Integrating mortality into poverty measurement through the Poverty Adjusted Life Expectancy index.*

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October 15, 2024

Abstract

Poverty measures typically do not account for mortality, resulting in counter-intuitive evaluations. The reason is that they (i) do not attribute intrinsic value to the lifespan and (ii) suffer from a mortality paradox. We propose the first poverty index that always attributes a positive value to lifespan and does not suffer from the mortality paradox. This index, called the poverty-adjusted life expectancy, follows an expected lifecycle utility approach à la Harsanyi and is based on a single normative parameter that transparently captures the trade-off between poverty and mortality. Empirically, we show that accounting for mortality substantially changes cross-country comparisons and trends. We also quantify the fraction of these comparisons that are robust to the choice of the normative parameter.

JEL: D63, I32, O15.

Keyworks: Multidimensional poverty, Poverty, Mortality, Mortality paradox.

*Acknowledgments : We express all our gratitude to Kristof Bosmans, Jed Friedman, Aart Kraay, Dilip Mookherjee and Jacques Silber for helpful discussions and suggestions. This work was supported by the Fonds de la Recherche Scientifique - FNRS under Grant n° 33665820 and Excellence of Science (EOS) Research project of FNRS n°O020918F. We are grateful to the audience at the World Bank seminar for providing insightful comments. All errors remain our own. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors and should not be attributed in any manner to the World Bank, to its affiliated organizations, or to members of its Board of Executive Directors or the countries they represent. The World Bank does not guarantee the accuracy of the data included in this paper and accepts no responsibility for any consequence of their use.

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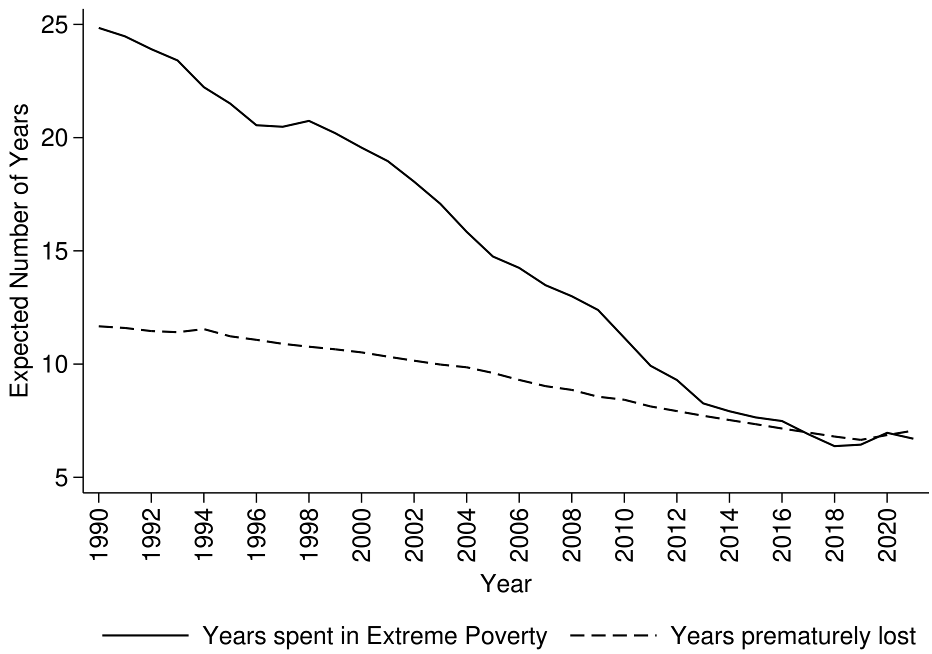
1 Introduction

Poverty measures are widely used for monitoring progress and guiding policies. However, most poverty measures do not account for the impact that mortality has on longevity.[1] The orders of magnitudes involved are staggering. As illustrated in Figure 1, in 2021, a new born expects to lose 7 years of life due to premature death[2] and to spend 7 years of life in poverty according to our Expected Deprivation index (which we define below). This represents, overall, 1.67 billion years of life either spent in extreme poverty or lost to premature death in that year.

Figure 1: Expected number of years spent in extreme poverty and prematurely lost for a newborn worldwide according to $ED_{1,70}$, 1990-2021.



Reading: in 1990, a newborn expected to spend 25 years in poverty and to lose 12 years due to premature death.

Mortality should be integrated into poverty measurement for several reasons. First, mortality reduces the lifespan of the deceased. As lifespan is a key resource, it should be attributed a positive *intrinsic* value. Second, mortality has a perverse *instrumental* impact on poverty measures. As observed by Kanbur and Mukherjee (2007), poverty measures face a "mortality paradox", since the death of poor individuals is measured as an improvement. Finally, an integrated indicator may be useful to guide policy decisions that require trading-off poverty and mortality. How to allocate a fixed budget between poverty alleviation and premature mortality reduction? How much should be spent on AIDS prevention programs? An integrated indicator that meaningfully reflects the relative impacts that poverty and mortality have on

[1] This remark also applies to measures of multidimensional poverty, which always ignore the impact of mortality on the deceased. While we refer to income poverty throughout the paper, our argument can be applied to these measures as well.

[2] We define a death as premature if it occured before 70 years old. The average life expectancy in 2021 is 71 in our data.

well-being may prove useful in guiding such choices.

Currently, there exists no indicator that integrates mortality into poverty measurement in a way that systematically attributes intrinsic value to longevity and always avoids the mortality paradox. Some poverty indicators avoid the mortality paradox but do not attribute intrinsic value to longevity (Kanbur and Mukherjee, 2007; Lefebvre et al., 2017). Some poverty indicators attribute intrinsic value to longevity only below some age threshold defining premature mortality, but do not avoid the paradox due to their "mash-up" aggregation of poverty and mortality (Watkins, 2006), or only avoid the mortality paradox below this age threshold (Baland et al., 2021). The Generated Deprivation proposed by Baland et al. (2021) is closest to satisfying the desired properties. However, this index is not easy to interpret and it exhibits non-monotonic behavior after a permanent mortality shock.

In this paper, we propose a new index, the poverty-adjusted life expectancy ($PALE_\theta^t$), that meaningfully integrates the poverty and mortality observed in a given year. This index systematically attributes intrinsic value to longevity and avoids the mortality paradox at all ages. We study the conditions under which its comparisons are robust to all plausible values of its normative parameter. We show that $PALE_\theta^t$ reacts straightforwardly to mortality shocks. Our empirical application shows that our indicators substantially change poverty comparisons and quantifies the cases for which reversed comparisons are robust.

Our main indicator, $PALE_\theta^t$, is normatively grounded on the expected lifecycle utility, the measure of social welfare proposed by Harsanyi (1953).[3] Under some conditions, $PALE_\theta^t$ normalizes the expected lifecycle utility of a newborn who *assumes* she will be confronted throughout her lifetime to the poverty and mortality prevailing in the current period.[4] This index simply counts the number of years that such newborn expects to live but weighs down the periods that she expects to live in poverty. Mathematically, our index is obtained by multiplying life expectancy at birth by a factor one minus the fraction of poor, with a lower weight being given to the latter. This (normative) weight $\theta > 0$, which captures the trade-off between poverty and mortality, corresponds to the fraction of the period utility lost when poor. When being poor has no utility cost, θ takes the value zero and $PALE_0^t$ corresponds to life expectancy at birth. When being poor (for one year) is as bad as losing one year of life, $\theta = 1$ and our index $PALE_1^t$ then corresponds to the poverty-free life expectancy at birth (Riumallo-Herl et al., 2018), i.e. the number of years of life a newborn expects to live out of poverty. $PALE_\theta^t$ does not suffer from the mortality paradox as long as $\theta \leq 1$. Besides its theoretical properties, $PALE_\theta^t$ enjoys two practical advantages. First, its data-requirement are minimal as only the life-expectancy at birth and the poverty head-count ratio are necessary. Second, $PALE_\theta^t$ has a simple interpretation, as it measures the equivalent number of years of life spent out of poverty.

To measure the real world relevance of our indexes, we combine data sets provided

[3]Following Harsanyi, social welfare in a given period can be understood as the lifecycle utility expected by a newborn when drawing at random a life that reflects the outcomes observed in that particular period.

[4]As we make clear later, our index is closely related to the concept of life expectancy, and its interpretation is based on similar assumptions. In particular, our index is not a forecast or a record of the actual average lifecycle utility of the cohort born in a particular period.

by the World Bank data on income poverty (Poverty and Inequality Platform (World Bank, 2024)) and an internationally comparable data set on mortality data (Global Burden of Disease Collaborative Network, 2021) from 1990 to 2021. We show that mortality is growing in relative importance and substantially affects global poverty comparisons: during the 2005-2019 period, at least 34% of $PALE_\theta^t$'s growth was due to the growth of life expectancy, as opposed to 17% from 1991 to 2004. For all possible values of θ, $PALE_\theta^t$ is able to solve in 2019 about half of the between country and 40% of the within country comparisons when focusing on these comparisons for which life expectancy and headcount are conflicting.

The remainder of the paper is organized as follows. In Section 2, we define the $PALE_\theta^t$ indicator, derive the conditions under which it corresponds to social welfare à la Harsanyi, and characterize the conditions under which its comparisons are robust to all the values of its normative parameter. In Section 3, we present our global empirical application, which quantifies the relevance of accounting for mortality and the frequency with which $PALE_\theta^t$ comparisons are robust. In Section 4, we compare the $PALE_\theta^t$ indicator with other indicators proposed in the literature. In particular, we provide a detailed comparison of $PALE_\theta^t$ with the Generated Deprivation index. We also contrast the empirical comparisons obtained with $PALE_\theta^t$ and the Generated Deprivation index. Section 5 concludes by discussing a key limitation of our indicators, namely that $PALE_\theta^t$ does not account for the unequal distribution of lifecycle utilities when the same individuals *cumulate* poverty and premature mortality.

2 The $PALE_\theta^t$ indicator: theory

2.1 Stationary societies

A toy example

The poverty head-count ratio does not account for mortality. As a result, this indicator can yield counterintuitive cross-society comparisons. The main reason is that the head-count ratio does not account for the high intrinsic value of longevity. Indeed, this indicator considers equally good two societies that have the same fraction of poor individuals, even if individuals in one society have a lifespan twice as long as the lifespan of their counterparts in the other society. The second reason is that mortality can have an instrumental impact on poverty. As a result, ignoring mortality can lead to paradoxical comparisons, as we illustrate by means of an example.

Consider two societies A and B, in which two individuals are born every year, one in the poor dynasty and one in the rich dynasty. Individuals born in the poor (resp. rich) dynasty remain poor (resp. non-poor) throughout their lives. The lifespan of a rich individual is four years. The only difference between societies A and B is the lifespan of a poor individual, which is one year in society A and three years in society B. These societies are **stationary** in the sense that natality is constant over time and the lifecycle outcomes of two individuals born in the same dynasty are the same. In a stationary society, all poverty and mortality outcomes are replicated identically every year, only the names of the affected individuals change.[5]

The relevant outcomes in societies A and B are summarized in Table 1. Consider

[5]Stationary societies are more formally defined in Appendix A.

society A. In any arbitrary year t, one individual is poor (P) and four individuals are non-poor (NP). The head-count ratio, which we denote by H^t, is thus 1/5. Individuals born in the poor dynasty before year t are already dead (D) and the same holds for individuals born in the non-poor dynasty before year $t-2$.

Table 1: Comparison of stationary societies A and B.

Age in year t Birth year	0 t	1 $t-1$	2 $t-2$	3 $t-3$
Poor dynasty A	P	D	D	D
Non-poor dynasty A	NP	NP	NP	NP
Poor dynasty B	P	P	P	D
Non-poor dynasty B	NP	NP	NP	NP

When mortality is ignored, the poverty comparison of societies A and B reveals a mortality paradox, since the head count ratio is larger in society B than in society A: $H^t(A) = 1/5$ and $H^t(B) = 3/7$. The longer lifespan of the poor dynasty in society B is thus recorded by H^t as a worsening. The problem is that, when comparing societies A and B, H^t does not take into account the different lifespans of individuals born in the poor dynasty.

Our proposed indicator, the poverty-adjusted life expectancy, solves both issues. Indeed, this indicator accounts for the intrinsic value of longevity and avoids the mortality paradox when comparing societies A and B.

Definition of $PALE_\theta$

The poverty-adjusted life expectancy index in year t is defined as

$$PALE_\theta^t = LE^t(1 - \theta H^t). \tag{1}$$

where $\theta > 0$ captures the normative trade-off between one year spent in poverty and one year spent out of poverty, H^t denotes the poverty head-count ratio and LE^t denotes life expectancy at birth, i.e., $LE^t = \sum_{a=0}^{a^*-1} \prod_{k=0}^{a-1}(1-\mu_k^t)$ where μ_k^t denotes the mortality rate at age k observed in year t and a^* denotes the maximal lifespan than can be reached and thus $\mu_{a^*-1}^t = 1$. The larger the value of $PALE_\theta^t$, the better. Two special cases are worth noting: $PALE_0^t$ corresponds to life expectancy at birth and $PALE_1^t$ corresponds to the Poverty Free Life Expectancy, an indicator proposed by Riumallo-Herl et al. (2018).

Our indicator accounts for the intrinsic value of longevity through life expectancy at birth. Also, it avoids the mortality paradox when assuming $\theta \leq 1$. Indeed, $PALE_\theta^t$ is the weighted sum of a number of years spent in poverty and a number of years spent out of poverty. Its mathematical expression can be written as

$$PALE_\theta^t = LE^t(1 - H^t) + (1-\theta)LE^t\ H^t.$$

For a newborn who expects to face throughout her life the poverty and mortality observed in year t, the term $LE^t(1-H^t)$ captures the number of years she expects to live out of poverty and the term $LE^t\ H^t$ captures the number of years she expects to live in poverty (its "poverty expectancy").[6] Years out of poverty receive weight 1

[6]As we explain below, $PALE_\theta^t$ is not a forecast on the life of a newborn. Rather, its purpose is

and years in poverty receive weight $(1-\theta)$. Consider again the example in Table 1. As $LE^t(A) = 2.5$ years and $LE^t(B) = 3.5$ years, we get from last expression that

$$PALE^t_\theta(A) = 2 + (1-\theta)0.5,$$
$$PALE^t_\theta(B) = 2 + (1-\theta)1.5.$$

We have $PALE^t_\theta(A) \leq PALE^t_\theta(B)$ when $(1-\theta) \geq 0$ and thus when $\theta \leq 1$. In other words, $PALE^t_\theta$ avoids the mortality paradox for values of θ for which being poor is no worse than being dead. This is more formally shown by Corollary 1 in Section 4.

Relationship with social welfare à la Harsanyi

We show that, in stationary societies, $PALE^t_\theta$ corresponds to social welfare à la Harsanyi under two assumptions. According to Harsanyi (1953), social welfare in a given year t corresponds to the lifecycle utility expected by a newborn given the outcomes observed in year t. Behind the veil of ignorance, the newborn faces a lottery whereby she ignores whether and when she will be poor and for how long she will live. When evaluating her life-cycle utility,[7] she considers the life of a randomly drawn individual in that society. Following the formulation of Jones and Klenow (2016), her expected life-cycle utility is given by

$$EU^t = \mathbb{E}^t \sum_{a=0}^{a^*-1} \beta^a u(c_a) V^t(a), \tag{2}$$

where $\beta \in [0,1]$ is the discount factor, $c_a \geq 0$ is consumption at age a, u is the period utility function, $V^t(a)$ is the unconditional probability that the newborn survives to age a given the mortality rates observed in year t, a^* is the maximal lifespan one can reach and the expectation operator $\mathbb{E}^t$ applies to the uncertainty with respect to c_a given the distribution of consumption observed in year t. The period utility when being dead is normalized to zero, i.e., $u(D) = 0$. As a result, mortality is valued through its opportunity cost: death reduces the number of periods during which a newborn can enjoy consumption.

For stationary societies, Eq. (2) simplifies into $PALE^t_\theta$ under two assumptions. Assumption A1 is to ignore discounting, i.e. $\beta = 1$. This assumption is necessary in order to assign equal weights to all individuals, regardless of their age.[8] Assumption A2 is to transform the consumption of alive individuals into a binary variable, i.e., c_a can be either being non-poor (NP) or being poor (P). This strong assumption implies that the impact on period utility of consumption differences within these two categories is ignored.[9] We denote the period utilities associated to being poor,

to jointly assess the mortality and poverty taking place in a given year.

[7]The rationality requirements of decision theory provide a structure on admissible life-cycle preferences. Rational preferences over streams of consumption have been axiomatized by Koopmans (1960) and later generalized by Bleichrodt et al. (2008). Such preferences must be represented by a discounted utility function, which aggregates these streams as a discounted sum of period utilities $U = \sum_{a=0}^{d} \beta^a u(c_a)$ where $d \in \mathbb{N}$ is the age at death, $\beta \in [0,1]$ is the discount factor, c_a is consumption at age a and u is the period utility function.

[8]Indeed, Eq. (2) equates a society's welfare in a given period to the expected life-cycle utility of individuals born in that period. Clearly, the expected life-cycle utility of newborns is related to the society's welfare in a given period only when one assumes that their expected lives reflect at each age the outcomes observed for individuals of that age during the period considered. Discounting with a factor less than one would give less weight to the outcomes of older individuals.

[9]Assumption A2 allows us to use the head-count ratio, the simplicity of which largely explains its

non-poor and dead respectively by $u_P = u(P)$, $u_{NP} = u(NP)$ and $u_D = u(D)$.

Proposition 1 shows that, under A1 and A2, in any stationary society, $PALE_\theta^t$ corresponds to expected life-cycle utility as expressed in Eq. (2).

Proposition 1 (Correspondence between Harsanyi and $PALE_\theta^t$)**.**
For any stationary society, assumptions A1 and A2 imply that

$$\frac{EU^t}{u_{NP}} = LE^t\left(1 - \underbrace{\frac{u_{NP} - u_P}{u_{NP} - u_D}}_{\theta} H^t\right)$$

and thus $PALE_\theta^t$ is ordinally equivalent to EU^t.

Proof. The proof is provided in Appendix B. □

Proposition 1 calls for several remarks. First, this result provides a mathematical expression for parameter θ. Parameter θ captures the fraction of her period utility that a non-poor individual loses when she becomes poor. This mathematical expression allows calibrating a value for parameter θ when selecting a particular period utility function and computing its value for the typical consumption of the poor and the non-poor, as we illustrate in Appendix C.

Second, Proposition 1 holds even when mortality is selective, that is when mortality rates affect differently poor and non-poor individuals, as in the case of societies A and B. The reason why $PALE_\theta^t$ is a simple normalization of EU^t even when mortality is selective is that EU^t is a *risk-neutral* social welfare function. Being risk-neutral, EU^t is unaffected by the distribution across individuals of periods spent in poverty or out of poverty. In the conclusion, we discuss the more general case of a social planner who cares for unequal *lifecycle utilities*, when some individuals combine poverty and short lifespans.[10]

Last but not least, societies are not stationary in practice. Indeed, societies are regularly affected by transitory or permanent shocks to their mortality and poverty. Crucially, non-stationarity breaks the equivalence between two alternative approaches for comparing societies A and B in Table 1. First, the "**cohort**" approach evaluates a society from the lifetime outcomes of a cohort of individuals. One can distinguish the "ex-ante" cohort approach, which considers the cohort born in year t, from the "ex-post" cohort approach, which considers the cohort that dies in year t. We illustrate the ex-ante approach using society A. The newborn in the poor dynasity will live year t in poverty and die at the end of year t. In turn, the newborn in the non-poor dynasity will live years t, $t+1$, $t+2$ and $t+3$ out of poverty and die in year $t+3$. The cohort approach thus considers outcomes happening in different years. Second, the "**population**" approach evaluates a society only from the outcomes observed in year t for its population alive in that year. For instance, in society A, there is one poor individual born in t and four non-poor individuals respectively born in t, $t-1$, $t-2$ and $t-3$. Regarding mortality, half of the newborns and all individuals who are 3 years old die at the end of year t.

popularity (as suggested by Table 1 in Kraay et al. (2023)). The headcount ratio remains however a crude indicator of poverty with well-known limitations (Sen, 1976).

