Guilt, Esteem, and Motivational Investments*

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February 9, 2018.

Abstract

In this paper we address the question what are the determinants of a firm's investment in the loyalty and motivation of its employees? We develop a simple principal-agent model where the standard optimal contract is to offer a bonus that trades off incentive provision vs rent extraction. We allow the principal to undertake two types of investments - one that reduces the marginal cost of effort of the agent and the other that increases the on-the-job satisfaction. We characterize the relationship between these investments and the level of the bonus. We also characterize how varying the outside option of the agent affects these choices.

1 Introduction

Why do firms invest in employee motivation, and what are the determinants of the level of "motivational" investment? The narrow self-interest driven view of individuals in the economic domain has increasingly come under question. Recent work in economics has moved beyond stylized models of motivation based on a narrow view of *homo economicus* who cares about only money and leisure, and been considering a richer set of motivations. Broadly speaking, this has been focused on different approaches to pro-social motivation. These include: commitment to a mission (e.g., Besley and Ghatak, 2005), the role of identity such as being a "good" or "responsible" employee, a good teacher or doctor (e.g., Akerlof and Kranton, 2005, 2011), commitment to "in-group" (e.g., family, community, tribe), intrinsic motivation, reputational concerns (e.g., Benabou and Tirole, 2006, 2010), social norms , status rewards, and pure altruism.[1]

*Preliminary draft. Thanks are due to Greg Fischer, Rocco Macchiavello, and LSE lunch workshop participants for helpful comments. The usual discalimer applies.

[1]See Besley and Ghatak (2014, 2017) for reviews of the empirical and theoretical literature relating to sources of non-pecuniary motivation and how they affect incentives and organization design.

In this paper we take up a set of questions that have received relatively little attention all having to do with the "endogenous" creation of motivation: how would organizations invest in increasing the motivation of their employees? Are explicit bonuses and investment in motivation complements or substitutes in the firm's choice? What governs the choice between causing agents to feel guilt at under-performing versus pride in being part of the organization? How do these outcomes depend on the level of competition in the labour market as captured by the outside option facing the employee?

What separates our work from the existing literature on this topic (e.g., Akerlof and Kranton, 2005, Besley and Ghatak, 2005, and Kvaløy and Schöttner, 2015) is our explicit attempt to separate investments that raise overall surplus (which could also be interpreted as investment in skills or training) from investments that reduce agency problems only, with no impact on the first-best level of effort choice by the agent.

An example will make this clear. Imagine you have hired a gardener to "dig holes in your garden". The gardener has an hourly rate which is determined by the market. The gardener has the ability to dig x holes per hour. Suppose you can measure how long the gardener is working because you don't have the time to inspect his work. Then he may not dig the number of holes you have specified because he incurs disutility from the effort. In this situation, if you could 'motivate' the gardener to 'take pride' in his work, then this would offset his disutility from effort and he would dig the number of holes you have asked for even if you can't inspect his work. But this kind of investment does not affect his capacity for digging holes, which remains x per hour.

On the other hand, you could train the gardener to dig more holes per hour. But this would not necessarily mean that he will dig more holes for you when you employ him because you can't inspect his work. It is worth thinking about what it means to 'motivate' the gardener to 'take pride' in his work. If this kind of motivation meant that he takes more pleasure in digging holes, then it may mean that, even if he were working in his own garden, he would choose to dig a few extra holes. In other words, it would affect the first-best. So the effect is not dissimilar to lowering his cost of digging holes, which is like improving his hole-digging skill. But if a motivated gardener takes more satisfaction in doing the task he has been assigned to do, and incurs disutility from not doing this work properly (a sense of shame) then the concept is relevant only in an agency relationship. And it wouldn't affect the number of holes he digs in his own garden.

We take a standard principal-agent problem with all parties being risk neutral, effort being non-contractible leading to moral hazard, and limited-liability preventing a fixed-price contract that would achieve full efficiency. We allow for the principal can invest in reducing the agent's cost of effort. We also allow an investment that increases the utility of the worker from working in the firm. The former type of investment will affect the marginal cost of eliciting effort and the latter will affect the on-the-job utility and therefore, the effective reservation option for the agent relative to other employment opportunities. We characterize conditions under which a decline in the cost of investing in reducing the cost of effort as well as a decline in the cost of investments that raise on-the-job rents would lead the principal to invest more in both types of investment and reduce the amount of bonus offered. We also show that if the agent's participation constraint is not binding, the principal makes no investment in on-the-job rents and the contract offered is independent of the agent's outside option. If the agent's participation constraint is binding, then we characterize conditions under which an increase in labour market tightness (i.e. an increase in the outside option) would lead the principal to invest less in reducing the cost of effort for the agent, increase the amount of bonus offered, and decrease the net amount of on-the-job rents (the level of on-the-job rents less the outside option) associated with the job.

