constant returns to scale (CRS) in production, CRRA utility of entrepreneurs which implies linear savings rules, and the linearity of the collateral constraint in the wealth of entrepreneurs. Together they result in optimal production and accumulation decisions that are linear in the wealth of the entrepreneurs, allowing for tractable aggregation and substantial reduction in the size of the state space. Indeed, in general, the state space of the model should include the time-varying joint distribution of endogenous wealth and exogenous productivity,  $\mathcal{G}_t(a, z)$ . Yet, with our assumptions, the state space reduces to a single variable—the aggregate (or average) wealth of all entrepreneurs  $x_t$ —and we only need to keep track of its dynamics characterized by (20). In the earlier literature, the enormous state space typical in the models with financial frictions and heterogeneity has been the main impediment to the

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<sup>50</sup>We additionally adopt a technical assumption that entrepreneurs are more impatient than households ( $\delta > \rho = r^*$ ) in order to insure the existence of a steady state. This assumption can be dropped if one is willing to stick to the analysis of the transition path in a model without a steady state. Alternatively, this assumption is not needed if workers are hand-to-mouth (in equilibrium) or subject to idiosyncratic income risk, in which case  $\delta = \rho > r^*$  is a natural assumption in a small open economy and would arise endogenously in a closed economy (Aiyagari, 1994).

optimal policy analysis outside of stationary equilibria.<sup>51</sup> While there is a sharp discontinuity in the tractability of the framework once one departs from CRS in production, there is no reason to expect such discontinuity in optimal policies, as the equilibrium allocation itself is continuous in returns to scale.<sup>52</sup> At a conceptual level, our policy implications are likely to remain relevant, as long as a non-trivial share of output is produced by financially constrained firms during the transition period.

**Heterogeneity** We develop a particularly tractable model of heterogeneity and aggregation. Many of our results can be illustrated in economies without heterogeneity, i.e. with a single productivity type of entrepreneurs. There are three main reasons why we opt in favor of a model with heterogeneity. First, this makes our framework closer to the canonical model of financial frictions used in the macro-development literature (see references in the Introduction) to which we want to relate our optimal policy analysis. Second, it allows us to capture misallocation and endogenous TFP dynamics, as well as their response to optimal policies, along the transition path. Third, and somewhat surprisingly, the model with a continuum of heterogeneous entrepreneurs is more tractable than its analogue without heterogeneity. This is because continuous heterogeneity is *regularizing*, adding smoothness to the equilibrium conditions without complicating them. This in turn allows us to capture the declining force of the financial frictions as the economy accumulates wealth and approaches the steady state, but at the same time avoids the need to keep track of different binding patterns of the financial constraint and the corresponding switches in the equilibrium regime.<sup>53</sup> In addition, in Appendix A.10 we show how to generalize our analysis from iid productivity draws to a persistent productivity process.

Finally, we rule out endogenous occupation choice, i.e. there are no transitions between the group of workers and the group of entrepreneurs. This is important for analytical

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<sup>51</sup>Some existing analyses of optimal policy do take into account transition dynamics but restrict tax instruments to be constant over time, making the optimal policy choice effectively a static problem (see e.g. Conesa, Kitao, and Krueger, 2009). Our result that the sign of the optimal policy differs according to whether an economy is close to or far away from steady state underlines the importance of examining time-varying optimal policy.

<sup>52</sup>For example, with CRS all active firms are financially constrained, while with returns to scale slightly below one, *almost* all firms are constrained. Therefore, the CRS economy is simply the tractable limiting case of a decreasing returns economy, and standard calibrated values for returns to scale are relatively close to one (see e.g. Atkeson and Kehoe, 2007; Buera and Shin, 2013). More formally, Moll (2014) shows, starting with DRS production functions  $y = ((zk)^\alpha \ell^{1-\alpha})^\beta$  with  $\beta < 1$ , that the solution for the production side of the economy under CRS can be obtained by taking the limit as  $\beta \rightarrow 1$ .

<sup>53</sup>In the case without heterogeneity, such regime switches would happen, for example, if the financial frictions stopped to bind altogether around or at the steady state. In the case with a discrete number of productivity types such regime switches occur more often throughout the transition. These regime switches make the complete characterization of dynamics substantially less tractable.

tractability of the model, in particular the savings policy function of the entrepreneurs. Furthermore and as already discussed, the wage subsidy prescribed in our analysis increases labor income and wealth accumulation of workers and therefore it may have additional beneficial effects in a richer environment with occupational choice. Therefore, we expect our results to also hold in a model with occupational choice, though such a model would be substantially more complicated and would have to be solved numerically.