[10] However, data on selective mortality is often not available. As a result, a social planner who cares for unequal utilities may do no better than integrating mortality into poverty measurement through indicators like $PALE_\theta^t$.

In a stationary society, both the cohort and the population approaches yield the same evaluation. As Table 1 makes it clear, as well as representing an individual, each of its cell is a unit of time. The cohort approach relies on units of time and the population approach on individuals. For instance, for the non-poor dynasty, the ex-ante cohort approach considers the four years that the individual born in year t will live out of poverty. In contrast, the population approach considers the four non-poor individuals who are alive in year t. The former approach thus considers four units of time for a single individual while the latter approach considers one unit of time for four different individuals. Thus, both approaches consider four units of time out of poverty. One particularity of a stationary society is that the poverty and mortality outcomes affecting the population that is alive in year t exactly reflect the future lifecycle outcomes expected by a newborn in year t. As a result, both approaches compare any two stationary societies in the same way.

2.2 Non-stationary societies

The cohort and population approaches yield different comparisons of non-stationary societies. Even though life expectancy at birth (LE^t) and social welfare à la Harsanyi (EU^t) have a cohort-like *interpretation*, they both follow the *population* approach since they are both only based on outcomes observed in year t. To be policy relevant, an index should account for the outcomes that take place *in the year* considered. For indicators aimed at tracking progress, the cohort approach has the key disadvantage of accounting for outcomes happening in different years. In the ex-post case, the fact that a transiant poverty or mortality shock happened a few decades ago seems of little relevance to evaluate well-being in year t. Similarly, in the ex-ante case, the fact that a newborn might still be poor or might die prematurely in a few decades after year t should also be considered irrelevant to year t's evaluation.

The population approach faces one key difficulty under non-stationary societies, namely how to aggregate the outcomes of individuals that have different ages. For instance, how should we weight mortality rates associated with different ages? For a stationary society, these mortality rates can be meaningfully weighed using the current population pyramid because the current population pyramid perfectly reflects these mortality rates. However, for a non-stationary society, the current population pyramid might not reflect the current mortality rates.

The typical solution to this difficulty is to aggregate outcomes *as-if the society was stationary*. This is for instance the solution underpinning the life expectancy at birth indicator. LE^t weights the age-specific mortality rates observed in year t using a counterfactual population pyramid that perfectly reflects these mortality rates. The interpretation of LE^t is the average lifespan of a (counterfactual) cohort of newborns that would be confronted throughout their lives to the mortality rates observed in year t. Crucially, this interpretation should not be understood as a forecast or a prediction for the average lifespan of a cohort born in year t. Indeed, mortality rates are unlikely to remain constant after year t. Rather, LE^t is simply a meaningful way of weighting the mortality rates observed in year t. Hence, LE^t has a cohort-like interpretation but follows the population approach. Social welfare à la Harsanyi also aggregates the outcomes observed in year t *as-if the society was stationary*. It takes the perspective of a (counterfactual) cohort of newborns that would be confronted throughout their lives to the mortality and poverty outcomes observed in year t. Again, the expected welfare of this counterfactual cohort should not be understood as a prediction, but only as a meaningful way of aggregating the various poverty and mortality outcomes observed in year t.

$PALE_\theta^t$ also aggregates outcomes as-if the society was stationary. However, under non-stationary societies, $PALE_\theta^t$ requires an additional condition to remain equivalent to social welfare à la Harsanyi. This condition is that the poverty rates observed in year t are the same for individuals of all ages. Formally, assumption A3 requires that $\mathbb{E}^t u(c_a) = H^t u_P + (1 - H^t) u_{NP}$ for all a, i.e., Harsanyi's newborn assumes that the probability she will be poor at any given age corresponds to H^t. Proposition 2 shows that, under A1, A2 and A3, $PALE_\theta^t$ corresponds to expected life-cycle utility as expressed in Eq. (2) even in societies that are non-stationary.

Proposition 2 (Correspondence between Harsanyi and $PALE_\theta^t$)**.**
Assumptions A1, A2 and A3 imply that

$$\frac{EU^t}{u_{NP}} = LE^t \left(1 - \underbrace{\frac{u_{NP} - u_P}{u_{NP} - u_D}}_{\theta} H^t \right)$$

and thus $PALE_\theta^t$ is ordinally equivalent to EU^t.

Proof. The proof is provided in Appendix D. □

Clearly, assumption A3 is unlikely to hold in practice, at least not exactly. Nevertheless, indicator $PALE_\theta^t = LE^t(1 - \theta H^t)$ has two main advantages. First, this indicator has very limited data requirements. Indeed, it can be applied from the two widely available indicators LE^t and H^t. Second, it has a relatively simple interpretation, namely the equivalent number of years of life spent out of poverty. Again, we cannot interpret $PALE_\theta^t$ as a projection or a forecast for the average lifecycle utility of a cohort born in year t. However, the validity of $PALE_\theta^t$ to evaluate a society in period t does *not* rely on its capacity to correctly forecast the future. Indeed, our objective is to aggregate the mortality and poverty observed in period t in a meaningful manner. This aggregation should not depend on the future evolutions of poverty and mortality.

We observe that $PALE_\theta^t$, like LE^t, aggregates mortality rates based on a counterfactual population pyramid. As we show in Section 4, this renders $PALE_\theta^t$ very reactive to permanent mortality shocks.

2.3 Robustness to normative parameter

We study the conditions under which comparisons by $PALE_\theta^t$ are robust to all the plausible values for parameter θ. As discussed above, these comparisons by $PALE_\theta^t$ avoids the mortality paradox when $\theta \leq 1$ (see Corollary 1). Yet, the comparison of two societies with $PALE_\theta^t$ may depend on the particular value assigned to $\theta \in (0, 1]$. We show that a nontrivial part of these comparisons does not depend on the value for θ *even for some pairs not related by domination.* In other words, there exist pairs of societies that are *robustly* ranked by $PALE_\theta^t$, that is, in the same way for all values of $\theta \in (0, 1]$ even though one is poorer but the other has higher mortality.

We illustrate this property in Figure 2. Without aggregation, domination alone allows comparing society A with the northwest quadrant (where societies have more poverty and more mortality) and the southeast quadrant (where societies have less poverty and less mortality). For any value of θ, we can draw the iso-PALE$_\theta^t$ curves passing through A. The iso-PALE$_0^t$ curve (associated to $\theta = 0$) is a vertical line since poverty has no welfare costs and life expectancy is the sole determinant of welfare. However, the iso-PALE$_1^t$ curve (associated to $\theta = 1$) is *not* a horizontal line. Therefore, the constraint $\theta \leq 1$ defines two additional areas for which welfare can be robustly compared with that of society A. The iso-PALE$_\theta^t$ curves associated to intermediate values of $\theta \in (0, 1]$ are indeed all located in the area between the iso-PALE$_0^t$ curve and the iso-PALE$_1^t$ curve. The area in the NE quadrant below the iso-PALE$_1^t$ curve yields a robustly higher social welfare than A, even though these societies have a higher poverty than A. The area in the SW quadrant above the iso-PALE$_1^t$ yields an robustly lower social welfare than A, even though these societies have a lower poverty than A. The size of these two new areas depends on the marginal rate of substitution of $PALE_1^t$ at A. For society A and $PALE_1^t$, this marginal rate of substitution is given by $\frac{LE^t(A)(1-H^t(A))}{(LE^t(A))^2}$. If $LE^t(A) = 70$ and $H^t(A) = 20\%$, this marginal rate of substitution is equal to 0.011, meaning that one additional year of life is exactly compensated by an increase in the head-count ratio H^t of 1.1 percentage points. These additional robust comparisons follow from (i) the fact that expected life-cycle utility sums period utilities and (ii) the assumption that a year of life spent in poverty is considered not worse than a year of life lost (i.e., $\theta \leq 1$, which is $u_D \leq u_P$).

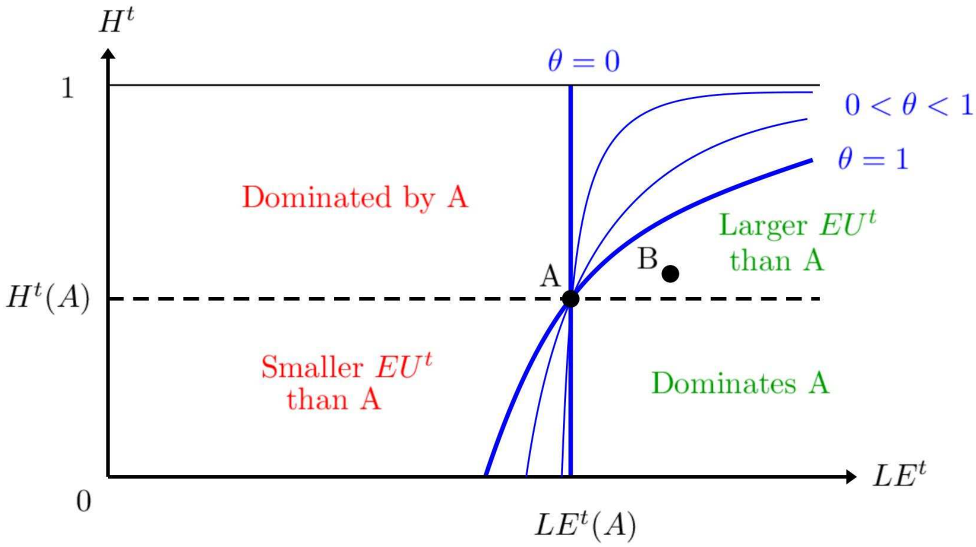


Figure 2: A and B are robustly ranked even though $H^t(A) < H^t(B)$ and $LE^t(A) < LE^t(B)$.

As an illustration, Table 2 below reports the situation of Pakistan and Bangladesh in 2021. Note that Life Expectancy can trivially be decomposed into Poverty Expectancy ($LE^t * H^t$) and Poverty Fee Life Expectancy ($LE^t * (1 - H^t)$). Pakistan has a lower headcount ratio than Bangladesh, but life expectancy is also lower in Pakistan. Therefore, it is a priori difficult to rank those two societies. Assuming that poverty and mortality remain unchanged, an individual born in Bangladesh can expect to spend 4.3 years of his life in poverty and 67.2 years out of poverty. In Pakistan, he can expect 2.7 years in poverty and 61.3 years out of poverty. Hence, a newborn in Bangladesh can expect not only to spend more years in poverty, but also more years out of poverty since the longer life expectancy there more than compensates for the higher poverty rate. As a result, $PALE_\theta^t$ ranks Bangladesh above Pakistan for all $\theta \in (0, 1]$.

Table 2: An example of robust comparison: Pakistan and Bangladesh in 2021.

	Headcount ratio	Life Expectancy	Poverty Expectancy $(LE^t * H^t)$	Poverty Free Life Expectancy $LE^t * (1 - H^t) = PALE_1^t$
Pakistan	4.2%	64.0	2.7	61.3
Bangladesh	6.0%	71.4	4.3	67.2

In the absence of domination (NE and SW quadrants in Figure 2), ignoring mortality, i.e., comparing two societies based on H^t, may lead to robustly erroneous comparisons. This happens when the ranking provided by $PALE_\theta^t$ is robust but differs from the ranking provided by H^t. Proposition 3 describes the conditions under which $PALE_\theta^t$ comparisons are robust.

Proposition 3. *(Robust comparisons with $PALE_\theta^t$)*
(i) For any two societies A and B, $PALE_\theta^t(A) \leq PALE_\theta^t(B)$ for all $\theta \leq 1$ if and only if

$$PALE_0^t(A) \leq PALE_0^t(B) \text{ and } PALE_1^t(A) \leq PALE_1^t(B) \qquad \text{(Condition C1)}$$

(ii) There exist societies A and B for which $PALE_\theta^t(A) \leq PALE_\theta^t(B)$ for all $\theta \leq 1$ even though $H^t(A) < H^t(B)$. These societies are such that $H^t(A) < H^t(B)$ and $LE^t(A) < LE^t(B)$.

Proof. See Appendix K. □

3 The $PALE_\theta^t$ indicator: empirics

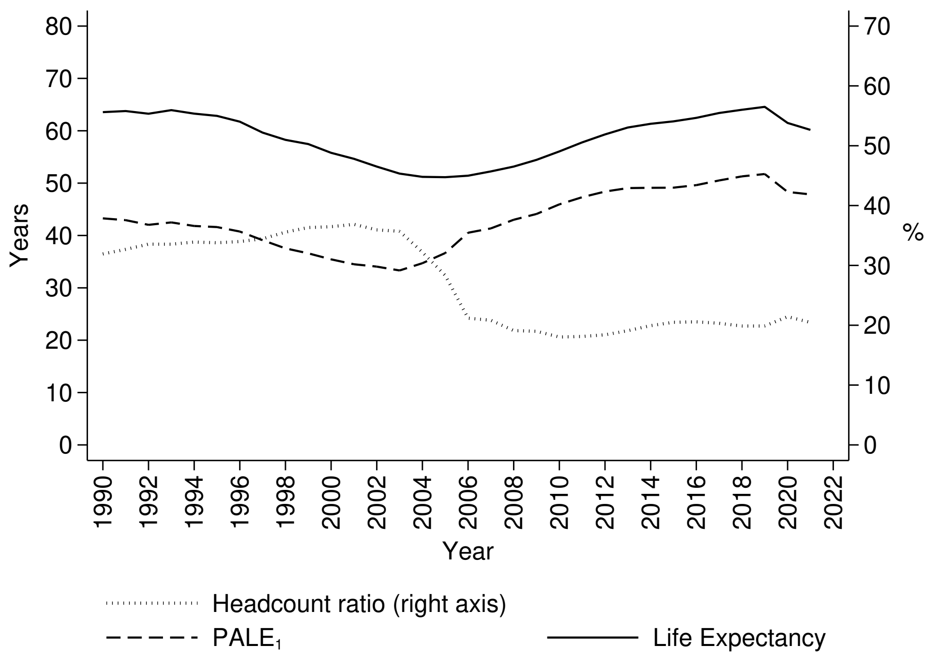
We now turn to data on poverty and life expectancy spanning the period 1990-2021. The data come respectively from the World Bank's Poverty and Inequality Platform (World Bank, 2024) and the Global Burden of Disease Project (Global Burden of Disease Collaborative Network, 2021). Our sample consists of 121 countries: non high income countries for which both poverty and mortality data are available. See Appendix N for the list of countries in the database as well as their descriptive statistics for the year 2021. Appendix O presents a practictioner guide to the construction of our index. In all our empirical exercises, the period t of reference is a year. Therefore to alleviate the notation, we will refer to $PALE_\theta$ instead of $PALE_\theta^t$.

3.1 A case study of South Africa

We first illustrate the relevance of our indices with the case of South Africa. Figure 3 reports the evolution of life expectancy, poverty rate and $PALE_1$ for South Africa from 1990 to 2021. From the perspective of poverty, the progress of South

Africa is impressive, with poverty rates decreasing from 32% to 20% over the period. However, life expectancy shows a different pattern. Following the AIDS epidemic, life expectancy decreased from the mid 90s onwards, to revert back to the pre-AIDS levels after 2017. Thus, in 2007, poverty rates are low, at 21%, but life expectancy is also low, at only 52 years. How then do we compare South Africa in 2007 to South Africa in 1990 when poverty rate was at 31% but life expectancy at 64 years old? $PALE_1$ indicates that deprivation is higher in 2007 than in was in 1990. Indeed, $PALE_1$ is equal to 43 years in 1990 as opposed to 41 years in 2007. We discuss in Section 3.3 the sensitivity of the comparisons made under $PALE_\theta$ to the choice of θ.

Figure 3: South Africa Evolution of $PALE_1$ and Life Expectancy, 1990-2021

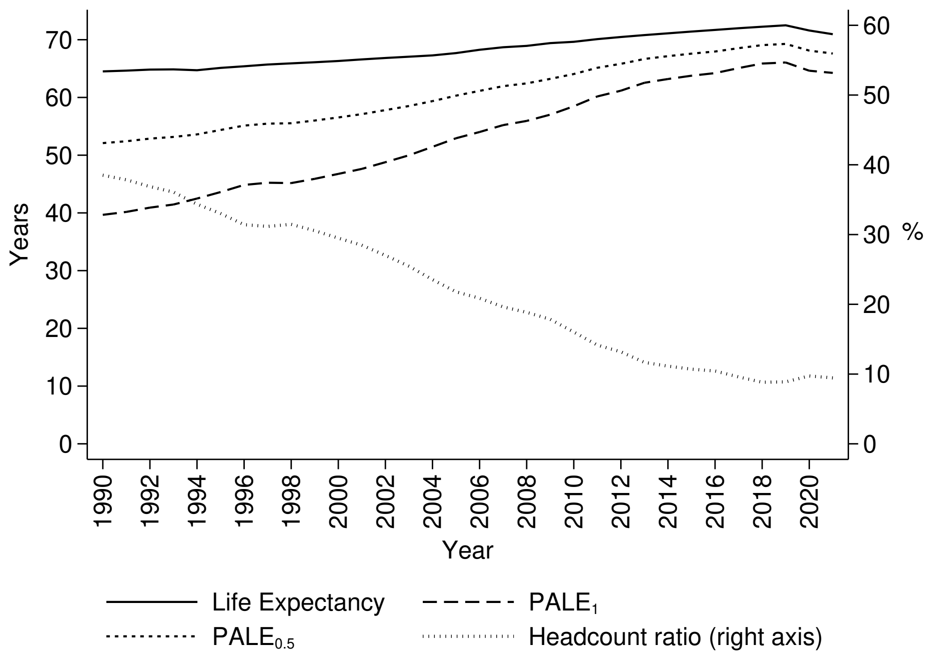


Reading: in 1990, life expectancy was 64 years, 32% of the population was living below the poverty line and poverty adjusted life expectancy was 43 years.

3.2 Life expectancy and poverty in the World, 1990-2021

At the world level, Figure 4 presents the evolution of life expectancy, the headcount ratio and $PALE_\theta$ between 1990 and 2021. Throughout this period, life expectancy increased from 64 to 71 but the decrease in poverty expectancy is even more spectacular, from 38% in 1990 to less than 10% in 2021. This decrease in poverty combined with an increase in life expectancy resulted in a large increase in $PALE_1$, from 40 in 1990 to 64 years in 2021. For $\theta < 1$, the corresponding $PALE_\theta$ curves all lie between life expectancy and the $PALE_1$ curve. For example, $PALE_{0.5}$ is higher in absolute value (52 years in 1990). However, its evolution is much slower than that of $PALE_1$: from 1990 to 2021, $PALE_{0.5}$ increased by 30% as opposed to 62% for $PALE_1$. Indeed $\theta = 1$ implies that one year spent in poverty is equivalent to one year spent dead. When instead one assumes that a year spent in poverty is equivalent to half a

Figure 4: Evolution of $PALE_\theta$ and Life Expectancy, 1990-2021



Reading: in 1990, Poverty-Adjusted Life Expectancy was about 40 years according to $PALE_1$ and 52 years according to $PALE_{0.5}$.