2 Model

Consider a simple principal-agent model where the agent provides effort $e \in [0,1]$ at cost $c(e)$. This produces output $A > 0$ with probability e and output zero otherwise. We assume $c(.)$ is strictly increasing, strictly convex, twice differentiable and $c(0) = 0$.

A contract between the principal and the agent specifies a 'bonus' in case the agent produces positive output and an effort level e_c. For simplicity, we normalize the wage to be zero in the case of no output. (This is not an innocuous assumption, but we will revisit this point at a later stage of the analysis). The agent's effort level is unobservable to the principal and non-contractible, but the agent experiences some 'guilt' – (i.e. disutility) in deviating from e_c. In particular, the agent solves

$$\max_e be - c(e) - \psi g(e_c - e) + \sigma.$$

where ψ is a positive constant and $g(x)$ is a twice differentiable U-shaped function, with $g''(x) > 0$, $g'(x) \lesseqgtr 0$ for $x \lesseqgtr 0$, and $g(0) = p \geq 0$, $g(x) = g(-x)$.

From the first-order condition, we obtain

$$b + \psi g'(e_c - e) = c'(e). \tag{1}$$

Given the assumptions of convexity on $c(.)$ and $g(.)$, this optimization problem also has a unique solution which we denote by $\hat{e}(b, e_c, \psi)$. Let $\hat{e}_0(b) = \hat{e}(b, 0, 0)$. In words, $\hat{e}_0(b)$ is the effort level that the agent would choose if he or she did not experience any guilt. We can show that $\hat{e}(b, e_c, \psi) > \hat{e}_0(b)$ for $e_c > \hat{e}_0(b)$, and $\hat{e}(b, e_c, \psi)$ is increasing (decreasing) in ψ for $e_c > \hat{e}_0(b)$ ($e_c < \hat{e}_0(b)$).

In addition to the payoff described above, the agent obtains an additional utility of σ from undertaking the task, but that does not depend on the level of effort. We will call it 'esteem'. The significance of esteem will become evident in what follows.

Our formulation is equivalent to one where the agent has a cost of effort function $C(e) = c(e) + \psi g(e_c - e) - \sigma$ that has two parameters, ψ and σ, with the former affecting its slope, i.e., the marginal cost of effort, and the latter being an additive parameter that affects its *level.* The principal can affect these two parameters in a way that will be made specific below.

3 First-Best Case

We postulate that changing ψ would not change the first-best level of effort. We can see this as follows: In the first-best case, we solve

$$\max_{e,e_c} Ae - c(e) - \psi g(e_c - e) + \sigma.$$

It should be possible to show that, given this problem, the unique solution is given by $e = e_c = \hat{e}_0(A)$. This is because the term $Ae - c(e)$ attains its maximum, by construction, at $e = \hat{e}_0(A)$. And the term $-g(e_c - e) \leq 0$. Therefore, the maximand is at most $A\hat{e}_0(A) - c(\hat{e}_0(A))$. This value is attained at $e = e_c = \hat{e}_0(A)$. For any other value of e, the term $Ae - c(e) < A\hat{e}_0(A) - c(\hat{e}_0(A))$. Therefore, the maximand necessarily attains a lower value. Given $e = \hat{e}_0(A)$, the maximand also attains a lower value if $e_c \neq \hat{e}_0(A)$. Thus, we have established that the unique solution is given by $e = e_c = \hat{e}_0(A)$. And it yields a payoff of $A\hat{e}_0(A) - c(\hat{e}_0(A))$. As this expression is independent of ψ, it follows that there are no gains from raising the level of motivation, as represented by ψ, in the first-best case.

4 The Principal's Problem

The principal will propose a contract that species the bonus b and a level of effort e_c. In addition, the principal can 'invest' in raising the agent's sense of guilt when he/she falls short of the specified level of effort (ψ) and the esteem associated with the job (σ). More precisely, we assume that the principal has to make an investment equal to $h(\sigma)$ so that the agent experiences sense of esteem with utility value σ if he/she decides to accept the job. And we suppose that inducing a psychological cost ψ requires the principal to make an investment of $m(\psi)$. We assume that $h(0) = 0$, and $h(.)$ is increasing, strictly convex and twice differentiable; $m(.)$ is strictly increasing, strictly convex, twice differentiable and $m(0) = 0$.