**Financial frictions** Following the tradition in the literature (see references in the Introduction), we model financial frictions as the interaction between incomplete markets and collateral constraints, both exogenously imposed. The particular form of the collateral constraint can be generalized in a number of ways, in particular letting the financial friction parameter  $\lambda$  be a function of time or other individual or aggregate variables. The key conceptual assumption, however, is that the use of capital and production require a certain minimal *skin in the game*, and thus the effects generalize to a model with a richer set of available assets, including equity.<sup>54</sup> More generally, the effects we emphasize are likely to be present as long as the scale of production of a non-trivial share of businesses is constrained by their net worth or the wealth of their owners.<sup>55</sup> A large development literature (see e.g. [Banerjee and Duflo, 2005](#), and the references cited therein) has documented the importance of such constraints for developing countries.

## 6 Conclusion

The presence of financial frictions opens the door for welfare-improving government interventions in product and factor markets. We develop a framework to study the Ramsey optimal interventions which accelerate economic development in financially underdeveloped economies. We first study a one-sector economy, and then generalize the framework to multiple sectors so as to relate to the popular discussion of exchange rate and industrial policies. In the one-sector economy, financial frictions justify a policy intervention that reduces wages and increases labor supply in the early stages of transition so as to speed up entrepreneurial wealth accumulation and to generate higher labor productivity and wages in the long-run.

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<sup>54</sup>Note that issuing equity does not replicate transfers between agents, which have the ability to sidestep the financial constraints. This is because equity does not increase the net worth (assets) of the entrepreneurs, which we assume to be the relevant variable for the collateral constraint. In other words, it is the net worth of entrepreneurs that limits borrowing, not the absence of markets in risky assets.

<sup>55</sup>Indeed, the model identifies the derivative of aggregate output with respect to aggregate wealth of the business sector,  $\partial y/\partial x = \gamma \cdot y/x$ , as the key statistic determining the benefits of a pro-business policy intervention (see [Appendix A.8](#)), an insight we expect to persist beyond the specific environment of our model.

In a multi-sector economy, optimal policy compresses wages in undercapitalized sectors. It is also optimal to subsidize sectors with a *latent* comparative advantage, which may differ from the sectors with a short-run comparative advantage due to the accumulated entrepreneurial wealth. Finally, if producers of tradables are undercapitalized relative to producers of non-tradables, optimal policy compresses wages in the tradable sectors thereby making tradables more competitive. However, such an improvement in competitiveness is not necessarily reflected in a depreciation of the CPI-based real exchange rate.

To gain a better understanding of the optimal development policies and their implications for a country's growth dynamics, we set up our Ramsey problem in as tractable an environment as possible. By making a number of strong assumptions, we obtain a sharp analytical characterization of the optimal policies and a precise qualitative understanding of the mechanisms at play, which are likely to persist in more detailed and complex quantitative models with financial frictions. Similarly, it is this tractability that allows us to extend our baseline one-sector economy to multiple tradable and non-tradable sectors. The framework could be extended further in a number of additional directions. For example, another natural application is an analysis of the optimal policy response to cyclical and transitory shocks, such as recessions, market liberalizations and trade integrations.