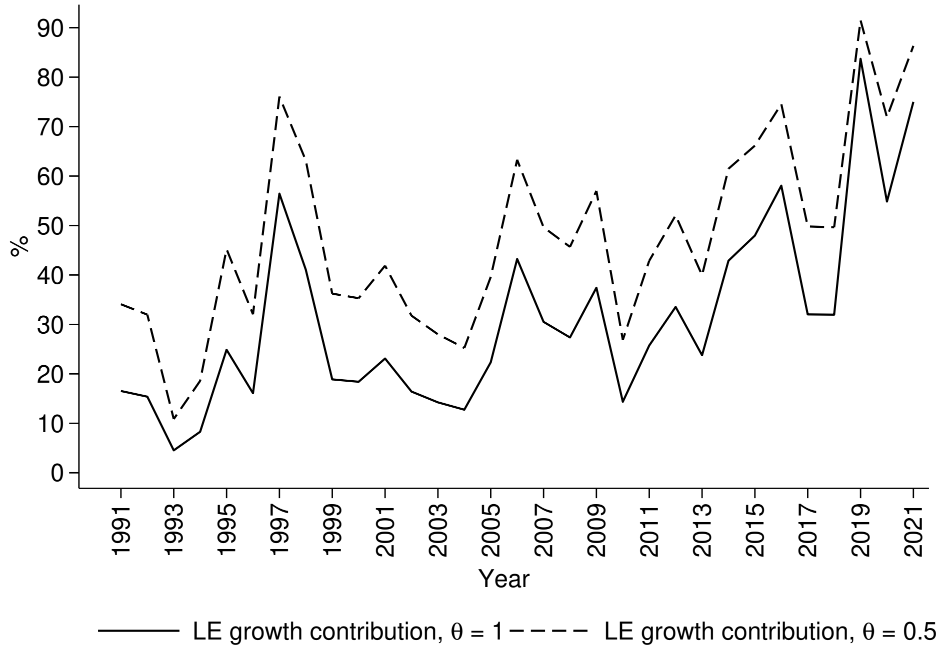
year lost to death, life expectancy has more weight in $PALE_{0.5}$ than in $PALE_1$. As the progress on life expectancy have been much slower than those against poverty, $PALE_{0.5}$ growth is slower.

Note that $PALE_\theta$ can not be directly decomposed into each of its components. However, it is possible to decompose its growth into the contribution of each of its component. Indeed, the growth rate of $PALE_\theta$ can be decomposed as follows:

$$\frac{\partial PALE_\theta}{\partial t} = \epsilon_{LE} * \frac{\partial LE}{\partial t} + \epsilon_H * \frac{\partial H}{\partial t}$$

where $\epsilon_{LE} = 1$ and $\epsilon_H = \frac{-\theta H}{(1-\theta H)}$ represent the elasticities of $PALE_\theta$ to life expectancy and poverty, respectively. Figure 5 shows the share of the growth of $PALE_\theta$ explained by changes in life expectancy, from 1991 to 2021. First, note that the choice of $\theta = 1$ is conservative: the contribution of life expectancy to $PALE_\theta$ is on average 16 percentage point smaller when $\theta = 1$ than when $\theta = 0.5$. Second, irrespective of the precise value given to θ, the contribution of life expectancy is growing over time. Life expectancy contributes on average to 21% (resp. 37%) of PALE's growth from 1991 to 2004, as opposed to 40% (resp. 57%) from 2005 onwards. Even though mortality has not decreased as much as poverty, changes in mortality play a substantial role in the trend of $PALE_\theta$.

Figure 5: Share of the growth of LE in the growth of $PALE_1$, 1990-2021



Reading: in 1991, the growth of life expectancy contributed to 17% of the growth of $PALE_1$ and to 34% to that of $PALE_{0.5}$.

3.3 Improving on comparisons based on the headcount only

Our indices also allow for comparing countries in which life expectancy and poverty evolve in opposite direction, as in the Pakistan-Bangladesh comparison presented in Table 2 or in the comparison between the years 1990 and 2007 in South Africa (Figure 3). However, the choice of θ may not be innocuous in these comparisons. We now focus on these cases. For these cases, we discuss the extent to which $PALE_\theta$ offers comparisons that are robust, that is, for which the ranking is not affected by the choice of θ. Note that if the ranking proposed by $PALE_\theta$ is robust to the choice of θ, this implies that $PALE_0$ and $PALE_1$ yield the same ranking. In that case, since $PALE_0$ corresponds to life expectancy, a measure solely based on the headcount provides a wrong ranking whenever life expectancy and headcount diverge (which is the case we focus on). The main interest of robust comparisons is to measure the extent to which $PALE_\theta$ allows to improve on a ranking based on the headcount only: in all these situations, irrespective of the value given to θ, the ranking under $PALE_\theta$ contradicts the poverty ranking.

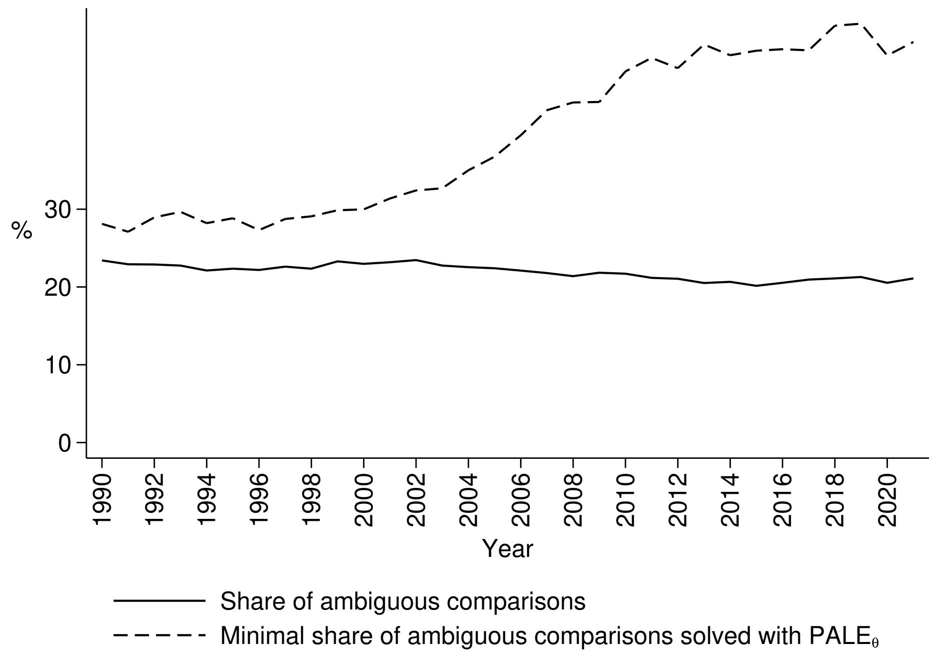
Inter country comparisons

To what extent does $PALE_\theta$ help in robustly ranking countries, as compared to a simple headcount? Figure 6 reports the proportion of all country-pairs comparisons whose ranking based on life expectancy and headcount ratio differs. There are 22% of them.[11] The share of these ambiguous cases for which $PALE_\theta$ provides a robust answer, i.e., independently of the value given to θ, is equal to 39%. In other

[11] These are the only situations in which $PALE_\theta$ can offer a different ranking than the headcount.

words, 39% of these "ambiguous" cases are wrongly classified by the headcount ratio. Note also that the share of ambiguous comparisons that our index unambiguously solves increases over time, owing to the falling incidence of absolute poverty in many countries.[12]

Figure 6: Evolution of the resolution of ambiguous inter-country comparisons, 1990-2021



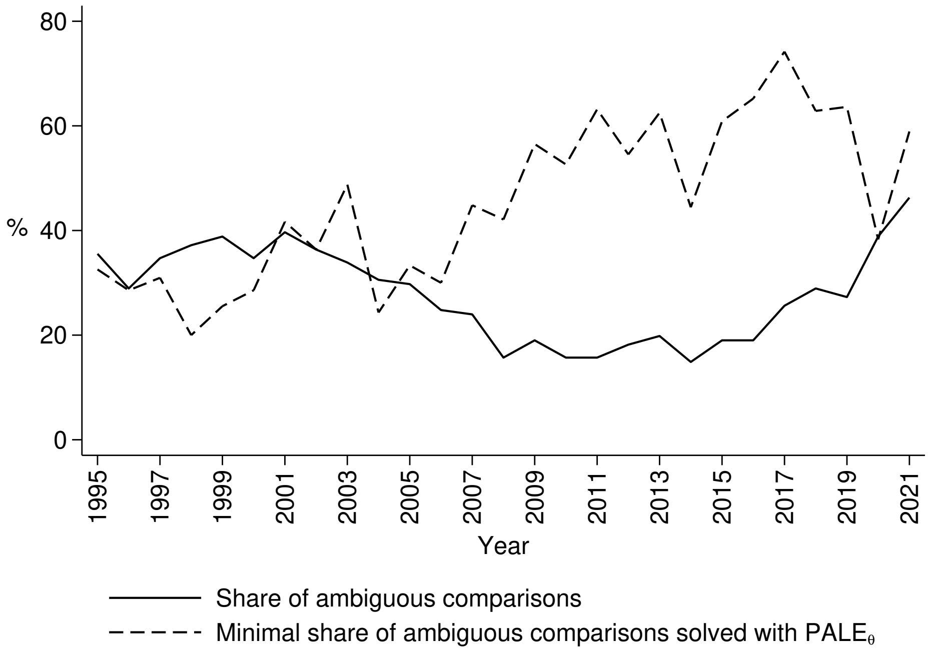
Reading: in 1990, countries had on average 23% of ambiguous comparisons, out of which at least 28% were unambiguously ranked by $PALE_\theta$.

Countries' trajectories

We now turn to individual trajectories of all countries, such as the South African case discussed earlier. For each country in our data, we computed the growth rate of H and LE over each 5-years period. Figure 7 presents the evolution of the share of ambiguous intra-country comparisons as well as the share that is robustly resolved by $PALE_\theta$. Over the period, the share of ambiguous trajectories oscillates between 20 and 40% of all cases. The share of these cases that $PALE_\theta$ ranks unambiguously varies between between 20 and 40% for the period 1995-2005 up to 40 to 60% in the 2005-2015 period. As above, $PALE_\theta$ corrects an increasing share of the rankings proposed by the headcount. In Appendix P, we present each country's evolution for the period 1990-2021 and its resolution in a graphical format reminiscent of the theoretical Figure 2.

[12]The falling incidence of absolute poverty implies that differences in H across countries in a given year become, on average, smaller over time. This explains why the share of ambiguous comparisons that our index unambiguously solves increases over time. This is easy to see when assuming that the differences in LE across countries in a given year remain constant over time. Indeed, a smaller difference in H can be "over-compensated" by a smaller difference in LE.

Figure 7: Evolution of the resolution of ambiguous countries' trajectories, 1990-2021



Reading: in the 1995, 36% of countries' trajectories was ambiguous. Among these, 33% can be assessed with $PALE_\theta$.

4 $PALE_\theta^t$ VS other indicators

$PALE_\theta^t$ is the first indicator to integrate mortality into poverty measurement in a way that always provides intrinsic value to longevity and is immune to the mortality paradox. We discuss similar indicators that integrate mortality with other dimensions of well-being.

4.1 $PALE_\theta^t$ vs other indicators

The poverty-adjusted life expectancy is reminiscent of several indicators proposed in health economics, like the quality-adjusted life expectancy (QALE) or the quality-adjusted life year (QALY).[13] Following Sullivan (1971), these two indicators account for the quality and quantity of life, by weighting down the quantity of life for periods with low quality. Propositions 1 and 2 show that they directly follow from the expected life-cycle utility approach à la Harsanyi. Our index, however, accounts for a major dimension of well-being other than health, which is poverty. In contrast, these indicators in the health economics literature ignore consumption.

We now turn to indicators that combine mortality and consumption. Several of these indicators do not dichotomize consumption and thus rely on a continuous consumption variable. One prominent is the Human Development Index (HDI), which aggregates mortality, consumption and education. Each of these three dimensions

[13] See for instance Whitehead and Ali (2010) for an economic interpretation of QALYs, or Heijink et al. (2011); Jia et al. (2011) for applications of the QALE index to comparisons of health outcomes across populations.

is normalized and then they are equally weighted. Ravallion (2012) calls "mash-up" this kind of aggregation that is not grounded in preference theory. Other indicators follow the social welfare à la Harsanyi and are thus grounded in preference theory. These indicators include proposals by Becker et al. (2005), Jones and Klenow (2016) or yet Boarini et al. (2022). These indicators enjoy deep conceptual foundations, but are little used in practice, perhaps due to their complexity. As illustrated in Table 1 of Kraay et al. (2023), simpler indicators are typically more used in practice than more sophisticated and thus more complex ones.

We now move to indicators aggregating poverty and mortality. Typically, these indicators consider binary consumption, i.e., individuals are either poor or non-poor (A2). This simplification sacrifices most of the consumption information. Therefore, these indicators are not ideal from a conceptual perspective. However, this is a widespread practice for policymaking. Perhaps this simplification allows constructing indicators that are easier to interpret.

An early index that integrates mortality into poverty is the Human Poverty Index proposed by Watkins (2006). This index weighs the fraction of poor individuals (H^t) with the fraction of individuals who die below the age of 40 years. This indicator gives intrinsic value to longevity only below 40 years. Also, the HPI is an example of "mash-up" index in the sense that its mathematical expression is ad-hoc. As a result, the HPI does not consistently compare years of life lost and years of life spent in poverty, as shown by Baland et al. (2021). The HPI thus falls victim to the mortality paradox.

Our indicator differs from those proposed in the mortality paradox literature (Kanbur and Mukherjee (2007); Lefebvre et al. (2013)), which aim at neutralizing the instrumental impact mortality has on poverty measurement. However, these indicators do not attribute intrinsic value to longevity (Decerf, 2023). These indicators are sometimes unaffected when the lifespan of all individuals is multiplied by a constant value. As a result, these indicators do not necessarily consider it an improvement when the life-cycle utility of a population is increased through longer lifespans. Another difference is that these indicators are based on counterfactual consumption outcomes, namely the poverty status that dead individuals would have enjoyed had they not died.[14]

The indicator most closely related to ours is the Generated Deprivation index ($GD^t_{\theta\hat{a}}$) proposed by Baland et al. (2021). Like the HPI, $GD^t_{\theta\hat{a}}$ accounts for the intrinsic value of longevity below an age threshold that defines premature mortality. Like $PALE^t_\theta$, $GD^t_{\theta\hat{a}}$ is immune to the mortality paradox for some range of values of its parameter, at least below this age threshold. As we further discuss below, $PALE^t_\theta$ improves on $GD^t_{\theta\hat{a}}$ along the following dimensions. First, $PALE^t_\theta$ also accounts for intrinsic value of longevity and is thus also immune to the mortality paradox above the age threshold. Second, $GD^t_{\theta\hat{a}}$ is less straightforward to interpret than $PALE^t_\theta$, perhaps hampering its diffusion in public debates. Third, $GD^t_{\theta\hat{a}}$ responds to mortality shocks with considerable inertia, reflecting long run adjustments in the population pyramid. Inertia is not, in general, a desirable feature for poverty measures.

[14] For indicators like $PALE^t_\theta$ that attribute an intrinsic value of longevity, the poverty status that a dead individual would have enjoyed is irrelevant.

4.2 $PALE_\theta^t$ vs $GD_{\theta\hat{a}}^t$

In this section, we study the relationships between $PALE_\theta^t$ and $GD_{\theta\hat{a}}^t$.

One key difference between the two is that $GD_{\theta\hat{a}}^t$ is based on an additional normative parameter: the age threshold $\hat{a}$ below which death is considered premature. For $GD_{\theta\hat{a}}^t$, mortality has negative intrinsic value only to the extent that death occurs below this age threshold. The rationale is that individuals whose death is too premature should be considered lifespan deprived. Hence, mortality matters in so far as it occurs below $\hat{a}$, which defines a minimally acceptable lifespan. We call this the "minimalist" view. This is in contrast to $PALE_\theta^t$, which gives negative intrinsic value to all deaths, even to those occuring above $\hat{a}$. We call this the "maximalist" view.[15]

Minimalism, maximalism and the mortality paradox

We define $GD_{\theta\hat{a}}^t$ using an example provided in Table 3. This example is based on the same two stationary societies A and B presented in Table 1. The only difference is that we now consider an age threshold $\hat{a} = 3$. There are now two types of dead individuals. A dead individual is considered prematurely dead (PD) if she is born less than 3 years before t and is considered dead (D) if she is born at least three years before t. There are two prematurely dead individuals in society A.

Table 3: Comparison of stationary societies A and B under the minimalist view.

Age in year t Birth year	0 t	1 $t-1$	2 $t-2$	3 $t-3$
Poor dynasty A	P	PD	PD	D
Non-poor dynasty A	NP	NP	NP	NP
Poor dynasty B	P	P	P	D
Non-poor dynasty B	NP	NP	NP	NP

Individuals born in the non-poor dynasty do not spent any year in deprivation while individuals born in the poor dynasty spend three years in deprivation. In society A, these individuals spend one year in poverty and prematurely lose two years. In society B, they spend three years in poverty.

Like $PALE_\theta^t$, $GD_{\theta\hat{a}}^t$ only depends on the outcomes observed in year t and thus follows the population approach. For the poor dynasty in society A, $GD_{\theta\hat{a}}^t$ records in year t a newborn who is poor and the same newborn who dies prematurely at the end of year t. The newborn spends year t in poverty. Also, her death implies she prematurely loses two years of life, namely $t+1$ and $t+2$, which she should have lived before reaching age 3. $GD_{\theta\hat{a}}^t$ collects all the future years of life lost to premature deaths in year t and attributes them to year t,

$$GD_{\theta\hat{a}}^t = \underbrace{\frac{YLL_{\hat{a}}^t}{N^t + YLL_{\hat{a}}^t}}_{mortality\ term} + \theta \underbrace{\frac{N^t H^t}{N^t + YLL_{\hat{a}}^t}}_{poverty\ term}, \tag{3}$$

where N^t denotes the number of alive individuals and $YLL_{\hat{a}}^t$ denotes the total number

[15] Note how these two views, while conceptually different, may in practice differ only parametrically: a "minimalist" approach using a very large age threshold is in practice "maximalist".

of years of life prematurely lost due to premature mortality in year t

$$YLL_{\hat{a}}^{t} = \sum_{a=0}^{\hat{a}-2} N_a^t * \mu_a^t * (\hat{a} - (a+1)),$$

where N_a^t is the number of alive individuals who have age a and μ_a^t is the mortality rate observed for individuals who have age a. Note that in society A, we have $YLL_{\hat{a}}^t = 2$. Observe that, because society A is stationary, $YLL_{\hat{a}}^t$ is equal to the number of individuals who prematurely miss year t, namely the two individuals born in the poor dynasty respectively in years $t-1$ and $t-2$. Again, this equality reflects the fact that each cell in Table 3 represents an individual as well as a unit of time.[16] The normative weight $\theta > 0$ is the same parameter as for $PALE_{\theta}^t$. Under $GD_{\theta\hat{a}}^t$, the ratio $1/\theta$ can be interpreted as the number of poverty years that are considered to be equally bad as one year of life prematurely lost. When assuming $\theta \leq 1$, we require that one poverty year cannot be worse than one year of life prematurely lost.