Then, we can write principal's optimization problem as follows:

$$\max_{b,e,e_c,\psi,\sigma} (A - b)e - m(\psi) - h(\sigma)$$

subject to

$$e = \hat{e}(b, e_c, \psi) \tag{IC}$$

$$\underline{u} \leq be - c(e) - \psi g(e_c - e) + \sigma \tag{PC}$$

where $\underline{u}$ is the agent's outside option.

4.1 Binding Participation Constraint

For the subsequent analysis, we assume that, in the optimal contract, the agent's participation constraint is binding. The rationale is that if it were not, the principal could increase e_c till the PC begins to bind, and thus obtain a higher level of effort at zero cost to herself. (There is a special case in which the contract may be optimal with $e_c = 1$ and the PC is not binding which we will need to deal with at a later stage).

Let us denote by $V(b, e_c; \psi, \sigma)$ the agent's expected utility from a contract with bonus b and specified effort e_c. Then we can write

$$V(b, e_c; \psi, \sigma) = b\hat{e}(b, e_c, \psi) - c(\hat{e}(b, e_c, \psi)) - \psi g(e_c - \hat{e}(b, e_c, \psi)) + \sigma.$$

If the PC is binding, we have

$$b\hat{e}(b, e_c, \psi) - c(\hat{e}(b, e_c, \psi)) - \psi g(e_c - \hat{e}(b, \varepsilon, \psi)) = \underline{u} - \sigma. \tag{2}$$

Note that increasing σ is equivalent to lowering the outside option $\underline{u}$ as far as the participation constraint is concerned. Let us denote by $\hat{e}_c(b, \psi, \underline{u} - \sigma)$ the value of e_c for which the equation above is satisfied with equality. By construction, $V(b, e_c; \psi, \sigma)$ is increasing in b and decreasing in ψ. Therefore, we have $\hat{e}_c(b, \psi, \underline{u} - \sigma)$ increasing in b and decreasing in ψ. Furthermore, $\hat{e}_c(..)$ is decreasing in $\underline{u} - \sigma$.

4.2 Principal's First-Order Conditions (PC Binding)

Let

$$\pi(A, b, \psi, \sigma, \underline{u}) = (A - b)\,\hat{e}(b, \hat{e}_c(b, \psi, \underline{u} - \sigma), \psi) - h(\sigma) - m(\psi).$$

Under the assumption that the agent's PC is binding, we can rewrite the principal's optimization problem as follows:

$$\max_{b,\sigma,\psi} \pi(A, b, \psi, \sigma, \underline{u}). \tag{3}$$

Then, from the first-order conditions, we obtain

$$\begin{aligned}
b &: \; -\hat{e}(b, \hat{e}_c(b, \psi, \sigma), \psi) + (A - b)\left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial b}\right) = 0 \\
\psi &: \; (A - b)\left(\frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \psi} + \frac{\partial \hat{e}}{\partial \psi}\right) - m'(\psi) = 0 \\
\sigma &: \; -(A - b)\frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \underline{u}} - h'(\sigma) = 0
\end{aligned}$$

We can use the agent's first-order condition in (1) and the binding participation constraint in (2) to derive expressions for $\frac{\partial \hat{e}}{\partial e_c}, \frac{\partial \hat{e}}{\partial b}, \frac{\partial \hat{e}_c}{\partial b}, \frac{\partial \hat{e}_c}{\partial \psi}, \frac{\partial \hat{e}_c}{\partial \psi}$, and $\frac{\partial \hat{e}_c}{\partial \underline{u}}$ as follows (the derivations are shown in the Appendix):

$$\begin{aligned}
\frac{\partial e}{\partial e_c} &= \frac{\psi g''(e_c - \hat{e}(b, e_c, \psi))}{c''(\hat{e}(b, e_c, \psi)) + \psi g''(..)} \in (0,1) \\
\frac{\partial e}{\partial b} &= \frac{1}{c''(\hat{e}(b, e_c, \psi)) + \psi g''(..)} > 0 \\
\frac{\partial e}{\partial \psi} &= \frac{g'(e_c - e)}{\psi g''(..) + c''(\hat{e}(b, e_c, \psi))} > 0 \\
\frac{\partial \hat{e}_c}{\partial b} &= \frac{\hat{e}(b, e_c, \psi)}{\psi g'(e_c - \hat{e}(b, e_c, \psi))} > 0 \\
\frac{\partial \hat{e}_c}{\partial \psi} &= -\frac{g(e_c - \hat{e}(b, e_c, \psi))}{\psi g'(e_c - \hat{e}(b, e_c, \psi))} < 0 \\
\frac{\partial \hat{e}_c}{\partial \underline{u}} &= -\frac{1}{\psi g'(e_c - \hat{e}(b, e_c, \psi))} < 0.
\end{aligned}$$