## References

- AGHION, P., AND P. BOLTON (1997): “A Theory of Trickle-Down Growth and Development,” *The Review of Economic Studies*, 64, 151–172.
- AIYAGARI, S. R. (1994): “Uninsured Idiosyncratic Risk and Aggregate Saving,” *The Quarterly Journal of Economics*, 109(3), 659–84.
- (1995): “Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting,” *Journal of Political Economy*, 103(6), 1158–75.
- AMARAL, P. S., AND E. QUINTIN (2010): “Limited Enforcement, Financial Intermediation, And Economic Development: A Quantitative Assessment,” *International Economic Review*, 51(3), 785–811.
- ANGELETOS, G.-M., F. COLLARD, H. DELLAS, AND B. DIBA (2013): “Optimal Public Debt Management and Liquidity Provision,” NBER Working Papers 18800.
- ATKESON, A., AND P. J. KEHOE (2007): “Modeling the Transition to a New Economy: Lessons from Two Technological Revolutions,” *American Economic Review*, 97(1), 64–88.
- BACCHETTA, P., K. BENHIMA, AND Y. KALANTZIS (2014): “Optimal Exchange Rate Policy in a Growing Semi-Open Economy,” *IMF Economic Review*, 62(1), 48–76.
- BANERJEE, A. V., AND E. DUFLO (2005): “Growth Theory through the Lens of Development Economics,” in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf, vol. 1 of *Handbook of Economic Growth*, chap. 7, pp. 473–552. Elsevier.
- BANERJEE, A. V., AND A. F. NEWMAN (1993): “Occupational Choice and the Process of Development,” *The Journal of Political Economy*, 101(2), 274–298.
- (2003): “Inequality, Growth and Trade Policy,” Working paper, MIT.
- BARRO, R. J. (1979): “On the Determination of the Public Debt,” *Journal of Political Economy*, 87(5), 940–71.
- BARTELSMAN, E. J., J. HALTIWANGER, AND S. SCARPETTA (2013): “Cross Country Differences in Productivity: The Role of Allocation and Selection,” *American Economic Review*, 103(1), 305–335.
- BASSETTO, M., M. CAGETTI, AND M. D. NARDI (2013): “Credit Crunches and Credit Allocation in a Model of Entrepreneurship,” Nber working papers, National Bureau of Economic Research, Inc.
- BENIGNO, G., AND L. FORNARO (2012): “Reserve Accumulation, Growth and Financial Crises,” CEPR Discussion Papers 9224.
- BIANCHI, J. (2011): “Overborrowing and Systemic Externalities in the Business Cycle,” *American Economic Review*, 101(7), 3400–3426.
- BRUNNERMEIER, M. K., T. M. EISENBACH, AND Y. SANNIKOV (2013): “Macroeconomics with Financial Frictions: A Survey,” in *Advances in Economics and Econometrics*, ed. by D. Acemoglu, M. Arellano, and E. Dekel, vol. II of *Tenth World Congress of the Econometric Society*, pp. 3–94. Cambridge University Press, Cambridge, UK.
- BUERA, F., AND J.-P. NICOLINI (2013): “Liquidity Traps and Monetary Policy: Managing a Credit Crunch,” Working paper, Federal Reserve Bank of Chicago.
- BUERA, F. J., J. P. KABOSKI, AND Y. SHIN (2011): “Finance and Development: A Tale of Two Sectors,” *American Economic Review*, 101(5), 1964–2002.
- (2012): “The Macroeconomics of Microfinance,” NBER Working Papers 17905, National Bureau of Economic Research, Inc.
- BUERA, F. J., AND B. MOLL (2012): “Aggregate Implications of a Credit Crunch,” NBER Working Papers 17775.
- BUERA, F. J., B. MOLL, AND Y. SHIN (2013): “Well-Intended Policies,” *Review of Economic Dynamics*, 16(1), 216–230.
- BUERA, F. J., AND Y. SHIN (2013): “Financial Frictions and the Persistence of History: A Quantitative Exploration,” *Journal of Political Economy*, 121(2), 221 – 272.
- CABALLERO, R. J., AND G. LORENZONI (2014): “Persistent Appreciations and Overshooting: A Normative Analysis,” *IMF Economic Review*, 62(1), 1–47.
- CAGETTI, M., AND M. DE NARDI (2006): “Entrepreneurship, Frictions and Wealth,” *Journal of Political Economy*, 114(5), 835–870.
- CASELLI, F., AND N. GENNAIOLI (2013): “Dynastic Management,” *Economic Inquiry*, 51(1), 971–996.