We are now equipped to contrast $GD_{\theta\hat{a}}^t$ and $PALE_{\theta}^t$. The main difference comes from the use of the age threshold $\hat{a}$, which has two implications. First, $GD_{\theta\hat{a}}^t$ does not provide intrinsic value to longevity above $\hat{a}$. To see this, consider a stationary society C where all individuals in the poor dynasty live for 6 years and individuals in the non-poor dynasty live for 8 years. Society C is thus obtained from society B by doubling everyone's lifespan. However, $GD_{\theta\hat{a}}^t$ does not record an improvement when moving from society B to society C. We have indeed $GD_{\theta 3}^t(C) = GD_{\theta 3}^t(B)$ because $YLL_{\hat{a}}^t(C) = YLL_{\hat{a}}^t(B) = 0$, $N^t(C) = 2N^t(B)$, and $H^t(C) = H^t(B)$. Second, $GD_{\theta\hat{a}}^t$ is still affected by the mortality paradox: any death of a poor individual occurring above the age threshold $\hat{a}$ is recorded as an improvement. To see this, consider the stationary society D whose only difference with society B is that the individuals born in the poor dynasty lives for four periods. Hence, poor individuals live one year longer in D than in B. We have $GD_{\theta 3}^t(D) = 4\theta/8$ and $GD_{\theta 3}^t(B) = 3\theta/7$, which shows that $GD_{\theta\hat{a}}^t$ can *improve* with the death of a poor individual. Our discussion illustrates that a maximalist indicator is required if one wishes to attribute intrinsic value to longevity at all ages and avoid the mortality paradox at all ages.

Reaction to mortality shocks

There is a second more subtle difference between $GD_{\theta\hat{a}}^t$ and $PALE_{\theta}^t$. This difference originates in the way in which these indicators aggregate mortality rates. In contrast to $PALE_{\theta}^t$, $GD_{\theta\hat{a}}^t$ is not based on a counterfactual population pyramid (associated to the mortality rates observed in year t), but rather on the actual population pyramid in year t. This difference has implications for the way in which these indicators respond to mortality shocks. This second difference thus kicks in when comparing non-stationary societies.

To best illustrate this difference, we define a minimalist version of $PALE_{\theta}^t$, which we call Expected Deprivation (ED). The only difference between $ED_{\theta\hat{a}}^t$ and $GD_{\theta\hat{a}}^t$ lies in the way in which they aggregate mortality rates and this difference only matters when comparing non-stationary societies. After a few results establishing

[16] Summing $YLL_{\hat{a}}^t$ with N^t in the denominator may seem strange until one realizes that the two terms capture years of human life, respectively prematurely lost or spent alive. Under this approach, it is natural to compute the share of units of time spent in deprivation among the total amount of units one ought to live out of deprivation.

these relationships between $ED^t_{\theta\hat{a}}$, $GD^t_{\theta\hat{a}}$ and $PALE^t_\theta$, we use simulations to illustrate how $ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$ differ when reacting to permanent mortality shocks.

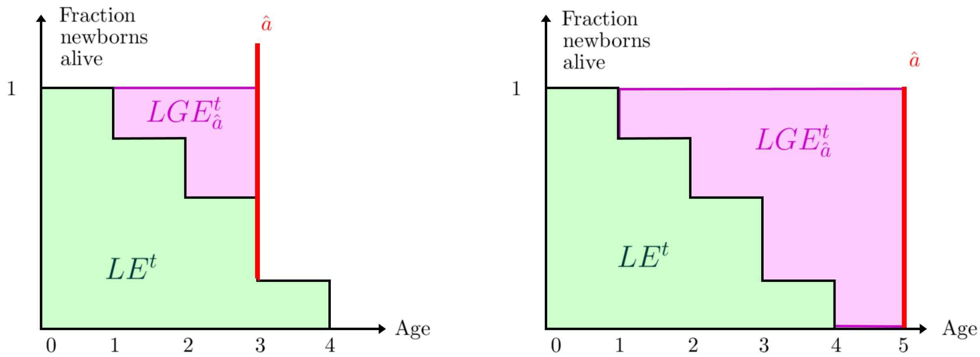
Definition of $ED^t_{\theta\hat{a}}$

We define a new indicator – the expected deprivation index ($ED^t_{\theta\hat{a}}$) – which generalizes $PALE^t_\theta$ under a minimalist view. Under this view, one should only give a (negative) intrinsic value to the years of life lost *before* reaching a minimal age threshold $\hat{a}$. $ED^t_{\theta\hat{a}}$ accounts for mortality through the lifespan gap expectancy ($LGE^t_{\hat{a}}$), which measures the number of years that a newborn expects to lose prematurely.[17]

$$LGE^t_{\hat{a}} = \sum_{a=0}^{\hat{a}-1} (\hat{a} - (a+1)) * \mu^t_a * \prod_{k=0}^{a-1} (1 - \mu^t_k),$$

We illustrate in Figure 8 the close connection between $LGE^t_{\hat{a}}$ and LE^t. The figure depicts for each age the fraction of newborns that are expected to still be alive at age a, assuming again that age-specific mortality rates are fixed. These fractions define a normalized counterfactual population pyramid. Indeed, the population pyramid of a stationary society confronted to these fixed mortality rates is obtained by multiplying these fractions by the fixed number of newborns.[18] In the left panel of Figure 8, LE^t is proportional to the area below the normalized population pyramid. By contrast, $LGE^t_{\hat{a}}$ is equal to the area between this normalized population pyramid and the age threshold. The right panel illustrates the property that, for large enough age thresholds, $LGE^t_{\hat{a}}$ is the complement of LE^t. Formally, when $\hat{a} \geq a^*$, where a^* is the maximal lifespan, we have $LGE^t_{\hat{a}} = \hat{a} - LE^t$.

Figure 8: Life Expectancy and Lifespan Gap Expectancy



Note: In the Left panel, the light green area below the normalized "stationary" population pyramid is equal to LE^t and the dark pink area is equal to $LGE^t_{\hat{a}}$.

The expected deprivation index ($ED^t_{\theta\hat{a}}$) aggregates the poverty and mortality observed in year t by taking the perspective of a newborn who expects to be confronted, throughout her life-cycle, to the poverty and mortality prevailing at the time of her

[17] $LGE^t_{\hat{a}}$ is a particular version of the Years of Potential Life Lost, an indicator used in medical research in order to quantify and compare the burden on society due to different causes of death (Gardner and Sanborn, 1990).

[18] In a stationary society, the current population pyramid can be obtained by successively applying the current age-specific mortality rates to each age group.

birth.

$$ED^t_{\theta\hat{a}} = \underbrace{\frac{LGE^t_{\hat{a}}}{LE^t + LGE^t_{\hat{a}}}}_{mortality\ term} + \theta \underbrace{\frac{LE^t * H^t}{LE^t + LGE^t_{\hat{a}}}}_{poverty\ term}, \tag{4}$$

with the same parameters $\theta > 0$ and $\hat{a} \geq 2$. The two normative parameters θ and $\hat{a}$ jointly define the respective importance attributed to poverty and mortality.[19]

Both terms have the same denominator, which measures a normative lifespan corresponding to the sum of LE^t and $LGE^t_{\hat{a}}$.[20] The numerator of each term measures the expected number of years characterized by one of the two dimensions of deprivation, again assuming that the society is stationary. The numerator of the mortality term measures the number of years that a newborn expects to lose prematurely (when observing mortality in the period) given the age threshold, $\hat{a}$. The numerator of the poverty term measures the number of years that a newborn expects to spend in poverty.

Relationship between $ED^t_{\theta\hat{a}}$ and $PALE^t_\theta$

We show that $PALE^t_\theta$ is a version of $ED^t_{\theta\hat{a}}$ that encapsulates the maximalist view. Indeed, as stated in Proposition 4, $ED^t_{\theta\hat{a}}$ ranks societies exactly in the same way (up to a sign) as $PALE^t_\theta$ as long as its age threshold $\hat{a}$ is at least as large as the maximal lifespan a^*. For such values, the age threshold is not binding, and all deaths become relevant in terms of deprivation.

Proposition 4 ($ED^t_{\theta\hat{a}}$ generalizes $PALE^t_\theta$)**.**
For all $\hat{a} \geq a^*$ we have $PALE^t{}_\theta = \hat{a}(1 - ED^t{}_{\theta\hat{a}})$, which implies that, for any two societies A and B,

$$PALE^t{}_\theta(A) \geq PALE^t{}_\theta(B) \Leftrightarrow ED^t_{\theta\hat{a}}(A) \leq ED^t_{\theta\hat{a}}(B).$$

Proof. See Appendix H. □

When the age threshold is binding (smaller than the maximal age a^*), the rankings obtained under $ED^t_{\theta\hat{a}}$ may not correspond to the rankings obtained under $PALE^t_\theta$. In Appendix I, we contrast the impact of mortality shocks on $PALE^t_\theta$ and $ED^t_{\theta\hat{a}}$.

Relationship between $ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$

Proposition 5 shows that $ED^t_{\theta\hat{a}}$ also compares stationary societies in the same way as $GD^t_{\theta\hat{a}}$.

Proposition 5 ($ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$ are identical in stationary societies)**.**
For any stationary society, we have $ED^t_{\theta\hat{a}} = GD^t_{\theta\hat{a}}$

Proof. See Appendix E. □

[19] Parameter θ determines the relative weights of being dead or being poor *for one period.* In contrast, parameter $\hat{a}$ determines the number of periods for which "being prematurely dead" is accounted for. Hence, $\hat{a}$ affects the relative size of the deprivation coming from mortality versus the deprivation coming from poverty.

[20] This normative lifespan can be interpreted as the (counterfactual) life expectancy at birth that would prevail if all premature deaths were postponed to the age threshold. It is at least as large as LE^t, and corresponds to LE^t if the age threshold is equal to 1.

Escaping the mortality paradox with $PALE^t_\theta$

As observed above, $GD^t_{\theta\hat{a}}$ is not immune to the mortality paradox. A paradox-free index should record an improvement when a stationary society is obtained from another stationary society by *an increment to the lifespan of a poor person.*[21] For instance, in Table 3, stationary society B is obtained from stationary society A by two successive increments to the lifespan of individuals in the poor dynasty.

Definition 1 (Paradox-free)**.**
$M^t_{\theta\hat{a}} \in \{ED^t_{\theta\hat{a}}, GD^t_{\theta\hat{a}}\}$ is paradox-free if for any two stationary societies A and B such that B is obtained from A by an increment to the lifespan of a poor person we have $M^t_{\theta\hat{a}}(A) \geq M^t_{\theta\hat{a}}(B)$.

Proposition 6 identifies the values for the two parameters θ and $\hat{a}$ under which $ED_{\theta\hat{a}}$ is Paradox-free. First, all deaths should matter, which implies that the age threshold $\hat{a}$ should be at least as large as the maximal lifespan a^*. Second, one year of life prematurely lost should be at least as bad as one year of life spent in poverty, which implies that $\theta \leq 1$ and thus $u_{NP} - u_P \leq u_{NP} - u_D$.

Proposition 6 ($ED^t_{\theta\hat{a}}$ and the mortality paradox)**.**
$M^t_{\theta\hat{a}} \in \{ED^t{}_{\theta\hat{a}}, GD^t{}_{\theta\hat{a}}\}$ is Paradox-free if and only if $\theta \leq 1$ and $\hat{a} \geq a^*$.

Proof. See Appendix J. □

An immediate corollary for Propositions 4 and 6 follows: the only way for $ED^t_{\theta\hat{a}}$ to be Paradox-free is to be ordinally equivalent to $PALE^t_\theta$ with $\theta \leq 1$.

Corollary 1 ($PALE^t{}_\theta$ and the mortality paradox)**.**
$ED^t_{\theta\hat{a}}$ is Paradox-free if and only if $ED^t_{\theta\hat{a}}$ is ordinally equivalent (up to a sign) to $PALE^t_\theta$ with $\theta \leq 1$.

Corollary 1 shows that the mortality paradox provides a justification for $PALE^t_\theta$.

In Appendix L, we study the conditions under which comparisons by $ED^t_{\theta\hat{a}}$ are robust to the plausible values for its parameters.

Simulated reactions to permanent mortality shocks

We now discuss more systematically the dynamic differences between $ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$. $ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$ rank non-stationary societies differently. A stationary society becomes non-stationary after it is affected by a poverty shock, a mortality shock or a shock affecting both poverty and mortality. For our purposes, we can ignore poverty shocks. The reason is that both $ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$ account for poverty in the same way, namely through H^t. As a result, they both account for the dynamic evolution of poverty through the way in which this evolution affects H^t. In other words, the difference between $ED^t_{\theta\hat{a}}$ and $GD^t_{\theta\hat{a}}$ in non-stationary societies comes from how they aggregate mortality rates. $GD^t_{\theta\hat{a}}$ records the number of years prematurely

[21] We define more formally the notion of an increment to the lifespan of a poor person in this footnote. Following our formal framework presented in Appendix A, the life of an individual i is a list of poverty statuses $l_i = (l_{i0}, \dots, l_{id_i})$ that she experiences between age 0 and the age at which she dies $d_i \in \{0, \dots, a^* - 1\}$, where $l_{ia} \in \{NP, P\}$. We say that stationary society B is obtained from stationary society A by an increment to the lifespan of a poor person when both societies have the same natality, $l^A_i = l^B_i$ for all individuals i except for some individual j such that $d^B_j = d^A_j + 1$, $l^A_{ja} = P$ for all $a \leq d^A_j$ and $l^B_{ja} = P$ for all $a \leq d^B_j$.

lost over all premature deaths actually taking place in year t. $ED^t_{\theta\hat{a}}$ also counts the number of years prematurely lost but, instead of being computed on the actual population pyramid, $ED^t_{\theta\hat{a}}$ uses a counterfactual population pyramid, which is the one that would prevail in a stationary society characterized by the age-specific mortality rates observed in the period.

A major implication of this difference is that $ED^t_{\theta\hat{a}}$ is more reactive to mortality shocks than $GD^t_{\theta\hat{a}}$. A toy example in Appendix F provides an illustration of this property for a *transitory* mortality shock. In the remainder, we focus on *permanent* mortality shocks, which are more relevant for policymaking. Thus, consider a permanent mortality shock. The population dynamics is such that a transition phase sets in during which the population pyramid slowly adjusts to the new mortality rates. This transition stops when a new stationary population pyramid is reached, typically after several decades (up to a^* periods). $GD^t_{\theta\hat{a}}$ records each step of this transition and therefore exhibits inertia in its response to a permanent mortality shock.[22] By contrast, $ED^t_{\theta\hat{a}}$ immediately refers to the new stationary population pyramid and thus disregards the inertia caused by these transitory demographic adjustments.

We illustrate this difference by simulating the consequences of a permanent mortality shock on a stationary population. For simplicity, we assume constant natality rates and no poverty. The age threshold is 50 and the maximal age is 100. Before the shock, the population pyramid is consistent with a mortality vector such that, at each age before 100, the mortality rate is equal to 2%. Figure 9 illustrates the relative evolution of the two indices for two types of permanent shocks: (1) the mortality rate falls from 2 to 1% only at age 40 and (2) the mortality rate falls from 2 to 1% only at age 5.

The upper graph illustrates the consequences of the mortality shock at age 40 and the bottom graph of the mortality shock at age 5. The two indices evolve differently over the transition period. Both $GD^t_{\theta\hat{a}}$ and $ED^t_{\theta\hat{a}}$ jump discretely in the period of the shock. $GD^t_{\theta\hat{a}}$ makes a partial jump and continues to slowly adjust to the induced changes in the population pyramid. In the long run, when society reaches stationarity, the two indices are again equal, once the shock is fully accounted for. $ED^t_{\theta\hat{a}}$ directly jumps to the long run equilibrium value. These simulations indicate that $ED^t_{\theta\hat{a}}$ is more reactive than $GD^t_{\theta\hat{a}}$ to a permanent mortality shock.

The bottom graph illustrates that the behavior of $GD^t_{\theta\hat{a}}$ during the adjustment periods needs not be monotonic, as observed in Baland et al. (2021). This non-monotonicity might be confusing and makes interpretation of the trend in $GD^t_{\theta\hat{a}}$ less straightforward than the trend in $ED^t_{\theta\hat{a}}$. There is thus another benefit of aggregating mortality rates through a counterfactual population pyramid, as done in $PALE^t_\theta$ and $ED^t_{\theta\hat{a}}$, namely that it simplifies the interpretation of its trend after permanent mortality shocks.

Note finally that $GD^t_{\theta\hat{a}}$ and $ED^t_{\theta\hat{a}}$ also differ in their axiomatic properties. Ba-

[22] For instance, assume that society A (see Table 3) undergoes a permanent mortality shock such that society A is subjected to her mortality vector in all years before t, namely $(\mu_0^A, \mu_1^A, \mu_2^A, \mu_3^A) = (1/2, 0, 0, 1)$, but from year t onwards society A is subjected to the mortality vector of society B, namely $(\mu_0^B, \mu_1^B, \mu_2^B, \mu_3^B) = (0, 0, 1/2, 1)$. There is a mechanical adjustment to the population pyramid, such that only two poor individuals live in year $t+1$. Only in year $t+2$ does the population pyramid reach the new equilibrium, with three poor individuals. The inertia of $GD^t_{\theta\hat{a}}$ may be deemed undesirable because it may complicate the analysis. Baland et al. (2021) show that the mechanical adjustments following a permanent mortality shock may lead to a non-monotonic trend in $GD^t_{\theta\hat{a}}$.

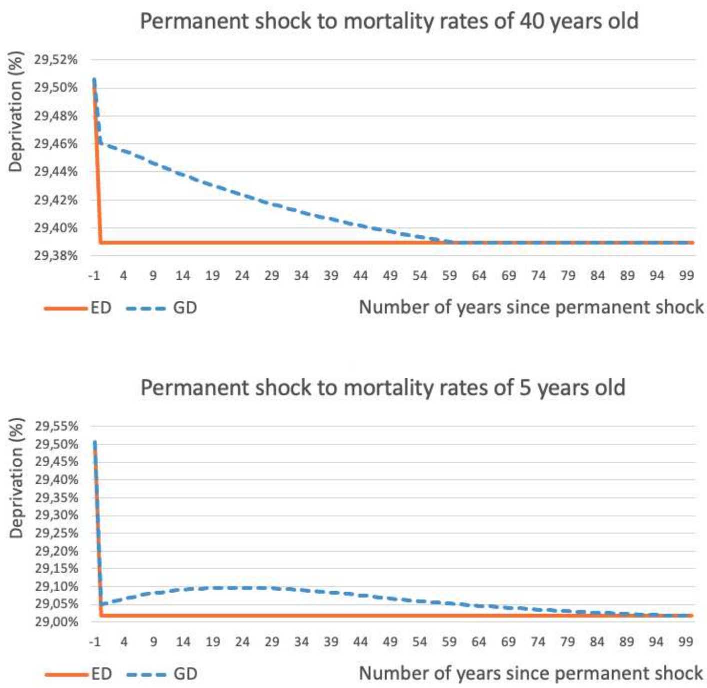


Figure 9: Simulation of permanent mortality shocks on a stationary population ($\hat{a} = 50$ and $H^t = 0$)

land et al. (2021) show that $GD^t_{\theta\hat{a}}$ is essentially the only index decomposable into subgroups to compare stationary societies in a way that satisfies some basic properties. As a result, $ED^t_{\theta\hat{a}}$ cannot be decomposable into subgroups.[23] This is no surprise given that $ED^t_{\theta\hat{a}}$ is based on life expectancy, which cannot be decomposed into subgroups. In Appendix G, we also show that $ED^t_{\theta\hat{a}}$ is the only index that is independent on the actual population pyramid (thereby avoiding the inertia associated to its mechanical adjustments after mortality shocks) and compares stationary populations in a way that respects basic properties.