Substituting into the principal's first-order conditions using these expressions, we obtain

$$b \quad : \quad \frac{A-b}{c''(..) + \psi g''(..)} + \hat{e}(..)\left\{\frac{A-b}{c''(..) + \psi g''(..)}\frac{g''(..)}{g'(..)} - 1\right\} = 0 \tag{4}$$

$$\psi \quad : \quad \left\{\frac{A-b}{c''(..) + \psi g''(..)}\right\}\left\{-\frac{g''(..)\,g(..)}{g'(..)} + g'(..)\right\} - m'(\psi) = 0 \tag{5}$$

$$\sigma \quad : \quad \frac{g''(..)(A-b)}{g'(..)\{c''(..) + \psi g''(..)\}} - h'(\sigma) = 0 \tag{6}$$

These three equations, together with the binding participation constraint in (2) and the agent's first-order condition in (1), fixes the principal's motivational investments and choice of contract.

5 Comparative Statics

We wish to investigate how the principal's choice of contract and motivational investments will vary with the cost of each type of investment, and relatedly whether investing in guilt and motivation are complements or substitutes. For this purpose, we first establish the following lemma.

Lemma 1 *: Under the assumption that $g(.)$ and $c(.)$ are quadratic, the function $\pi(A, b, \psi, \sigma, \underline{u})$ is supermodular in ψ, σ, and $-b$ if $\frac{\hat{e}(b,e_c,\psi)}{g'(x)} < \frac{\psi + g''(x)\psi^2}{c''(\hat{e}(b,e_c,\psi)) + \psi g''(x)}$ and $g''(x)\,g(x) < \{g'(x)\}^2$ where $x = \hat{e}_c(b, \psi, \underline{u} - \sigma) - \hat{e}(b, e_c, \psi)$.*

Proof. See the Appendix. ■

We briefly discuss the assumptions and implications of Lemma 1. We make the assumption that $g(.)$ and $c(.)$ are quadratic to ensure that the properties of the principal's objective function do not depend on the third derivative of the agent's cost functions. The result would remain valid as long as the third derivatives were relatively small.

The first inequality in the statement of Lemma 1 ensures that increasing the bonus in the contract reduces the gap between the level of effort specified in the contract and the actual level of effort exerted by the worker. The reason why this is not always true is that, starting from a contract in which the agent's PC is binding, increasing the bonus relaxes the constraint and allows the principal to specify a higher level of effort in the contract.

The second inequality in the statement of Lemma 1 ensures that an increase in the guilt parameter induces the worker to exert more effort. The reason why this is not always true is that, starting from a contract in which the agent's PC is binding, increasing guilt tightens the participation constraint and thus forces the principal to specify a lower level of effort in the contract to persuade the worker to accept it.

Next, we consider how exogenous changes in the cost of investments in guilt and esteem affect motivational investments. For the comparative statics exercise, we replace the investment functions $m(\psi)$ and $h(\sigma)$ by $\mu m(\psi)$ and $\lambda h(\sigma)$ where μ and λ are positive parameters. Then we can establish the following results.

Proposition 2 *: Under the conditions stated in Lemma 1 a decline in the cost of guilt investments (i.e. a decline in* μ*) as well as a decline in the cost of esteem investments (i.e a decline in* λ*) would lead the principal to invest more in both guilt and esteem and reduce the amount of bonus offered.*

Proof. See the Appendix. ■

The intuition behind the first part of Proposition 2 is as follows. It is natural that the principal would make higher investments in the worker's sense of guilt in deviating from a specified level of effort when the cost of making such investments is lower. A worker with a stronger sense of guilt exerts higher effort which increases the principal's expected costs from any given level of bonus. Therefore, the principal would want to make less use of the bonus. Additionally, a worker with a stronger sense of guilt has a tighter participation constraint. Investing in esteem – the direct utility the worker derives from the job – is a way of slackening the worker's participation constraint. Therefore, the principal is more inclined to invest in esteem when the level of guilt is high. Therefore, esteem is a complement of guilt while financial rewards are a substitute of guilt.