- COLE, H. L., AND L. E. OHANIAN (2004): “New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis,” *Journal of Political Economy*, 112(4), 779–816.
- CONESA, J. C., S. KITAO, AND D. KRUEGER (2009): “Taxing Capital? Not a Bad Idea after All,” *American Economic Review*, 99(1), 25–48.
- COSTINOT, A., G. LORENZONI, AND I. WERNING (2013): “A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation,” *Journal of Political Economy*, forthcoming.
- DÁVILA, J., J. HONG, P. KRUSELL, AND J.-V. RÍOS-RULL (2012): “Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks,” *Econometrica*, 80(6), 2431–2467.
- DIAZ-ALEJANDRO, C. (1985): “Good-bye Financial Repression, Hello Financial Crash,” *Journal of Development Economics*, 19(1), 1–24.
- EROSA, A., AND A. HIDALGO-CABRILLANA (2008): “On Finance As A Theory Of TFP, Cross-Industry Productivity Differences, And Economic Rents,” *International Economic Review*, 49(2), 437–473.
- FEINSTEIN, C. H. (1998): “Pessimism perpetuated: real wages and the standard of living in Britain during and after the industrial revolution,” *The Journal of Economic History*, 58(03), 625–658.
- GALOR, O., AND J. ZEIRA (1993): “Income Distribution and Macroeconomics,” *The Review of Economic Studies*, 60(1), 35–52.
- GREENWALD, B. C., AND J. E. STIGLITZ (1986): “Externalities in Economies with Imperfect Information and Incomplete Markets,” *The Quarterly Journal of Economics*, 101(2), 229–64.
- GROWTH COMMISSION (2008): *The Growth Report: Strategies for Sustained Growth and Inclusive Development*.
- HAMILTON, A. (1791): *Report on manufactures*.
- HARRISON, A., AND A. RODRIGUEZ-CLARE (2010): *Trade, Foreign Investment, and Industrial Policy for Developing Countries* pp. 4039–4214, Handbook of Development Economics. Elsevier.
- HIDALGO, C. A., B. KLINGER, A.-L. BARABÁSI, AND R. HAUSMANN (2007): “The Product Space Conditions the Development of Nations,” *Science*, 317(5837), 482–487.
- HSIEH, C.-T., AND P. KLENOW (2009): “Misallocation and Manufacturing TFP in China and India,” *Quarterly Journal of Economics*, 124(4), 1403–1448.
- JEANNE, O. (2012): “Capital Account Policies and the Real Exchange Rate,” NBER Working Papers 18404.
- JEANNE, O., AND A. KORINEK (2010): “Managing Credit Booms and Busts: A Pigouvian Taxation Approach,” NBER Working Papers 16377, National Bureau of Economic Research, Inc.
- JEONG, H., AND R. TOWNSEND (2007): “Sources of TFP Growth: Occupational Choice and Financial Deepening,” *Economic Theory*, 32, 179–221.
- JUDD, K. L. (1985): “Redistributive taxation in a simple perfect foresight model,” *Journal of Public Economics*, 28(1), 59–83.
- KEHOE, T. J., AND D. K. LEVINE (2001): “Liquidity Constrained Markets versus Debt Constrained Markets,” *Econometrica*, 69(3), 575–598.
- KIM, D.-I., AND R. H. TOPEL (1995): “Labor Markets and Economic Growth: Lessons from Korea’s Industrialization, 1970-1990,” in *Differences and Changes in Wage Structures*, NBER Chapters, pp. 227–264. National Bureau of Economic Research, Inc.
- KIM, K., AND D. M. LEIPZIGER (1997): “Korea: A Case of Government-Led Development,” in *Lessons from East Asia*, ed. by D. M. Leipziger, pp. 155–212. University of Michigan Press, Ann Arbor.
- KIYOTAKI, N., AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105(2), 211–48.
- KORINEK, A., AND L. SERVEN (2010): “Undervaluation through foreign reserve accumulation : static losses, dynamic gains,” Policy Research Working Paper Series 5250, The World Bank.
- KRUGMAN, P. R. (1987): “The narrow moving band, the Dutch disease, and the competitive consequences of Mrs. Thatcher: Notes on trade in the presence of dynamic scale economies,” *Journal of Development Economics*, 27(1-2), 41–55.
- LA PORTA, R., F. L. DE SILANES, A. SHLEIFER, AND R. W. VISHNY (1997): “Legal Determinants of External Finance,” *Journal of Finance*, 52(3), 1131–50.
- (1998): “Law and Finance,” *Journal of Political Economy*, 106(6), 1113–1155.
- LEIPZIGER, D. M. (1997): *Lessons from East Asia*. University of Michigan Press, Ann Arbor, MI.
- LEVINE, R. (2005): “Finance and Growth: Theory and Evidence,” in *Handbook of Economic Growth*, ed. by P. Aghion, and S. N. Durlauf. Elsevier B.V.