4.3 Empirics

How often, in practice, do $GD_{\theta\hat{a}}$, $ED_{\theta\hat{a}}$ and $PALE_{\theta}$ provide different rankings of countries? Figure 10 presents the degree of agreement between these indices for inter country comparisons for several definitions of the parameters θ and $\hat{a}$. Figure 10 focuses on ambiguous cases, when the headcount ratio and life expectancy provide a different ranking. The degree of agreement between $GD_{1,70}$, $ED_{1,70}$ is extremely high, at around 93% throughout the period. By Proposition 5, these 7% cases of disagreement between $GD_{1,70}$ and $ED_{1,70}$ can entirely be attributed to the fact that the societies being compared are not stationary. These 7% of remaining cases are due to the different manner in which they react to mortality shocks, as discussed above. Note also how the agreement between $GD_{1,\hat{a}}$, $ED_{1,\hat{a}}$ is almost identical with another age threshold $\hat{a}$, such as 50. Comparing Figures 10a and 10b, we see that the

[23]In other words, if decomposability into subgroups is seen as a key property, one should use $GD^t_{\theta\hat{a}}$. Indeed, this index yields the same ranking as $ED^t_{\theta\hat{a}}$ in stationary populations. In those populations, $GD^t_{\theta\hat{a}}$ thus yields the same ranking as $PALE^t_{\theta}$ when all deaths are normatively relevant ($\hat{a} \geq a^*$).

level of agreement between $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$, while still high, decreases when moving from $\theta = 1$ to $\theta = 0.5$. Indeed, the only difference between our indicators is the way in which they incorporate mortality. The larger θ, the lower the relative weight given to mortality compared to poverty, and the closer the diagnostics proposed by the three indexes.

Now comparing the diagnostics of $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$ with that of $PALE_\theta$, the agreement, while still very high, is much lower, and decreasing over time. The disagreements between $PALE_\theta$ and $ED_{\theta 70}$ arise because the former accounts for all deaths while the latter does not account for the deaths occurring above the age threshold (Proposition 4). Proposition 6 shows that this difference between $PALE_\theta$ and $ED_{\theta 70}$ is precisely the reason why the former avoids the mortality paradox while the latter does not. As a result, we can attribute the disagreements between $PALE_\theta$ and $ED_{\theta 70}$ to the fact that $PALE_\theta$ avoids the mortality paradox. Together, the disagreements between $PALE_\theta$ and $GD_{\theta\hat{a}}$ in Figure 10 are mostly due to the fact that $PALE_\theta$ avoids the mortality paradox. The role of non-stationarity plays a more limited role in these disagreements.

We emphasize that, the smaller the age threshold $\hat{a}$, the more numerous the disagreements between $PALE_\theta$ and $GD_{\theta\hat{a}}$. When $\hat{a}$ switches from 70 to 50, agreement between the three indices moves from 82% to 73% on average for $\theta = 1$ (Figure 10a). The smaller the age threshold $\hat{a}$, the more $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$ are exposed to the mortality paradox, the more numerous the disagreements with $PALE_\theta$, which is immune from it. Remember how the agreement between $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$ remains similar no matter the level of $\hat{a}$. This suggests that the driving force behind the disagreement between the three indices originates in the mortality paradox rather than non-stationarity. The decreasing trend in agreement between $PALE_\theta$ and the other two indices is due to the increasing life expectancy trend. As life expectancy increases throughout the world, a larger share of deaths occurs above $\hat{a}$ and are ignored by $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$ so that their diagnostic differs more and more from $PALE_\theta$. The increasing agreement during the COVID pandemic reflects the opposite mechanism: as life expectancy decreased, more deaths occured prior to $\hat{a}$ and were therefore accounted for by all indices.

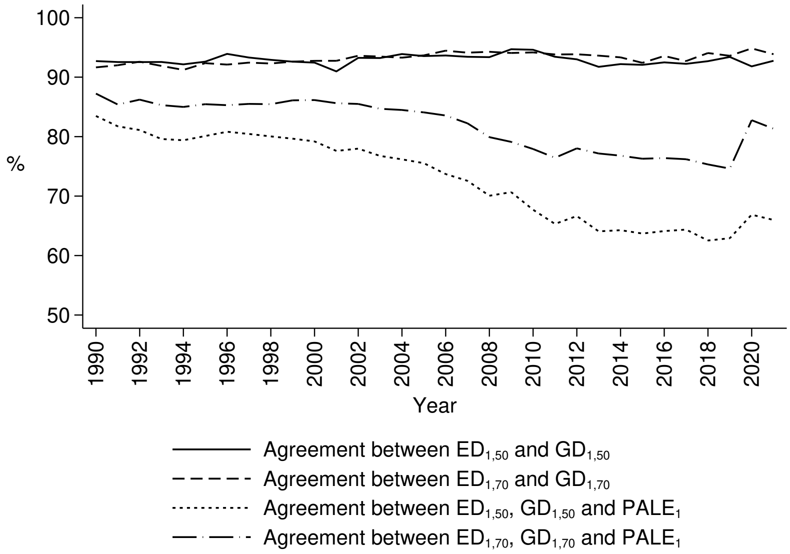
As $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$ offer similar diagnostics in our data, the use of any of these two indices may be driven by the specific needs of researchers and practitioners. Indeed, $ED_{\theta\hat{a}}$ has the benefit of a relatively simple interpretation. $GD_{\theta\hat{a}}$, however, is decomposable across sub-groups, which may be a useful property for analytical purposes. Finally, $PALE_\theta$ offers the advantage of not suffering from the mortality paradox and of having the most straightforward interpretation. Its diagnostic, however, differs from that of $GD_{\theta\hat{a}}$ and $ED_{\theta\hat{a}}$. Its use will be driven by the conception of lifespan deprivation preferred by researchers and practitioners: should all deaths matter or should only the deaths occuring below an age threshold be taken into account?

5 Concluding remarks

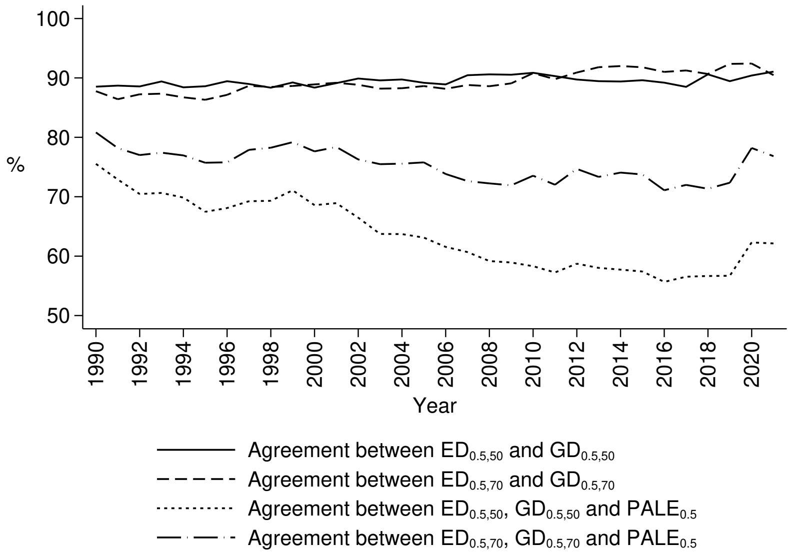
An important limitation of the two indices proposed in this paper, $PALE_\theta^t$ and $ED_{\theta\hat{a}}^t$, is that they account for the distribution of outcomes "dimension-by-dimension". More precisely, they account for the distribution of quality of life and for the distribution of

Figure 10: Agreement between $GD_{\theta\hat{a}}$, $ED_{\theta\hat{a}}$ and $PALE_\theta$

(a) $\theta = 1$



(b) $\theta = 0.5$



Reading: In 2021, $ED_{1,70}$ and $GD_{1,70}$ provide the same ranking of countries in 94% of all pair wise comparisons. In 2021, $PALE_1$, $ED_{1,70}$ and $GD_{1,70}$ provide the same ranking for 81% of all pair wise comparisons.

quantity of life, but not for the distribution of life-cycle utilities. Indeed, our indices are insensitive to the allocation of years of life prematurely lost between the poor and the non-poor. This allocation may however have implications for the distribution of life-cycle utilities. When poor individuals die early, they cumulate low achievements in the two dimensions and the difference between their life-cycle utility and that of non-poor individuals increases.

Accounting for the distribution of lifecycle utilities requires data that are typically not available. The necessary data include not only information on the correlation between poverty and premature mortality, but also information on mobility in and out of poverty. When such data is not available, $PALE^t_\theta$ and $ED^t_{\theta\hat{a}}$ can be used as a second-best solution, as they improve over the widespread practice of entirely ignoring the impact of mortality on longevity. This is particularly relevant for societies in which premature mortality is highly selective, i.e., for which premature mortality disproportionately affects poorer individuals. In those societies, the premature mortality term of $ED^t_{\theta\hat{a}}$ essentially captures these negative outcomes.

If one cares about the distribution of life-cycle utilities and the necessary data is available, our indicators would need to be adjusted. Let us define as "life-cycle poor" the individuals whose life-cycle utility is smaller than that of a reference life, e.g., a life characterized by a lifespan of 40 years with no period of poverty. One index combining mortality and poverty that would account for the distribution of life-cycle utilities is the expected fraction of newborns who will be "life-cycle poor", again assuming constant poverty and mortality.[24]

Our paper calls for future research on the value that the normative parameter θ should take. Its mathematical expression based on social welfare à la Harsanyi allows calibrating its value, as we show in Appendix C. However, the calibrated values are highly sensitive to the parametric values selected for the period utility function. Survey-based estimates for θ may provide a firmer base for narrowing the plausible range of values for this central parameter.

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[24] Note that the indicators proposed in the literature on the mortality paradox are typically not appropriate to capture the distribution of life-cycle utilities, as they do not attribute an intrinsic value to the quantity of life. The may therefore miss improvements when the lifecycle utility of all individuals increase, for instance if the lifespan of all individuals is multiplied by a common factor.

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Appendices

A Notation and definition of a stationary society

We present here the formal notation used for the proofs.

There is a discrete set of periods $\{\dots, t-1, t, t+1, \dots\}$. In each period, some individuals are born (at the beginning of the period) and some individuals die (at the end of the period). All alive individuals are assigned a consumption status for the period (P or NP) . We define the life of an individual i as the list of consumption statuses $l_i = (l_{i0}, \dots, l_{id_i})$ she enjoys between age 0 and age $d_i \in \{0, \dots, a^* - 1\}$ at which she dies, where $l_{ia} \in \{NP, P\}$ for all $a \in \{0, \dots, d_i\}$. The set of lives is thus $L = \cup_{d \in \{0,\dots,a^*-1\}} \{NP, P\}^{d+1}$.

The number of newborns in period t is denoted by n_t. The profile of lives for the cohort born in t is denoted by $C_t = (l_i)_{i \in \{1,\dots,n_t\}}$, where $\{1, \dots, n_t\}$ is the set of newborns in t. For any $i \in \{1, \dots, n_t\}$, l_{i0} is the consumption status i enjoys in t, l_{i1} is the consumption status i enjoys in $t+1$, etc.

Let $n_t(a)$ denote the number of individuals born in period t who are still alive when reaching age a. In particular, we have $n_t(0) = n_t$. Let $p_t(a)$ denote the number of individuals born in period t who are poor at age a, with $p_t(a) \leq n_t(a)$. By definition, the probability that an individual born in t survives to age a is given by $V^t(a) = \frac{n_t(a)}{n_t}$, and the conditional probability that an individual born in t will be poor when reaching age a is $\pi_t(a) = \frac{p_t(a)}{n_t(a)}$. We denote the probability distribution on the set of lives that C_t implicitly defines by $\Gamma_t : L \to [0,1]$, with $\sum_{l \in L} \Gamma_t(l) = 1$.

In period t, we cannot observe the full profile of lives for the cohort born in t. The only elements of C_t that we observe in period t are $n_t(0)$, $p_t(0)$ and $n_t(1)$. However, we also have information about the profile of lives of the cohorts born *before* t. Formally, let a **society** S_t be the list of profiles of lives for all cohorts born during the a^* periods in $\{t - (a^* - 1), \dots, t\}$, i.e. $S_t = (C_{t-a^*+1}, \dots, C_t)$. In period t, we observe (i) the number N^t of individuals who are alive in t:

$$N^t = \sum_{a=0}^{a^*-1} n_{t-a}(a),$$

(ii) the fraction H^t of alive individuals who are poor in t:

$$H^t = \frac{\sum_{a=0}^{a^*-1} p_{t-a}(a)}{\sum_{a=0}^{a^*-1} n_{t-a}(a)},$$

and (iii) the age-specific mortality vector $\mu^t = (\mu_0^t, \dots, \mu_{a*-1}^t)$ in period t where for each $a \in \{0, \dots, a^* - 1\}$ we have

$$\mu_a^t = \frac{n_{t-a}(a) - n_{t-a}(a+1)}{n_{t-a}(a)},$$

with $\mu_{a^*-1}^t = 1$ (by definition of a^*).

The particularity of stationary societies is to have their natality, mortality and poverty constant over time, so that outcomes in a given period t are replicated over the next period $t+1$. The only change between t and $t+1$ is the identity of individuals who face these outcomes. More formally, a society is stationary if both the distribution of lives and the size of generations are constant over the last a^* periods.

Definition 2 (Stationary Society)**.**
A society S_t is stationary if, at any period $t' \in \{t-a^*+1, \ldots, t\}$, we have

- $\Gamma_{t'} = \Gamma_t$ *(constant distribution of lives),*
- $n_{t'} = n_t$ *(constant size of cohorts).*

It follows from this definition that a stationary society S_t is such that $n_t(a) = n_{t-a}(a)$ and $p_t(a) = p_{t-a}(a)$ for all $a \in \{1, \ldots, a^*-1\}$.[25]

B Proof of Proposition 1

The proof is based on Lemma 1, which shows that, in a stationarity society, the poverty and mortality observed in a given period completely reflects the lifecycle outcomes of newborns.

Lemma 1. *If society S_t is stationary, then*

$$V^t(a) = \Pi_{k=0}^{a-1}(1-\mu_k^t) \qquad \text{for all } a \in \{0, \ldots, a^*-1\}, \tag{5}$$

$$N^t = n_t * LE^t, \tag{6}$$

$$N^t * H^t = n_t * \sum_{a=0}^{a^*-1} V^t(a)\pi_t(a). \tag{7}$$

Proof. We first prove Eq (5). As S_t is stationary, we have $n_t(k) = n_{t-k}(k)$ for all $k \in \{1, \ldots, a^*-1\}$ and $n_t(k+1) = n_{t-k}(k+1)$ for all $k \in \{0, \ldots, a^*-2\}$. Therefore, we have for all $a \in \{1, \ldots, a^*-1\}$ that

$$\begin{aligned} V^t(a) &= \frac{n_t(a)}{n_t}, \\ &= \Pi_{k=0}^{a-1}\frac{n_t(k+1)}{n_t(k)}, \\ &= \Pi_{k=0}^{a-1}\frac{n_{t-k}(k+1)}{n_{t-k}(k)}, \\ &= \Pi_{k=0}^{a-1}(1-\mu_k^t). \end{aligned}$$

We then prove Eq (6). As S_t is stationary, we have $n_t(a) = n_{t-a}(a)$ for all $a \in$

[25] Clearly, a constant distribution of lives is not sufficient for these equalities, one also needs a constant size of cohorts.

$\{1,\ldots,a^*-1\}$. Recalling Eq (5) and $V^t(a)=\frac{n_t(a)}{n_t}$, we can successively write

$$\begin{aligned}
LE^t &= \sum_{a=0}^{a^*-1} \Pi_{k=0}^{a-1}(1-\mu_k^t),\\
&= \sum_{a=0}^{a^*-1} V^t(a),\\
&= \frac{\sum_{a=0}^{a^*-1} n_t(a)}{n_t},\\
&= \frac{\sum_{a=0}^{a^*-1} n_{t-a}(a)}{n_t},\\
&= N^t/n_t.
\end{aligned}$$

Finally, we prove Eq. (7). As S_t is stationary, we have $p_t(a)=p_{t-a}(a)$ for all $a\in\{1,\ldots,a^*-1\}$. Given that $\pi_t(a)=\frac{p_t(a)}{n_t(a)}$ and $V^t(a)=\frac{n_t(a)}{n_t}$, we can successively write

$$\begin{aligned}
H^t &= \frac{\sum_{a=0}^{a^*-1} p_{t-a}(a)}{\sum_{a=0}^{a^*-1} n_{t-a}(a)},\\
&= \frac{\sum_{a=0}^{a^*-1} p_t(a)}{N_t},\\
&= \frac{\sum_{a=0}^{a^*-1} \pi_t(a)V^t(a)n_t}{N^t}.
\end{aligned}$$

□

We use Lemma 1 to prove Proposition 1.

The assumption that individuals only enjoy binary consumption statuses implies that $\mathbb{E}^t u(c_a)=\pi_t(a)u_P+(1-\pi_t(a))u_{NP}$ where $u_{NP}=u(NP)$ and $u_P=u(P)$. By Eq. (5), life expectancy at birth can be written as $LE^t=\sum_{a=0}^{a^*-1}V^t(a)$. We can thus rewrite Eq. (2) as

$$EU^t = u_{NP}LE^t - (u_{NP}-u_P)\sum_{a=0}^{a^*-1} V^t(a)\pi_t(a). \tag{8}$$

The result follows directly when substituting Eq. (6) and (7) into Eq. (8).

C Calibrating values for θ

Proposition 1 shows that $\theta=\frac{u_{NP}-u_P}{u_{NP}}$, where u_P and u_{NP} respectively denote the period utility of being poor and being non-poor (and the utility of being dead is normalized to zero). Consider the constant elasticity of substitution period utility function defined as

$$u(c)=\frac{c^{1-\epsilon}-\hat{c}^{1-\epsilon}}{1-\epsilon}, \tag{9}$$

where $\hat{c}$ denotes the "subsistence" consumption level, for which $u(\hat{c})=0$, and ϵ is the coefficient of relative risk aversion that captures the curvature of utility function u. A parametric value for θ requires defining the representative consumption for the (consumption) poor and non-poor statuses such that $u_P=u(c_{poor})$ and $u_{NP}=$

$u(c_{non-poor})$. Typically, c_{poor} and $c_{non-poor}$ could respectively be defined as mean or median consumption among the poor and non-poor.