A similar type of reasoning can help to understand the logic behind the second part of Proposition 2. It is natural that the principal would make higher investments in the worker's sense of esteem from the job when the cost of making such investments is lower. This type of investment slackens the worker's participation constraint, which allows the principal to specify a higher level of effort in the contract. Then, the worker would exert a higher level of effort to minimize his/her sense of guilt. This increases the principal's expected costs from any given level of bonus. Therefore, the principal would want to make less use of the bonus. Although the worker responds to the higher level of effort specified in the contract by working harder, the gap between the specified and actual level of effort becomes larger. Consequently, the worker becomes more responsive to changes in the guilt parameter ψ. Therefore, the principal's marginal benefit from guilt-related investments go up and she becomes more inclined to use such investments. Therefore, guilt is a complement of esteem while financial rewards are a substitute of esteem.

6 Labour Market Tightness

How does labour market tightness affect motivational investments? We can address this question by investigating the effects of varying the agent's outside option on the principal's optimal decision.

First note that if $\underline{u}$ is sufficiently low, then the agent's participation constraint would not bind, as is standard in models with limited liability. In this case, the constrained optimum contract must involve $e_c = 1$ because the principal can extract higher effort at zero cost by increasing e_c when $\psi > 0$ and the agent's participation constraint is slack. Then the principal's maximization problem can be written as

$$\max_{b,\sigma,\psi} (A - b)\, \hat{e}\,(b, 1, \psi) - h\,(\sigma) - m\,(\psi)\,.$$

We see immediately that the principal does not gain from investing in σ. Therefore, we have $\sigma = 0$. Furthermore, the maximand is independent of $\underline{u}$. Therefore, increasing the agent's outside option would not affect the principal's choice of b and ψ. When $\underline{u}$ is sufficiently high for the participation constraint to bind, the principal will potentially make positive investments in both σ and ψ. In particular, the choice of bonus and motivational investments will respond to labour market tightness as described in the next proposition.

For the purpose of the next proposition, a variable which we call 'relative esteem'

$$\hat{\sigma} = \sigma - \underline{u}.$$

It is the esteem obtained from the job relative to the worker's outside option. Then we have:

Proposition 3 *(i) If the agent's participation constraint is non-binding, the principal makes no investment in esteem and the contract offered is independent of the agent's outside option. (ii) Under the conditions stated in Lemma 1, if the agent's participation constraint is binding, then an increase in labour market tightness (i.e. an increase in $\underline{u}$) would lead the principal to invest less in guilt, increase the amount of bonus offered, and decrease 'relative esteem' associated with the job.*

Proof. (i) Evident from the discussion above. (ii) See the Appendix. ■

The intuition for the second part of Proposition 3 is as follows. The esteem associated with the job (σ) and the utility that the worker derives from the outside option ($\underline{u}$) affect the participation constraint only to the extent that they affect *relative* esteem. In fact, her outside option affects the contract offered and her effort level only through its effect on relative esteem.

When the worker's participation constraint binds, a higher level of relative esteem means that the principal can specify a higher level of effort in the contract. This, in turn, increases the returns to investments in guilt and lowers the return on increasing the bonus. Therefore, when the agent has a higher outside option, the principal needs to lower the level of effort specified in the contract for any combination of (ψ, σ, b), which translates into lower investments in guilt and a higher level of monetary incentives.

We are left with the question how an increase in $\underline{u}$ affects the principal's investment in the absolute level of esteem (σ), as distinct from relative esteem, $\sigma - \underline{u}$, if the agent's participation constraint is initially binding. The proposition above shows as $\underline{u}$ goes up starting with the participation constraint being binding, $\sigma - \underline{u}$ falls but that is consistent with σ going up, down or staying constant. Intuition suggests that in a tighter labour market, the principal would want to invest more in esteem associated with the job to slacken the agent's participation constraint. But according to Lemma 1, increasing the bonus (b) and decreasing investment in guilt (ψ) both reduce the principal's marginal return from investing in esteem. Given that, according to Proposition 3, b is increasing and ψ decreasing in $\underline{u}$, in theory, it is possible that investment in esteem is, in fact, lower in a tighter labour market.

7 Conclusion

The framework developed here can help formulate empirical tests regarding expenditure of firms on "motivation" and "worker satisfaction". Agency problems vary across sectors and the relationship between bonuses, motivation and effort likely to vary, for which our framework is useful. We also plan to further extend the theoretical analysis by varying the extent of observability of output, the importance of multi-tasking considerations, and possible differences between for-profit and non-profit firms in terms of their behaviour along the dimensions we study.