- LIN, J. Y. (2012): *The Quest for Prosperity: How Developing Economies Can Take Off*. Princeton, NJ: Princeton University Press.
- LORENZONI, G. (2008): “Inefficient Credit Booms,” *Review of Economic Studies*, 75(3), 809–833.
- LUCAS, JR., R. E., AND N. L. STOKEY (1983): “Optimal fiscal and monetary policy in an economy without capital,” *Journal of Monetary Economics*, 12(1), 55–93.
- MATSUYAMA, K. (1992): “Agricultural productivity, comparative advantage, and economic growth,” *Journal of Economic Theory*, 58(2), 317–334.
- (2008): “Aggregate Implications of Credit Market Imperfections,” in *NBER Macroeconomics Annual 2007*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford, vol. 22, pp. 1–60. Chicago, IL: University of Chicago Press.
- MCKINNON, R. I. (1981): “Financial Repression and the Liberalization Problem within Less Developed Countries,” in *The World Economic Order: Past and Prospects*, ed. by S. Grassman, and E. Lundberg, pp. 365–386. St. Martin’s Press, New York.
- MIDRIGAN, V., AND D. Y. XU (2014): “Finance and Misallocation: Evidence from Plant-Level Data,” *American Economic Review*, forthcoming.
- MOLL, B. (2014): “Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?,” *American Economic Review*, forthcoming.
- MURPHY, K. M., A. SHLEIFER, AND R. W. VISHNY (1989): “Industrialization and the Big Push,” *Journal of Political Economy*, 97(5), 1003–26.
- PIKETTY, T. (1997): “The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing,” *The Review of Economic Studies*, 64, 173–189.
- QUADRINI, V. (2011): “Financial Frictions in Macroeconomic Fluctuations,” *Economic Quarterly*, 97(3), 209–254.
- RESTUCCIA, D., AND R. ROGERSON (2008): “Policy Distortions and Aggregate Productivity with Heterogeneous Plants,” *Review of Economic Dynamics*, 11(4), 707–720.
- RODRIK, D. (2008): “The Real Exchange Rate and Economic Growth,” *Brookings Papers on Economic Activity*, 2008(2), 365–412.
- SCHUMAN, M. (2010): *The Miracle: The Epic Story of Asia’s Quest for Wealth*. Harper Collins.
- SHIN, Y. (2006): “Ramsey Meets Bewley: Optimal Government Financing with Incomplete Markets,” Working paper, Washington University St. Louis.
- SONG, Z., K. STORESLETTEN, AND F. ZILIBOTTI (2011): “Growing Like China,” *American Economic Review*, 101(1), 196–233.
- (2014): “Growing (with Capital Controls) like China,” *IMF Economic Review*, 62(3), 327–370.
- STIGLITZ, J. E., AND S. YUSUF (2001): *Rethinking the East Asian Miracle*. World Bank Publications.
- STOKEY, N. L. (2009): *The Economics of Inaction*. Princeton, NJ: Princeton University Press.
- STRAUB, L., AND I. WERNING (2014): “Positive Long Run Capital Taxation: Chamley-Judd Revisited,” NBER Working Papers 20441, National Bureau of Economic Research, Inc.
- TOWNSEND, R. M. (2010): “Financial Structure and Economic Welfare: Applied General Equilibrium Development Economics,” *Annual Review of Economics*, 2, 507–546.
- VENTURA, J., AND J. VOTH (2013): “Debt into Growth: How Government Borrowing Accelerated the First Industrial Revolution,” unpublished.
- VOTH, H.-J. (2001): “The longest years: new estimates of labor input in England, 1760–1830,” *The Journal of Economic History*, 61(04), 1065–1082.
- WEITZMAN, M. L. (1974): “Prices vs. Quantities,” *Review of Economic Studies*, 41(4), 477–91.
- WERNING, I. (2007): “Optimal Fiscal Policy with Redistribution,” *The Quarterly Journal of Economics*, 122(3), 925–967.
- WOODFORD, M. (2008): “Comment on “The Real Exchange Rate and Economic Growth”,” *Brookings Papers on Economic Activity*, 2008(2), 420–437.
- WYNNE, J. (2005): “Wealth as a Determinant of Comparative Advantage,” *American Economic Review*, 95(1), 226–254.
- YOUNG, A. (1991): “Learning by Doing and the Dynamic Effects of International Trade,” *The Quarterly Journal of Economics*, 106(2), 369–405.