Parametric values for θ are sensitive to the values selected for the parameters $\hat{c}$ and ϵ. We illustrate this by providing values for $1/\theta$ for India in 2019 for different values of these parameters. We define poverty using the the International Poverty Line, whose value is \$ 2.15 per person per day (2017 PPPs). We assume that c_{poor} and $c_{non-poor}$ are defined as mean consumption among the poor and non-poor, which we extract from the Poverty and Inequality Platform from the World Bank.

Table 4: Parametric values for $1/\theta$ for India in 2019 using the International Poverty Line.

		$\hat{c}$ 0.5 (\$ a day)	$\hat{c}$ 0.75 (\$ a day)	$\hat{c}$ 1.0 (\$ a day)	$\hat{c}$ 1.25 (\$ a day)
ϵ	2.5	7.8	4.1	2.6	1.8
ϵ	2.0	4.7	3.0	2.1	1.6
ϵ	1.5	3.0	2.2	1.7	1.4
ϵ	1.0	2.1	1.7	1.5	1.3

Note: According to the Poverty and Inequality Platform, mean consumption in India in 2019 was \$ 5.13 per person per day (2017 PPPs). For the International Poverty Line, mean consumption among the poor is \$ 1.75 and mean consumption among the non-poor is \$ 5.51. The utility function considered is CES. $1/\theta$ can be interpreted as the number of years spent in poverty yielding the same well-being loss as one year of life lost.

D Proof of Proposition 2

From assumption A3 we have that $\mathbb{E}^t u(c_a) = H^t u_P + (1 - H^t)u_{NP}$ for all a. Thus, the assumed probability to be poor at age a is $\pi_t(a) = H^t$ for all a. We thus get from Eq. (8) that

$$EU^t = u_{NP}LE^t - (u_{NP} - u_P)H^t \sum_{a=0}^{a^*-1} V^t(a). \tag{10}$$

Social welfare à la Harsanyi computes the unconditional probability to survive to age a, denoted by $V^t(a)$, by considering fixed mortality rates equal to those observed in year t. This implies that $\sum_{a=0}^{a^*-1} V^t(a) = LE^t$ and the result directly follows.

E Proof of Proposition 5

The proof builds on the framework presented in Appendix A.

We prove that $GD^t_{\theta\hat{a}}(S_t) = ED^t_{\theta\hat{a}}(S_t)$ for any stationary society S_t. By definition,

$$GD^t_{\theta\hat{a}} = \frac{YLL^t_{\hat{a}}}{N^t + YLL^t_{\hat{a}}} + \theta\frac{N^t H^t}{N^t + YLL^t_{\hat{a}}}.$$

As society S_t is stationary, Lemma 1 applies and $N^t = n_t LE^t$ (Eq. (6)). Substituting this expression for N^t into the definition of $GD^t_{\theta\hat{a}}$ yields the desired result, at least if we have $YLL^t_{\hat{a}} = n_t LGE^t_{\hat{a}}$, which remains to be shown. By definition, we have

$n_{t-a}(a) = N_a^t$ and thus

$$YLL_{\hat{a}}^t = \sum_{a=0}^{\hat{a}-2} n_{t-a}(a) * \mu_a^t * (\hat{a} - (a+1)).$$

As society S_t is stationary, Lemma 1 applies and we have $\frac{n_{t-a}(a)}{n_t} = \prod_{k=0}^{a-1}(1-\mu_k^t)$ (Eq. (5)). Substituting this expression for $n_{t-a}(a)$ into the definition of $YLL_{\hat{a}}^t$ gives:

$$YLL_{\hat{a}}^t = n_t \sum_{a=0}^{\hat{a}-2} (\hat{a} - (a+1)) * \mu_a^t * \prod_{k=0}^{a-1}(1-\mu_k^t),$$

which shows that $YLL_{\hat{a}}^t = n_t LGE_{\hat{a}}^t$ (recall that $\hat{a} - (a+1) = 0$ when $a = \hat{a} - 1$), the desired result.

F $ED_{\theta\hat{a}}^t$ and $GD_{\theta\hat{a}}^t$ under a transitory mortality shock

We illustrate the difference between $ED_{\theta\hat{a}}^t$ and $GD_{\theta\hat{a}}^t$ in their reaction to a transitory mortality shock with the help of a toy example. Consider a population with a fixed natality $n_t(0) = 2$ for all periods t. At each period, all alive individuals are non-poor, implying that $H^t = 0$. For all $t \neq 0$, we assume a constant mortality vector $\mu^t = \mu^* = (0, 1, 1, 1)$, so that before t each individual lives exactly two periods. Let us assume $\hat{a} = 4$, so that an individual dies prematurely if she dies before her fourth period of life. Before period $t = 0$, the population pyramid is stationary, and the two indices are equal to $1/2$ because there is no poor and individuals live for two periods instead of four. Consider now a transitory mortality shock in period 0, such that half of the newborns die after their first period of life: $\mu^0 = (1/2, 1, 1, 1)$. The population pyramid in period 1 is thus different than the population pyramid in all periods $t \neq 1$. The population pyramid returns to its stationary state in period 2, after a (mechanical) transition in period 1. This example is illustrated in Figure 11.

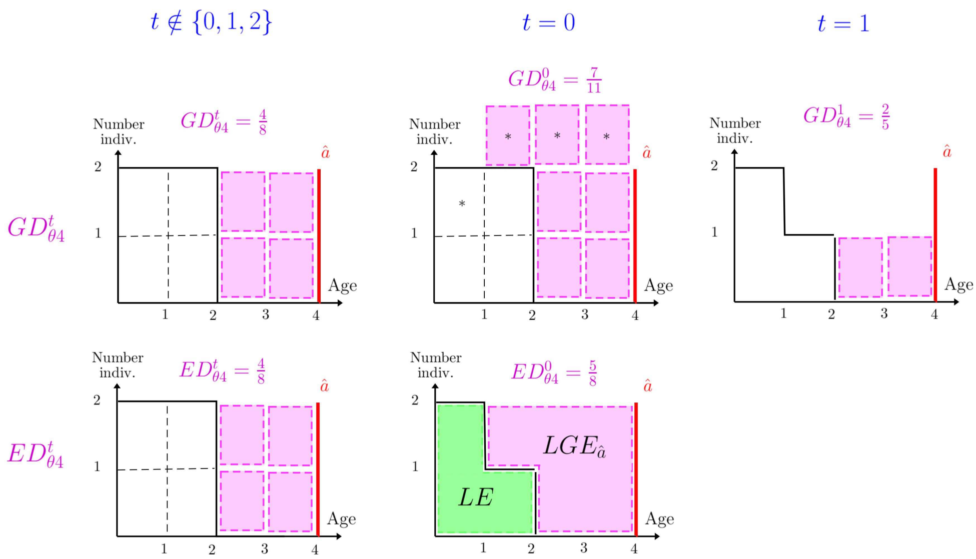


Figure 11: Response of $GD_{\theta\hat{a}}^t$ and $ED_{\theta\hat{a}}^t$ to a transitory mortality shock in $t = 0$. The years prematurely lost are shaded.

Consider first $GD_{\theta\hat{a}}^t$. In period 0, the actual population pyramid does not reflect the new mortality vector μ^0. Two one-year old individuals die in period 0, each losing two years of life. Yet, the transitory mortality shock implies the premature death of one newborn, which leads to the loss of three additional years of life. There are thus 7 years of life prematurely lost in period 0, and $GD_{\theta\hat{a}}^{t=0}$ takes value 7/11. In period 1, the population pyramid is different than in other periods because of the death of one newborn in period 0. The mortality vector reverses $\mu^1 = \mu^* = (0, 1, 1, 1)$, which implies that nobody dies except the only one-year old individual. As this one-year old loses two years of life, we have that $GD_{\theta\hat{a}}^{t=1}$ is equal to 2/5. In period 2, both the population pyramid and the mortality vector are back to their stationary values and so does $GD_{\theta\hat{a}}^{t=2}$.

We now turn to $ED_{\theta\hat{a}}^{t}$. In period 0, $ED_{\theta\hat{a}}^{t=0}$ considers the counterfactual population pyramid associated to mortality vector μ^0. As a result, $ED_{\theta\hat{a}}^{t=0}$ is equal to 5/8. Indeed, $ED_{\theta\hat{a}}^{t=0}$ focuses on the newborn and the one-year old who die prematurely in the counterfactual population pyramid, ignoring that there are two one-year old dying in the actual population pyramid in period 0. (The presence of two one-year old individuals in period 0 is a legacy of the past mortality vectors equal to μ^*.) In period 1, $ED_{\theta\hat{a}}^{t=1}$ considers the counterfactual population pyramid associated to mortality vector μ^*, which corresponds to the actual population pyramids in period $t \neq 1$. As a result, $ED_{\theta\hat{a}}^{t=1}$ reverses to its stationary value for μ^* equal to 4/8.

The transitory mortality shock only takes place in period 0. This is directly reflected in the evolution of $ED_{\theta\hat{a}}^{t}$, which registers a worsening in period 0 and then reverts back to its long run value in period 1 when the mortality vector reverses to μ^*. $GD_{\theta\hat{a}}^{t}$ also registers a worsening in period 0, but it also registers an improvement in period 1, before reverting to its intermediate long run value in period 2. This illustrates that $GD_{\theta\hat{a}}^{t}$ responds with more inertia to mortality shocks.

G Characterization of the $ED_{\theta\hat{a}}$ index

We first introduce the set-up provided by Baland et al. (2021), which we will use to charcterize $ED_{\theta\hat{a}}^{t}$.

Each individual i is associated to a birth year $b_i \in \mathbb{Z}$. In period t, each individual i with $b_i \leq t$ is characterized by a **bundle** $x_i = (a_i, s_i)$, where $a_i = t - b_i$ is the age that individual i would have in period t given her birth year b_i, and s_i is a categorical variable capturing individual status in period t, which can be either non-poor (NP), poor (P) or dead (D), i.e. $s_i \in S = \{NP, P, D\}$. (This set-up only considers outcomes in year t and therefore the notation in Appendix G omits reference to t, e.g., writing s_i instead of s_i^t.) We consider again that births occur at the beginning while deaths occur at the end of a period. As a result, an individual whose status in period t is D died before period t.[26]

An individual "dies prematurely" if she dies before reaching the minimal lifespan $\hat{a} \in \mathbb{N}$. The age threshold $\hat{a}$ must respect a lower-bound $\underline{\hat{a}} \in \mathbb{N}_0$, such that $\hat{a} \geq \underline{\hat{a}} \geq 0$. Formally, period t is "prematurely lost" by any individual i with $s_i = D$ and $a_i < \hat{a}$. A **distribution** $x = (x_1, \dots, x_{n(x)})$ specifies the age and the status in period t of all $n(x)$ individuals. Excluding trivial distributions for which no individual is alive or

[26] All newborns have age 0 during period t and some among these newborns may die at the end of period t. This implies that $b_i = t \Rightarrow s_i \neq D$.

prematurely dead, the set of distributions in period t is given by:

$$X = \{x \in \cup_{n \in \mathbb{N}} (\mathbb{Z} \times S)^n \mid \text{there is } i \text{ for whom either } s_i \neq D \text{ or } s_i = D \text{ and } \hat{a} > t - b_i\}.$$

Baland et al. (2021) show that the most natural *consistent* index to rank distributions in X is the inherited deprivation index ($ID_{\theta\hat{a}}$). Let $d(x)$ denote the number of *prematurely dead* individuals in distribution x, which is the number of individuals i for whom $s_i = D$ and $\hat{a} > t - b_i$, $p(x)$ the number of individuals who are poor and $f(x)$ the number of non-poor individuals. The $ID_{\theta\hat{a}}$ index is defined as:

$$ID_{\theta\hat{a}}(x) = \underbrace{\frac{d(x)}{f(x) + p(x) + d(x)}}_{\textit{quantity deprivation}} + \theta \underbrace{\frac{p(x)}{f(x) + p(x) + d(x)}}_{\textit{quality deprivation}}, \tag{11}$$

where $\theta \in [0, 1]$ is a parameter weighing the relative importance of poverty and premature death. An individual losing prematurely period t matters $1/\theta$ times as much as an individual spending period t in poverty.

We introduce additional notation for the mortality taking place in period t. Consider the population pyramid in period t, and let $n_a(x)$ be the number of *alive* individuals of age a in distribution x, i.e. the number of individuals i for whom $a_i = a$ and $s_i \neq D$. (The definition of $n_a(x)$ corresponds to $n_{t-a}(a)$ in the notation used in the main text of the paper. In this section, we adopt the notation of Baland et al. (2021), which does not require to mention period t.) The age-specific mortality rate $\mu_a \in [0, 1]$ denotes the fraction of alive individuals of age a dying at the end of period t: the number of a-year-old individuals dying at the end of period t is $n_a(x) * \mu_a$. Letting $a^* \in \mathbb{N}$ stand for the maximal lifespan (which implies $\mu_{a^*-1} = 1$), the **vector of age-specific mortality rates** in period t is given by $\mu = (\mu_0, \ldots, \mu_{a^*-1})$. Vector μ summarizes mortality in period t, while distribution x summarizes poverty in period t as well as mortality before period t. The set of mortality vectors is defined as:

$$M = \left\{ \mu \in [0, 1]^{a^*} \middle| \mu_{a^*-1} = 1 \right\}.$$

We consider pairs (x, μ) for which the distribution x is a priori unrelated to vector μ. We assume that the age-specific mortality rates μ_a must be feasible given the number of alive individuals $n_a(x)$. Given that distributions have finite numbers of individuals, mortality rates cannot take irrational values, i.e. $\mu_a \in [0, 1] \cap \mathbb{Q}$, where $\mathbb{Q}$ is the set of rational numbers. The set of pairs considered is given by:

$$O = \left\{ (x, \mu) \in X \times M \middle| \text{for all } a \in \{0, \ldots, a^*\} \text{ we have } \mu_a = \frac{c_a}{n_a(x)} \text{ for some } c_a \in \mathbb{N} \right\}.$$

Letting $d_a(x)$ be the number of *dead* individuals born a years before t in distribution x, the total number of individuals born a years before t is then equal to $n_a(x) + d_a(x)$. Formally, the **pair** (x, μ) is **stationary** if, for some $n^* \in \mathbb{N}$ and all $a \in \{0, \ldots, a^*\}$, we have:

- $n_a(x) + d_a(x) = n^* \in \mathbb{N}$ (constant natality),
- $n_{a+1}(x) = n_a(x) * (1 - \mu_a)$ (identical population pyramid in $t + 1$).

In a stationary pair, the population pyramid is such that the size of each cohort can be obtained by applying to the preceding cohort the current mortality rate. The pair associated to a stationary society (as defined in the main text) is stationary. An index is a function $P : O \times \mathbb{N} \to \mathbb{R}_+$. We simplify the notation $P(x, \mu, \hat{a})$ to $P(x, \mu)$ as a fixed value for $\hat{a}$ is assumed.

We now introduce the properties characterizing $ED_{\theta\hat{a}}$. $ID_{\theta\hat{a}}$ Equivalence requires that, since the current mortality (in period t) is the same as the mortality prevailing in the previous periods in stationary societies, any index defined on current mortality rates is equivalent to $ID_{\theta\hat{a}}$ in the case of a stationary pair:[27]

Deprivation axiom 1 ($ID_{\theta\hat{a}}$ Equivalence)**.** *There exists some $\theta \in (0, 1]$ and $\hat{a} \geq \underline{\hat{a}}$ such that for all $(x, \mu) \in O$ that are stationary we have $P(x, \mu) = ID_{\theta\hat{a}}(x)$.*

Independence of Dead requires that *past* mortality does not affect the index. More precisely, the presence of an additional dead individual in distribution x does not affect the index:

Deprivation axiom 2 (Independence of Dead)**.** *For all $(x, \mu) \in O$ and $i \leq n(x)$, if $s_i = D$, then $P((x_i, x_{-i}), \mu) = P(x_{-i}, \mu)$.*

Independence of Birth Year requires that the index does not depend on the birth year of individuals, i.e. only their status matters. As Independence of Dead requires to disregard dead individuals, the only relevant information in x is whether an alive individual is poor or not.

Deprivation axiom 3 (Independence of Birth Year)**.** *For all $(x, \mu) \in O$ and $i \leq n(x)$, if $s_i = s'_i$, then $P((x_i, x_{-i}), \mu) = P((x'_i, x_{-i}), \mu)$.*

Replication Invariance requires that, if a distribution is obtained by replicating another distribution several times, they both have the same deprivation when associated to the same mortality vector. By definition, a k-replication of distribution x is a distribution $x^k = (x, \ldots, x)$ for which x is repeated k times.

Deprivation axiom 4 (Replication Invariance)**.** *For all $(x, \mu) \in O$ and $k \in \mathbb{N}$, $P(x^k, \mu) = P(x, \mu)$.*

Proposition 7 shows that these properties jointly characterize the $ED_{\theta\hat{a}}$ index.

Proposition 7 (Characterization of $ED_{\theta\hat{a}}$)**.**
$P = ED_{\theta\hat{a}}$ if and only if P satisfies Independence of Dead, $ID_{\theta\hat{a}}$ Equivalence, Replication Invariance and Independence of Birth Year.

Proof. We first prove sufficiency. Proving that the $ED_{\theta\hat{a}}$ index satisfies Independence of Dead, Replication Invariance and Independence of Birth Year is straightforward and left to the reader. Finally, $ED_{\theta\hat{a}}$ index satisfies $ID_{\theta\hat{a}}$ Equivalence because $ED_{\theta\hat{a}}$ is equal to $GD_{\theta\hat{a}}$ in stationary populations (Proposition 5) and $GD_{\theta\hat{a}}$ satisfies $ID_{\theta\hat{a}}$ Equivalence (Proposition 2 in Baland et al. (2021)). (The pairs associated to stationary societies are stationary).

[27]Recall that past mortality is recorded in distribution x while current mortality is recorded in vector μ. As vector μ is redundant in stationary pairs, in the sense that μ can be inferred from the population pyramid, the index can be computed on distribution x only. See Baland et al. (2021) for a complete motivation for this axiom.

We now prove necessity. Take any pair $(x, \mu) \in O$. We construct another pair (x''', μ) that is stationary and such that $P(x''', \mu) = P(x, \mu)$ and $ED_{\theta\hat{a}}(x''', \mu) = ED_{\theta\hat{a}}(x, \mu)$. Given that (x''', μ) is stationary, we have by $ID_{\theta\hat{a}}$ Equivalence that $P(x''', \mu) = ID_{\theta\hat{a}}(x''', \mu)$ for some $\theta \in (0, 1]$. As $ID_{\theta\hat{a}} = ED_{\theta\hat{a}}$ for stationary pairs, we have $P(x''', \mu) = ED_{\theta\hat{a}}(x''', \mu)$ for some $\theta \in (0, 1]$. If we can construct such pair (x''', μ), then $P(x, \mu) = ED_{\theta\hat{a}}(x, \mu)$ for some $\theta \in (0, 1]$, the desired result.

We turn to the construction of the stationary pair (x''', μ), using two intermediary pairs (x', μ) and (x'', μ). One difficulty is to ensure that the mortality rates μ_a can be achieved in the stationary population given the number of alive individuals $n_a(x''')$, that is $\mu_a = \frac{c}{n_a(x''')}$ for some $c \in \mathbb{N}$.