8 Appendix

We derive the expressions for $\frac{\partial \hat{e}}{\partial \varepsilon}, \frac{\partial \hat{e}}{\partial b}, \frac{\partial \hat{e}_c}{\partial b}, \frac{\partial \hat{e}_c}{\partial \psi}, \frac{\partial \hat{e}}{\partial \psi}$, and $\frac{\partial \varepsilon}{\partial \underline{u}}$ as follows.

Differentiating throughout (1) w.r.t. e_c, we obtain

$$\psi g''(..)\left(1 - \frac{\partial e}{\partial e_c}\right) = c''(e)\frac{\partial e}{\partial e_c} \tag{7}$$

$$\Longrightarrow \quad c''(e)\frac{\partial e}{\partial e_c} + \psi g''(..)\frac{\partial e}{\partial e_c} = \psi g''(..)$$

$$\Longrightarrow \quad \frac{\partial e}{\partial e_c} = \frac{\psi g''(..)}{c''(e) + \psi g''(..)} \in (0,1)$$

Differentiating throughout the first-order condition w.r.t. b, we obtain

$$1 + \psi g''(e_c - e)\left(-\frac{\partial e}{\partial b}\right) = c''(e)\frac{\partial e}{\partial b} \tag{8}$$

$$\Longrightarrow \quad \{c''(e) + \psi g''(e_c - e)\}\frac{\partial e}{\partial b} = 1$$

$$\Longrightarrow \quad \frac{\partial e}{\partial b} = \frac{1}{c''(e) + \psi g''(..)}$$

Differentiating throughout the first-order condition w.r.t. ψ, we obtain

$$g'(e_c - e) + \psi g''(e_c - e)\left(-\frac{\partial e}{\partial \psi}\right) = c''(e)\left(\frac{\partial e}{\partial \psi}\right)$$

$$\Longrightarrow \quad \{\psi g''(e_c - e) + c''(e)\}\left(\frac{\partial e}{\partial \psi}\right) = g'(e_c - e)$$

$$\Longrightarrow \quad \frac{\partial e}{\partial \psi} = \frac{g'(e_c - e)}{\psi g''(..) + c''(e)}$$

To derive expressions for $\frac{\partial e_c}{\partial b}, \frac{\partial e_c}{\partial \psi}$ and $\frac{\partial e_c}{\partial \underline{u}}$, we can start with the equation for the binding participation constraint:

$$b\hat{e}(b, e_c, \psi) - c(\hat{e}(b, e_c, \psi)) - \psi g(e_c - \hat{e}(b, e_c, \psi)) = \underline{u} \tag{9}$$

Differentiating throughout (9) w.r.t. b, we obtain

$$\hat{e}(b, e_c, \psi) + b\left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial b}\right) - c'(.)\left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial b}\right) - \psi g'(.)\left(\frac{\partial e_c}{\partial b} - \frac{\partial \hat{e}}{\partial b} - \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial b}\right) = 0$$

$$\Longrightarrow \hat{e}(b, e_c, \psi) + \{b + \psi g'(.) - c'(.)\}\left(\frac{\partial \hat{e}}{\partial b} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial b}\right) - \frac{\partial e_c}{\partial b}\psi g'(.) = 0$$

From the first-order condition, we obtain $b + \psi g'(.) - c'(.) = 0$. Therefore, the equation above simplifies to

$$\hat{e}(b, e_c, \psi) = \frac{\partial \hat{e}_c}{\partial b}\psi g'(.)$$

$$\Longrightarrow \frac{\partial \hat{e}_c}{\partial b} = \frac{\hat{e}(b, e_c, \psi)}{\psi g'(e_c - \hat{e}(b, e_c, \psi))} > 0$$

Differentiating throughout (9) w.r.t. ψ, we obtain

$$b\left(\frac{\partial \hat{e}}{\partial \psi} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \psi}\right) - c'(.)\left(\frac{\partial \hat{e}}{\partial \psi} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \psi}\right) - g(e_c - \hat{e}(b, e_c, \psi)) - \psi g'(.)\left(\frac{\partial \hat{e}_c}{\partial \psi} - \frac{\partial \hat{e}}{\partial \psi} - \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \psi}\right) = 0$$

$$\Longrightarrow \{b + \psi g'(.) - c'(.)\}\left(\frac{\partial \hat{e}}{\partial \psi} + \frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \psi}\right) - g(e_c - \hat{e}(b, e_c, \psi)) - \psi g'(.)\frac{\partial \hat{e}_c}{\partial \psi} = 0$$

$$\Longrightarrow \frac{\partial \hat{e}_c}{\partial \psi} = -\frac{g(e_c - \hat{e}(b, e_c, \psi))}{\psi g'(e_c - \hat{e}(b, e_c, \psi))} < 0$$

Differentiating throughout (9) w.r.t. $\underline{u}$, we obtain

$$\{b - c'(.) + \psi g'(.)\}\frac{\partial \hat{e}}{\partial e_c}\frac{\partial \hat{e}_c}{\partial \underline{u}} - \psi g'(.)\frac{\partial \hat{e}_c}{\partial \underline{u}} = 1$$

From the first-order condition, we obtain $b + \psi g'(.) - c'(.) = 0$. Therefore, the equation above simplifies to

$$\Longrightarrow \quad -\frac{\partial \hat{e}_c}{\partial \underline{u}}\psi g'(.) = 1$$

$$\Longrightarrow \quad \frac{\partial \hat{e}_c}{\partial \underline{u}} = -\frac{1}{\psi g'(.)}$$

Proof. of Lemma 1: Under the assumption that the functions $c(.)$ and $g(.)$ are quadratic, we can write $\alpha = g''(.)$ and $\kappa(\psi) = c''(.) + \psi g''(..)$.

Then, using the expressions derived for the first-order conditions in (4)-(6), we obtain

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial \psi \partial \sigma} &= \frac{\partial}{\partial \sigma}\left[\left\{\frac{A - b}{\kappa(\psi)}\right\}\left\{-\frac{\alpha g(..)}{g'(..)} + g'(..)\right\} - m'(\psi)\right] \\
&= \left\{\frac{A - b}{\kappa(\psi)}\right\}\frac{\partial}{\partial \sigma}\left\{-\frac{\alpha g(..)}{g'(..)} + g'(..)\right\} \\
&= \left\{\frac{A - b}{\kappa(\psi)}\right\}\left\{-\alpha\frac{g'(..)}{g'(..)} + \alpha\frac{g(..)\,g''(..)}{\{g'(..)\}^2} + g''(..)\right\}\frac{\partial}{\partial \sigma}\{e_c(b, \psi, \underline{u} - \sigma) - \hat{e}(b, e_c(..), \psi)\} \\
&= \alpha^2\left\{\frac{A - b}{\kappa(\psi)}\right\}\left\{\frac{g(..)}{\{g'(..)\}^2}\right\}\left(-\frac{\partial e_c}{\partial \underline{u}}\right)\left(1 - \frac{\partial \hat{e}}{\partial e_c}\right) \\
&> 0
\end{aligned}$$

since $\frac{A-b}{\kappa(\psi)} > 0$, $\alpha^2 > 0$, $g(.) > 0$, $g'(.) > 0$, $\frac{\partial \hat{e}_c}{\partial \underline{u}} < 0$ and $\left(1 - \frac{\partial \hat{e}}{\partial e_c}\right) > 0$.

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial \psi \partial b} &= \frac{\partial}{\partial b}\left[\left\{\frac{A-b}{\kappa(\psi)}\right\}\left\{-\frac{\alpha g(..)}{g'(..)} + g'(..)\right\} - m'(\psi)\right] \\
&= -\frac{1}{\kappa(\psi)}\left\{-\frac{\alpha g(..)}{g'(..)} + g'(..)\right\} \\
&\quad + \left\{\frac{A-b}{\kappa(\psi)}\right\}\left\{\alpha^2 \frac{g(..)}{\{g'(..)\}^2}\right\}\frac{\partial}{\partial b}\{\hat{e}_0(b) + \varepsilon(b,\psi,\underline{u}-\sigma) - \hat{e}(b,e_c,\psi)\} \\
&= -\frac{1}{\kappa(\psi)}\left\{-\frac{\alpha g(..)}{g'(..)} + g'(..)\right\} \\
&\quad + \left\{\frac{A-b}{\kappa(\psi)}\right\}\left\{\alpha^2 \frac{g(..)}{\{g'(..)\}^2}\right\}\left\{\frac{\hat{e}(..)}{\psi g'(..)} - \frac{1+\alpha\psi}{\kappa(\psi)}\right\}
\end{aligned}$$

Under the assumption $g''(..)\,g(..) < \{g'(..)\}^2$, the first term is negative. If $\frac{\hat{e}(..)}{g'(..)} < \frac{\psi+\alpha\psi^2}{\kappa(\psi)}$, the second term is negative. Therefore, if both conditions hold, we obtain $\frac{\partial^2 \pi}{\partial \psi \partial b} < 0$.