We first construct a $n'-$replication of x that has sufficiently many alive individuals to meet this constraint. For any $a \in \{0, \ldots, a^* - 1\}$, take any naturals c_a and e_a such that $\mu_a = \frac{c_a}{e_a}$. Let $e = \prod_{j=0}^{a^*-1} e_j$, $n'_a = e \prod_{j=0}^{a-1}(1 - \frac{c_j}{e_j})$ and $n' = \sum_{j=0}^{a^*-1} n'_j$.[28] Let x' be a $n'-$replication of x. Letting $n^x = \sum_{j=0}^{a^*-1} n_j(x)$ be the number of alive individuals in distribution x, we have that x' has $n' * n^x$ alive individuals. We have $P(x', \mu) = P(x, \mu)$ by Replication Invariance.

We define x'' from x' by changing the birth years of *alive* individuals in such a way that (x'', μ) has a population pyramid that is stationary. Formally, we construct x'' with $n(x'') = n(x')$ such that

- dead individuals in x' are also dead in x'',
- alive individuals in x' are also alive in x'' and have the same status,
- the birth year of alive individuals are changed such that, for each $a \in \{0, \ldots, a^* - 1\}$, the number of a-years old individuals is $n' * n^x * \frac{\prod_{j=0}^{a-1}(1-\frac{c_j}{e_j})}{\sum_{k=0}^{a^*-1}\prod_{j=0}^{k-1}(1-\frac{c_j}{e_j})}$.[29]

One can check that (x'', μ) has a population pyramid corresponding to a stationary population and that each age group has a number of alive individuals in $\mathbb{N}$. We have $P(x'', \mu) = P(x', \mu)$ by Independence of Birth Year.

Define x''' from x'' by changing the number and birth years of *dead* individuals in such a way that (x''', μ) is stationary. To do so, place exactly $n_0(x'') - n_a(x'')$ dead individuals in each age group a. We have $P(x''', \mu) = P(x'', \mu)$ by Independence of Dead.

Together, we have that $P(x''', \mu) = P(x, \mu)$. Finally, by construction we have $H(x''') = H(x)$, which implies that $ED_{\theta\hat{a}}(x''', \mu) = ED_{\theta\hat{a}}(x, \mu)$. □

H Proof of Proposition 4

The proof builds on the framework presented in Appendix A.

We first show that $LE^t + LGE^t_{\hat{a}} = \hat{a}$ when $\hat{a} \geq a^*$. By definition, LE^t and $LGE^t_{\hat{a}}$ only depend on the age-specific mortality vector μ^t. Thus, the values for LE^t and $LGE^t_{\hat{a}}$ do not depend on whether the society is stationary or not. Consider any stationary society S_t whose constant mortality vector is μ^t. We show for this stationary society S_t that $LE^t + LGE^t_{\hat{a}} = \hat{a}$ when $\hat{a} \geq a^*$.

[28] These numbers imply that a constant natality of e newborns leads to a stationary population of n' alive individuals.

[29] Observe that $\sum_{k=0}^{a^*-1}\prod_{j=0}^{k-1}(1 - \frac{c_j}{e_j}) = LE$, implying that $e = \frac{n' * n^x}{\sum_{k=0}^{a^*-1}\prod_{j=0}^{k-1}(1-\frac{c_j}{e_j})}$.

As society S_t is stationary, Lemma 1 applies and we have $N^t = n_t * LE^t$ (Eq. (6)). As by definition $N^t = \sum_{a=0}^{a^*-1} n_t(a)$, we get

$$LE^t = \sum_{a=0}^{a^*-1} \frac{n_t(a)}{n_t}. \tag{12}$$

As society S_t is stationary, Lemma 1 applies and we have $V^t(a) = \Pi_{k=0}^{a-1}(1 - \mu_k^t)$ (Eq. (5)). Using the definition of age-specific mortality rate, namely $\mu_a^t = \frac{n_{t-a}(a) - n_{t-a}(a+1)}{n_{t-a}(a)}$, we can rewrite $LGE_{\hat{a}}^t$ as

$$LGE_{\hat{a}}^t(S_t) = \sum_{a=0}^{\hat{a}-1}(\hat{a} - (a+1)) * \frac{n_{t-a}(a) - n_{t-a}(a+1)}{n_{t-a}(a)} * V^t(a).$$

As society S_t is stationary, we have that $n_t(a) = n_{t-a}(a)$ and $n_t(a+1) = n_{t-a}(a+1)$ for all $a \in \{0, \dots, a^* - 1\}$. We can thus successively write

$$\begin{aligned}
LGE_{\hat{a}}^t &= \sum_{a=0}^{\hat{a}-1}(\hat{a} - (a+1)) * \frac{n_t(a) - n_t(a+1)}{n_t(a)} * \frac{n_t(a)}{n_t}, \\
&= \sum_{a=0}^{\hat{a}-1} \hat{a} * \frac{n_t(a) - n_t(a+1)}{n_t} - \sum_{a=0}^{\hat{a}-1}(a+1) * \frac{n_t(a) - n_t(a+1)}{n_t}, \\
&= \frac{1}{n_t}\left(\hat{a} * (n_t(0) - n_t(\hat{a})) - \sum_{a=0}^{\hat{a}-1} n_t(a) + \hat{a} * n_t(\hat{a})\right), \\
&= \hat{a} - \sum_{a=0}^{\hat{a}-1} \frac{n_t(a)}{n_t}.
\end{aligned}$$

By definition of a^*, we have $n_t(a) = 0$ for all $a \geq a^*$. When $\hat{a} \geq a^*$, this implies that $\sum_{a=0}^{\hat{a}-1} \frac{n_t(a)}{n_t} = \sum_{a=0}^{a^*-1} \frac{n_t(a)}{n_t}$. We have shown that $LGE_{\hat{a}}^t = \hat{a} - \sum_{a=0}^{a^*-1} \frac{n_t(a)}{n_t}$, which together with Eq. (12) proves that $LE^t + LGE_{\hat{a}}^t = \hat{a}$ when $\hat{a} \geq a^*$.

The fact that $LE^t + LGE_{\hat{a}}^t = \hat{a}$ implies that $PALE^t{}_\theta = \hat{a}(1 - ED^t{}_{\theta\hat{a}})$ because

$$\begin{aligned}
\hat{a}(1 - ED^t{}_{\theta\hat{a}}) &= (LE^t + LGE_{\hat{a}}^t)(1 - ED^t{}_{\theta\hat{a}}) \\
&= LE^t(1 - \theta H^t), \\
&= PALE^t{}_\theta.
\end{aligned}$$

Thus, when $\hat{a} \geq a^*$, $PALE_\theta^t$ is a linear function of $ED_{\theta\hat{a}}^t$ that depends negatively on $ED_{\theta\hat{a}}^t$. Therefore, these two indicators yields opposite ranking of any two societies A and B, i.e. $PALE^t{}_\theta(A) \geq PALE^t{}_\theta(B) \Leftrightarrow ED_{\theta\hat{a}}^t(A) \leq ED_{\theta\hat{a}}^t(B)$.

I Mortality shocks and the evolution of $ED_{\theta\hat{a}}^t$ and $PALE_\theta^t$

We briefly contrast the impact of mortality shocks on $PALE_\theta^t$ and $ED_{\theta\hat{a}}^t$, assuming that these mortality shocks are independent of the poverty status. Consider a mortality shock that equalizes individual lifespans across the age threshold $\hat{a}$ while keeping life expectancy LE^t constant. This lower dispersion in mortality does not affect $PALE_\theta^t$, which only accounts for mortality through LE^t. By contrast, this shock reduces $ED^t_{\theta\hat{a}}$, since $LGE^t_{\hat{a}}$ is thereby reduced. It is indeed easy to check that $\frac{\partial ED^t_{\theta\hat{a}}}{\partial LGE^t_{\hat{a}}} > 0$ (for $\theta H^t < 1$).

Consider instead a mortality shock that reduces mortality above the age threshold $\hat{a}$. Such shock increases LE^t but does not affect $LGE^t_{\hat{a}}$. As a result, $PALE^t_\theta$ mechanically increases. It is also easy to show that deprivation, as measured by $ED^t_{\theta\hat{a}}$, decreases: $\frac{\partial ED^t_{\theta\hat{a}}}{\partial LE^t} < 0$, for $\theta H^t < 1$. Moreover, $PALE^t_\theta$ is more sensitive to this kind of shock than $ED^t_{\theta\hat{a}}$, as the elasticity of $PALE^t_\theta$ to LE^t is equal to 1 while the elasticity of $ED^t_{\theta\hat{a}}$ to LE^t lies in $(-1, 0)$. If the mortality shock is such that it reduces mortality below the age threshold $\hat{a}$, this shock simultaneously increases LE^t and reduces $LGE^t_{\hat{a}}$. Again, $PALE^t_\theta$ improves and $ED^t_{\theta\hat{a}}$ decreases since both LE^t increases and $LGE^t_{\hat{a}}$ decreases.

J Proof of Proposition 6

The proof builds on the framework presented in Appendix A.

By Proposition 5, $M^t_{\theta\hat{a}}$ is Paradox-free if and only if $ED^t_{\theta\hat{a}}$ is Paradox-free.

First, we prove that $ED^t_{\theta\hat{a}}$ is Paradox-free only if $\theta \leq 1$ and $\hat{a} \geq a^*$. The proof is by contradiction.

Assume first that $\theta > 1$ and $\hat{a} \geq 2$. Consider two alternative stationary societies A and B that both feature only one newborn i every year. The life of i is respectively $l^A_i = (P)$ and $l^B_i = (P, P)$, namely i lives in poverty for one year in society A and for two years in society B. Society B is obtained from A by a lifespan increment to the poor person i. However, we have $ED^t_{\theta\hat{a}}(A) = \frac{(\hat{a}-1)+\theta}{\hat{a}}$ and $ED^t_{\theta\hat{a}}(B) = \frac{(\hat{a}-2)+2\theta}{\hat{a}}$, which yields $ED^t_{\theta\hat{a}}(A) < ED^t_{\theta\hat{a}}(B)$, which shows that $ED^t_{\theta\hat{a}}$ is not Paradox-free.

Assume then that $\theta > 0$ and $\hat{a} < a^*$. Consider two alternative stationary societies A' and B' that both feature two newborns i and j every year. Their lives are respectively $l^{A'}_i = (P, \ldots, P)$ and $l^{B'}_i = (P, \ldots, P, P)$, where i's lifespan is $\hat{a}$ years in society A' and $\hat{a}+1$ years in society B', while $l^{A'}_j = l^{B'}_j = (NP, \ldots, NP)$, where j's lifespan is $\hat{a}$ years in both societies. Society B' is obtained from A' by a lifespan increment to the poor person i. By construction, $LGE^t_{\hat{a}}(A') = LGE^t_{\hat{a}}(B') = 0$. However, we have $ED^t_{\theta\hat{a}}(A') = \frac{\theta}{2}$ and $ED^t_{\theta\hat{a}}(B') = \frac{\theta(\hat{a}+1)}{\hat{a}+\hat{a}+1}$, which yields $ED^t_{\theta\hat{a}}(A') < ED^t_{\theta\hat{a}}(B')$, which shows that $ED^t_{\theta\hat{a}}$ is not Paradox-free.

We have thus proven that $ED^t_{\theta\hat{a}}$ is Paradox-free only if $\theta \leq 1$ and $\hat{a} \geq a^*$.

Second, we prove that $ED^t_{\theta\hat{a}}$ is Paradox-free if $\theta \leq 1$ and $\hat{a} \geq a^*$. Take any two stationary societies A" and B" such that B" is obtained from A" by a lifespan increment to the poor person i. Poor person i dies prematurely in society A" because $\hat{a} \geq a^*$. The only difference between societies A" and B" is thus that i spends an additional year in poverty in society B", instead of prematurely loosing that year in society A". This relationship between the two stationary societies A" and B" implies that the following two equalities hold

$$LE^t(A'') + LGE^t_{\hat{a}}(A'') = LE^t(B'') + LGE^t_{\hat{a}}(B''), \tag{13}$$

and

$$LGE^t_{\hat{a}}(A'') - LGE^t_{\hat{a}}(B'') = LE^t(B'')H^t(B'') - LE^t(A'')H^t(A''), \tag{14}$$

where each of the two sides of Eq. (14) is equal to $\frac{1}{n_t}$ where the number of newborns $n_t = n_t(A'') = n_t(B'')$ (B" is obtained from A" by a lifespan increment to the poor person i).[30] Eq. (13) implies that $ED^t_{\theta\hat{a}}(A'') \geq ED^t_{\theta\hat{a}}(B'')$ if and only if $LGE^t_{\hat{a}}(A'') + \theta LE^t(A'')H^t(A'') \geq LGE^t_{\hat{a}}(B'') + \theta LE^t(B'')H^t(B'')$. This inequality is equivalent to $LGE^t_{\hat{a}}(A'') - LGE^t_{\hat{a}}(B'') \geq \theta(LE^t(B'')H^t(B'') - LE^t(A'')H^t(A''))$, which further simplifies to $1 \geq \theta$ given Eq. (14), which proves that $ED^t_{\theta\hat{a}}$ is Paradox-free.

K Proof of Proposition 3

Proof of (i). We start by the "only if" part. Assume to the contrary that $PALE^t_0(A) > PALE^t_0(B)$ or $PALE^t_1(A) > PALE^t_1(B)$. This directly implies that $PALE^t_\theta(A) > PALE^t_\theta(B)$ for some $\theta \in (0,1]$ and therefore we cannot have $PALE^t_\theta(A) \leq PALE^t_\theta(B)$ for all $\theta \in (0,1]$.

We now turn to the "if" part. By definition of the $PALE^t_\theta$ index, we have to show that

$$LE^t(B) - LE^t(A) \geq \theta * (LE^t(B)H^t(B) - LE^t(A)H^t(A)), \tag{15}$$

for all $\theta \in (0,1]$. As $PALE^t_0(A) \leq PALE^t_0(B)$, we directly have that $LE^t(B) - LE^t(A) \geq 0$ because $PALE^t_0 = LE^t$. As $PALE^t_1(A) \leq PALE^t_1(B)$, we have $LE^t(B) - LE^t(A) \geq LE^t(B)H^t(B) - LE^t(A)H^t(A)$. It immediately follows that inequality (15) is verified for all $\theta \in (0,1]$.

Proof of (ii). From (i), proving (ii) only requires providing two societies A and B with $H^t(A) < H^t(B)$ such that $PALE^t_0(A) \leq PALE^t_0(B)$ and $PALE^t_1(A) \leq PALE^t_1(B)$. If $H^t(A) = 0.2$, $H^t(B) = 0.4$, $LE^t(A) = 50$ and $LE^t(B) = 75$ we have $PALE^t_1(A) = 40$ and $PALE^t_1(A) = 45$, the desired result because $PALE^t_0 = LE^t$.

L Robust $ED^t_{\theta\hat{a}}$ comparisons

$ED^t_{\theta\hat{a}}$ is Paradox-free when $\theta \in (0,1]$ and $\hat{a} \geq a^*$. However, $ED^t_{\theta\hat{a}}$ no longer encapsulates the minimal view when $\hat{a} \geq a^*$. To ease the impossibility between paradox-freeness and the minimal view, we define a weaker notion of paradox-freeness. A deprivation index is *minimally paradox free* when the index does not record a worsening for increments to the lifespan of a poor person *who dies prematurely*, i.e., whose lifespan is smaller than $\hat{a}$. In that case, $ED^t_{\theta\hat{a}}$ is minimally paradox free when $\theta \in (0,1]$.

We assume that the age threshold $\hat{a} \in \mathbb{N}_0$ must respect a lower-bound $\underline{\hat{a}} \in \mathbb{N}_0$, such that $\hat{a} \geq \underline{\hat{a}} \geq 0$. Clearly, the value for the lower bound $\underline{\hat{a}}$ influences the set of comparisons that are robust to the values selected for θ and $\hat{a}$. Proposition 8 provides the conditions under which the ranking by $ED^t_{\theta\hat{a}}$ is robust for all $\theta \in (0,1]$ and all $\hat{a} \geq \underline{\hat{a}}$.

[30]The RHS of Eq. (14) is equal to $\frac{1}{n_t}$ because $N^t(B'')H(B'') = N^t(A'')H(A'')+1$ and $LE^t = \frac{N^t}{n_t}$ for stationary societies (Eq. (6)). The LHS of Eq. (14) is equal to $\frac{1}{n_t}$ because by Eq. (13) we have $LGE^t_{\hat{a}}(A'') - LGE^t_{\hat{a}}(B'') = LE^t(B'') - LE^t(A'')$ and by construction $N^t(B'') = N^t(A'') + 1$ and $LE^t = \frac{N^t}{n_t}$ for stationary societies (Eq. (6)).

Proposition 8 (Robust comparisons with $ED^t_{\theta\hat{a}}$).
(i) For any two societies A and B we have $ED^t_{\theta\hat{a}}(A) \geq ED^t_{\theta\hat{a}}(B)$ *for all* $\theta \leq 1$ *and all* $\hat{a} \geq \underline{\hat{a}}$ *if and only if*

$$ED^t_{0\hat{a}}(A) \geq ED^t_{0\hat{a}}(B) \text{ for all } \hat{a} \geq \underline{\hat{a}}, \text{ and}$$
$$ED^t_{1\hat{a}}(A) \geq ED^t_{1\hat{a}}(B) \text{ for all } \hat{a} \geq \underline{\hat{a}} \qquad \text{(generalized Condition C1)}$$

(ii) For any $\underline{\hat{a}} \geq 2$*, there exist societies A and B for which* $ED^t_{\theta\hat{a}}(A) \geq ED^t_{\theta\hat{a}}(B)$ *for all* $\theta \leq 1$ *and all* $\hat{a} \geq \underline{\hat{a}}$ *even though* $H^t(A) < H^t(B)$*. These societies are such that* $LE^t(A) < LE^t(B)$*.*

Proof. See Appendix M for the straightforward proof. □

We illustrate Proposition 8 in Figure 12.[31] The vertical axis represents the share of pairs of societies for which H^t and LE^t provide identical (at the top) or opposite rankings (at the bottom). By definition, rankings by H^t and LE^t are insensitive to the age threshold $\hat{a}$ considered. The horizontal axis represents all possible values of $\underline{\hat{a}}$, the lower bound on the age threshold.

The left panel describes the share of pairs for which $ED^t_{\theta\hat{a}}$ provides robust rankings as a function of $\underline{\hat{a}}$. Lower values of $\underline{\hat{a}}$ imply a fall in the share of cases that $ED^t_{\theta\hat{a}}$ can rank robustly. Indeed, a larger age interval of values of $\hat{a}$ over which $ED^t_{\theta\hat{a}}$ has to be computed implies a larger number of comparisons for $ED^t_{\theta\hat{a}}$. As a result, the number of pairs for which it can provide the same ranking for all age thresholds falls.[32] Second, if H^t and LE^t provide the same ranking, $ED^t_{\theta\hat{a}}$ provides the same ranking as H^t when $\underline{\hat{a}} = a^*$. Finally, as discussed above, when H^t and LE^t disagree, a larger value of $\underline{\hat{a}}$ implies that the share of cases for which H^t provides an robustly wrong ranking gets larger.

The right panel reports, for all values of $\underline{\hat{a}}$, the share of pairs of societies for which $PALE^t_\theta$ and $ED^t_{\theta\hat{a}}$ provide robust rankings. Since $PALE^t_\theta$ does not depend on the age threshold, it is able to rank a larger set of comparisons. As shown in Proposition 4, when $\underline{\hat{a}} = a^*$, the two indices are equivalent.

M Proof of Proposition 8

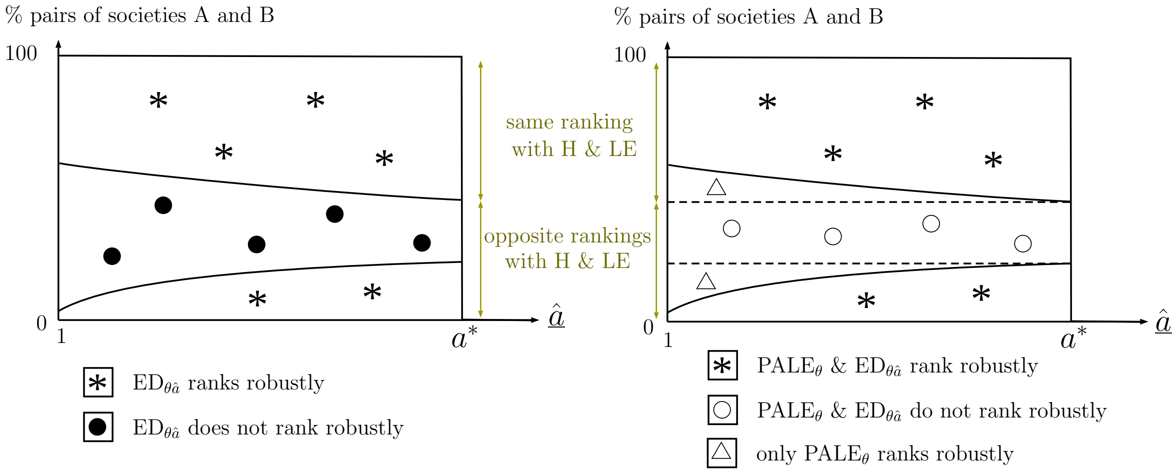
We first prove the following: for any $\hat{a} \geq \underline{\hat{a}}$ and any two societies A and B, we have $ED^t_{\theta\hat{a}}(A) \geq ED^t_{\theta\hat{a}}(B)$ for all $\theta \in (0, 1]$ if and only if

$$ED^t_{0\hat{a}}(A) \geq ED^t_{0\hat{a}}(B) \text{ and } ED^t_{1\hat{a}}(A) \geq ED^t_{1\hat{a}}(B).$$

We start with the "only if" part. Assume on the contrary that $ED^t_{0\hat{a}}(A) < ED^t_{0\hat{a}}(B)$ or $ED^t_{1\hat{a}}(A) < ED^t_{1\hat{a}}(B)$. This implies that $ED^t_{\theta\hat{a}}(A) < ED^t_{\theta\hat{a}}(B)$ for some $\theta \in (0, 1]$ and therefore we cannot have $ED^t_{\theta\hat{a}}(A) \geq ED^t_{\theta\hat{a}}(B)$ for all $\theta \in (0, 1]$.

[31] All graphs that follow are constructed using a lower bound on $\hat{a}$ equal to 1. Indeed, for $\theta = 0$ and $\hat{a} = 0$, $ED^t_{\theta\hat{a}}$ is equal to zero for all societies and cannot therefore deliver robust comparisons.
[32] It is not a sufficient condition that the rankings by H and LE are identical for the ranking by $ED^t_{\theta\hat{a}}$ to be robust. The reason is that, when $\underline{\hat{a}} < a^*$, LE^t no longer contains all the relevant information on mortality: for instance, two societies can share the same life expectancy at birth but one with several deaths occurring below $\hat{a}$ while the other has all deaths occurring above $\hat{a}$.

Figure 12: Share of robust $ED^t_{\theta\hat{a}}$ comparisons as a function of $\underline{\hat{a}}$.



Reading: Left: The smaller the lower-bound $\underline{\hat{a}}$, the lower the share of societies pairs robustly ranked by $ED^t_{\theta\hat{a}}$. Right: The higher the lower-bound $\underline{\hat{a}}$, the higher the share of societies pairs robustly ranked by both $ED^t_{\theta\hat{a}}$ and $PALE^t_\theta$.

We turn to the "if" part. By definition of the $ED^t_{\theta\hat{a}}$ index, we have to show that

$$\frac{LGE^t_{\hat{a}}(A)}{LE^t(A)+LGE^t_{\hat{a}}(A)}-\frac{LGE^t_{\hat{a}}(B)}{LE^t(B)+LGE^t_{\hat{a}}(B)} \geq \theta\left(\frac{LE^t(B)*H^t(B)}{LE^t(B)+LGE^t_{\hat{a}}(B)}-\frac{LE^t(A)*H^t(A)}{LE^t(A)+LGE^t_{\hat{a}}(A)}\right) \text{ for all } \theta \in (0,1]. \tag{16}$$

As $ED^t_{0\hat{a}}(A) \geq ED^t_{0\hat{a}}(B)$, the left hand side of Eq. (16) is non-negative. As $ED^t_{1\hat{a}}(A) \geq ED^t_{1\hat{a}}(B)$, Eq. (16) holds for $\theta = 1$. As a result, inequality (16) holds for all $\theta \in (0,1]$.

Proof of (i). This is an immediate implication of the statement proven above.

Proof of (ii). Consider two societies A and B with $H^t(A) < H^t(B)$ for which the generalized condition C1 holds.

Society A is such that $H^t(A) = 0.4$ and all its individuals die in their first year of life, which implies that $LE^t(A) = 1$ and $LGE^t_{\hat{a}}(A) = \hat{a} - 1$. Therefore, society A is such that

- $ED^t_{0\hat{a}}(A) = \frac{\hat{a}-1}{\hat{a}}$ and $ED^t_{1\hat{a}}(A) = 1 - \frac{0.6}{\hat{a}}$ for all $\hat{a} \geq \underline{\hat{a}}$.

Society B is such that $H^t(B) = 0.5$ and all its individuals die at the maximal age $a^* - 1$, which implies that $LE^t(B) = a^*$ and

- $LGE^t_{\hat{a}}(B) = 0$ if $\hat{a} \in \{2, \ldots, a^*\}$,
- $LGE^t_{\hat{a}}(B) = \hat{a} - a^*$ if $\hat{a} > a^*$.

Therefore, society B is such that

- $ED^t_{0\hat{a}}(B) = 0$ and $ED^t_{1\hat{a}}(B) = 0.5$ for all $\hat{a} \in \{2, \ldots, a^*\}$,

- $ED^t_{0\hat{a}}(B) = \frac{\hat{a}-a^*}{\hat{a}}$ and $ED^t_{1\hat{a}}(B) = 1 - \frac{0.5a^*}{\hat{a}}$ for all $\hat{a} > a^*$.

By statement (i), we get $ED^t_{\theta\hat{a}}(A) \geq ED^t_{\theta\hat{a}}(B)$ for all $\theta \in (0,1]$ and all $\hat{a} \geq \underline{\hat{a}}$ if we have $ED^t_{0\hat{a}}(A) \geq ED^t_{0\hat{a}}(B)$ and $ED^t_{1\hat{a}}(A) \geq ED^t_{1\hat{a}}(B)$ for all $\hat{a} \geq \underline{\hat{a}}$. Recalling that $\hat{a} \geq 2$, one can then easily check that we have $ED^t_{0\hat{a}}(A) \geq ED^t_{0\hat{a}}(B)$ and $ED^t_{1\hat{a}}(A) \geq ED^t_{1\hat{a}}(B)$ both for all $\hat{a} \in \{2, \ldots, a^*\}$ and for all $\hat{a} > a^*$.

N Descriptive statistics

Table 5 lists all the countries present in our data set as well as the 2019 values of the main variables of interest.

Table 5: Countries used in the dataset and descriptive statistics

Country	H %	LE Years	LGE_{70} Years	LE*H Years	$PALE^t_1$ Years	$ED_{1,70}$ %
Albania	0	75	4	0	75	5
Algeria	0	73	5	0	73	7
Angola	35	60	13	21	39	47
Armenia	1	74	4	0	74	6
Azerbaijan	0	69	7	0	69	9
Bangladesh	6	71	6	4	67	13
Belarus	0	70	6	0	70	8
Belize	20	72	6	15	58	26
Benin	14	62	12	9	53	29
Bhutan	0	73	6	0	73	7
Bolivia	2	65	9	1	64	13
BosniaandHerzegovina	0	74	4	0	74	5
Botswana	14	59	14	8	51	30
Brazil	6	73	6	4	69	13
Bulgaria	1	69	6	0	69	9
BurkinaFaso	26	59	14	15	44	41
Burundi	62	61	12	38	23	68
CaboVerde	5	72	5	4	69	12
Cameroon	23	60	13	14	46	37
CentralAfricanRepublic	66	50	21	33	17	76
Chad	32	57	16	18	39	47
China	0	77	3	0	77	4
Colombia	7	75	5	6	70	13
Comoros	18	66	9	12	54	29
CongoDemRep	78	61	12	47	13	82
CongoRep	45	61	12	28	33	54
CostaRica	1	77	4	1	76	7
CotedIvoire	11	62	12	7	55	25
Czechia	0	77	3	0	77	4
Djibouti	17	63	11	11	53	29
DominicanRepublic	1	73	6	1	72	9
Ecuador	4	73	5	3	71	10

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Country	H %	LE Years	LGE_{70} Years	LE*H Years	$PALE_1^t$ Years	$ED_{1,70}$ %
EgyptArabRep	1	68	6	1	67	10
ElSalvador	4	72	6	3	69	12
Eswatini	32	52	19	16	35	50
Ethiopia	18	64	11	12	52	30
Fiji	3	65	9	2	63	14
Gabon	3	63	11	2	61	17
GambiaThe	17	62	11	10	52	30
Georgia	5	71	6	4	67	12
Ghana	23	64	11	14	49	34
Grenada	0	69	6	0	69	9
Guatemala	7	69	8	5	64	16
Guinea	13	59	14	7	52	29
GuineaBissau	25	57	15	15	43	41
Guyana	4	64	10	3	61	17
Haiti	31	59	14	18	41	44
Honduras	13	68	7	9	59	21
India	14	68	8	9	59	23
Indonesia	4	69	7	2	66	12
IranIslamicRep	1	74	4	1	73	6
Iraq	0	69	6	0	69	8
Jamaica	0	73	5	0	73	7
Jordan	0	75	4	0	75	5
Kazakhstan	0	69	7	0	69	9
Kenya	36	63	11	23	40	45
Kiribati	1	63	11	1	62	16
KyrgyzRepublic	1	71	6	1	71	8
LaoPDR	7	67	9	5	62	17
Lebanon	0	74	4	0	74	5
Lesotho	36	48	23	17	30	57
Liberia	31	62	12	19	43	42
Madagascar	81	61	12	50	12	84
Malawi	70	58	14	41	17	76
Malaysia	0	72	5	0	72	7
Maldives	0	78	3	0	78	4
Mali	20	58	15	12	46	37
MarshallIslands	1	64	10	1	64	14
Mauritania	6	68	8	4	64	15
Mauritius	0	72	6	0	72	7
Mexico	2	70	7	1	69	10
Moldova	0	71	6	0	71	7
Mongolia	0	69	7	0	69	9
Montenegro	2	72	4	1	70	7
Morocco	1	72	5	1	71	8

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Country	H %	LE Years	LGE_{70} Years	LE*H Years	$PALE_1^t$ Years	$ED_{1,70}$ %
Mozambique	75	55	17	42	14	81
Myanmar	3	67	9	2	65	14
Namibia	16	59	13	10	50	31
Nepal	3	68	8	2	66	13
Nicaragua	3	72	5	2	70	10
Niger	53	61	13	32	29	61
Nigeria	32	62	13	20	42	44
NorthMacedonia	3	71	5	2	69	9
Pakistan	4	64	11	3	61	18
PapuaNewGuinea	36	62	11	22	40	46
Paraguay	1	71	6	0	71	8
Peru	3	71	6	2	69	10
Philippines	7	67	8	5	63	16
Romania	0	72	5	0	72	6
RussianFederation	0	69	7	0	69	9
Rwanda	45	64	10	29	35	53
SaoTomeandPrincipe	15	70	6	10	59	22
Senegal	10	65	9	6	59	21
Serbia	1	73	4	1	72	6
SierraLeone	26	60	14	15	44	40
SolomonIslands	29	65	9	19	46	38
SouthAfrica	20	60	13	12	48	35
SriLanka	1	76	4	1	75	6
StLucia	0	72	6	0	72	7
Sudan	25	67	8	17	50	33
Suriname	1	70	7	1	69	11
SyrianArabRepublic	21	72	5	15	56	26
TaiwanChina	0	80	3	0	80	3
Tajikistan	3	68	7	2	66	13
Tanzania	44	63	11	27	35	52
Thailand	0	75	5	0	75	6
Togo	27	62	12	17	45	39
Tonga	0	72	5	0	72	7
Tunisia	0	73	5	0	73	6
Turkiye	0	74	4	0	74	6
Turkmenistan	5	67	8	3	64	15
Uganda	42	60	13	25	35	52
Ukraine	0	70	6	0	70	8
UnitedArabEmirates	0	74	3	0	74	4
Uzbekistan	2	72	6	2	70	10
Vanuatu	12	65	9	8	57	23
Vietnam	1	73	5	1	73	7
WestBankandGaza	1	73	4	1	72	6

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Country	H %	LE Years	LGE_{70} Years	LE*H Years	$PALE^t_1$ Years	$ED_{1,70}$ %
YemenRep	52	64	10	33	31	58
Zambia	64	57	15	37	20	72
Zimbabwe	41	54	17	22	32	55

O Building $PALE^t_\theta$ and $ED^t_{\theta\hat{a}}$ in practice

How should a practitionner build our different indices with available data ? Table 6 presents the different step required to build our $PALE^t_\theta$ and $ED^t_{\theta\hat{a}}$ indices, the data source as well as their 2019 value.

Table 6: $PALE^t_\theta$ and Expected Deprivation in the developing world in 1990 and 2021, with $\hat{a} = 70$.

	Unit	1990 Value	2021 Value	Computation
Life Expectancy (LE)	Years	64.5	71.0	Source: Global Burden of Disease Collaborative Network (2021)
Poverty Headcount (H)	%	38.5	9.5	Source: World Bank (2024)
LE*H	Years	24.9	6.7	LE* H
$PALE^t_1$	Years	39.7	64.3	LE-θH*LE
Lifespan Gap Expectancy$_{70}$	Years	11.7	7.1	See Section 4.2
Expected Deprivation$_{1,70}$	%	47.9	17.7	$\frac{LGE_{\hat{a}}}{LE+LGE_{\hat{a}}} + \theta\frac{LE*H}{LE+LGE_{\hat{a}}}$

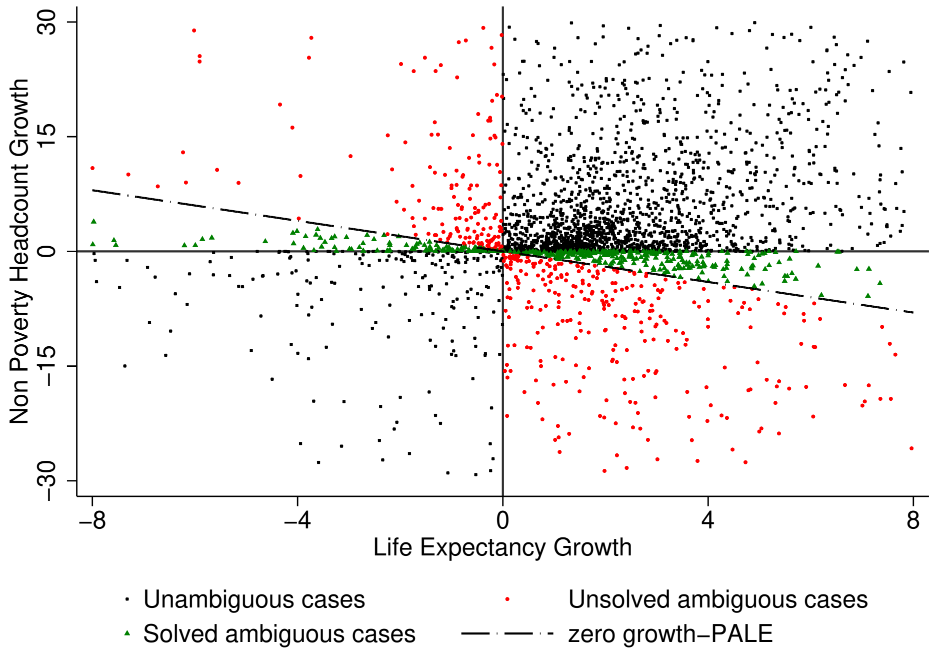
P Ambiguous countries' trajectories

In Figure 13, we provide $PALE^t_\theta$ comparisons within countries between present and past situations. More precisely, for each year, we compare the situation in period t to the situation prevailing in the same country five years earlier. Given that each country's situation changed over time, we need to adapt our graphical presentation to represent the set of situations for which $PALE^t_\theta$ stays constant over time. We conservatively assume θ equal to one.

By definition, $PALE^t_1 = LE(1-H)$, and thus $PALE^t_1$ increases if and only if $dLE/LE > d(1-H)/(1-H)$. This simple expression allows us to contruct a figure in the $(dLE/LE, d(1-H)/(1-H))$ plan, in which the rate of growth of LE is measured on the horizontal axis, and the rate of growth of $(1-H)$, which we refer to as the "Non-poverty Headcount", on the vertical axis. We define the "zero-growth $PALE^t_1$" curve, which represents all the combinations of the two growth rates such that $PALE^t_1$ remains unchanged: $dLE/LE = d(1-H)/(1-H)$. Above this curve, $PALE^t_1$ increases and below this curve $PALE^t_1$ decreases.

The situations of interest are located in the northwest and in southeast quadrants in which the two indicators move in opposite directions. In these quadrants, there are two regions, one in the triangle below the curve in the northwest quadrant, and one in the triangle above the curve in the southeast quadrant for which $PALE_{\theta}^{t}$ is able to provide a clear welfare comparison. In these two areas, the shaded triangles represent situations in which, in a particular country, the situation either strictly improved (in the southeast quadrant) or deteriorated (in the northwest quadrant) compared to the situation prevailing in the same country five years earlier.[33]

Figure 13: Resolution of ambiguous countries' trajectories, 1990-2021



Reading: Each dot represents a country-year. Countries located in the southwest (northeast) quadrant are worse (better) off than they were 5 years earlier. Countries' evolution located in the other quadrants can not be unambiguously assessed with a dashboard approach. Countries' trajectories located between the zero growth-PALE curve and the zero non poverty headcount growth line can be unambiguously assessed with $PALE_{\theta}^{t}$.
Note: for readibility, the graph only shows the points situated between a growth rate of +/- 30% in non poverty headcount and of +/-8% in life expectancy. These are 85% of all observations.

[33] Again, if being dead is strictly worse than being poor, so that θ is always strictly lower than one, more situations can be strictly signed. They are located in the triangle above the "zero-growth $PALE_{1}^{t}$" in the NW quadrant, and in the triangle below the "zero-growth $PALE_{1}^{t}$" in the SE quadrant.