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial \sigma \partial b} &= \frac{\partial}{\partial b}\left[\frac{g''(..)(A-b)}{g'(..)\{c''(..) + \psi g''(..)\}} - h'(\sigma)\right] \\
&= \frac{\alpha}{\kappa(\psi)}\frac{\partial}{\partial b}\left\{\frac{(A-b)}{g'(..)}\right\} \\
&= \frac{\alpha}{\kappa(\psi)}\left[-\frac{1}{g'(..)} - \frac{(A-b)\,g''(..)}{\{g'(..)\}^2}\left(\frac{\partial \hat{e}_c}{\partial b}\right)\left(1 - \frac{\partial \hat{e}}{\partial e_c}\right)\right] \\
&= -\frac{1}{g'(..)}\frac{\alpha}{\kappa(\psi)}\left[1 + \frac{(A-b)\,\alpha}{g'(..)}\left(\frac{\partial \hat{e}_c}{\partial b}\right)\left(1 - \frac{\partial \hat{e}}{\partial e_c}\right)\right] \\
&< 0
\end{aligned}$$

Therefore, if $g''(..)\,g(..) < \{g'(..)\}^2$ and $\frac{\hat{e}(..)}{g'(..)} < \frac{\psi+\alpha\psi^2}{\kappa(\psi)}$, then the function $\pi(A, b, \psi, \sigma, \underline{u})$ is supermodular in ψ, σ, and $-b$. ■

Proof. of Proposition 2: We define

$$\tilde{\pi}(A, b, \psi, \sigma, \underline{u}; \lambda_\psi, \lambda_\sigma) = (A-b)\,\hat{e}(b, \hat{e}_c(b, \psi, \underline{u}-\sigma), \psi) - \lambda_\sigma h(\sigma) - \lambda_\psi m(\psi)$$

It is straightforward to establish that $\frac{\partial \tilde{\pi}}{\partial \lambda_\psi}, \frac{\partial \tilde{\pi}}{\partial \lambda_\sigma} < 0$, and $\frac{\partial^2 \tilde{\pi}}{\partial \psi \partial \lambda_\psi}, \frac{\partial^2 \tilde{\pi}}{\partial \sigma \partial \lambda_\sigma} < 0$ and that all other cross-partials involving λ_ψ and λ_σ are equal to zero. Then, using Topkis' theorem (Topkis 1998) and Lemma 1, we obtain the results in the statement of Proposition 2. ■

Proof. of Proposition 3: (ii) Recall that

$$\pi(A, b, \psi, \sigma, \underline{u}) = (A-b)\,\hat{e}(b, \hat{e}_c(b, \psi, \underline{u}-\sigma), \psi) - h(\sigma) - m(\psi)$$

For the following analysis, we define the function $\hat{\pi}(.)$ as follows:

$$\hat{\pi}(A, b, \psi, \hat{\sigma}, \underline{u}) = (A - b)\, \hat{e}(b, \hat{e}_c(b, \psi, -\hat{\sigma}), \psi) - h(\hat{\sigma} + \underline{u}) - m(\psi)$$

By construction, $\hat{\pi}(A, b, \psi, \hat{\sigma}, \underline{u}) = \pi(A, b, \psi, \sigma, \underline{u})$ at $\hat{\sigma} = \sigma - \underline{u}$. Therefore, the principal's optimsation problem in (3) is identical to

$$\max_{b, \hat{\sigma}, \psi} \hat{\pi}(A, b, \psi, \hat{\sigma}, \underline{u}) \tag{10}$$

for $\hat{\sigma} = \sigma - \underline{u}$. It is straightforward to show that $\frac{\partial^2 \hat{\pi}}{\partial \hat{\sigma} \partial \psi}, \frac{\partial^2 \hat{\pi}}{\partial \hat{\sigma} \partial b}$, and $\frac{\partial^2 \hat{\pi}}{\partial \psi \partial b}$ are identical to $\frac{\partial^2 \pi}{\partial \sigma \partial \psi}, \frac{\partial^2 \pi}{\partial \sigma \partial b}$, and $\frac{\partial^2 \pi}{\partial \psi \partial b}$ at $\hat{\sigma} = \sigma - \underline{u}$. Therefore, using Lemma 1, $\hat{\pi}(A, b, \psi, \hat{\sigma})$ is supermodular in ψ, $\hat{\sigma}$ and $-b$.

Furthermore, we can show that $\frac{\partial^2 \hat{\pi}}{\partial \underline{u} \partial \psi} = \frac{\partial^2 \hat{\pi}}{\partial \underline{u} \partial b} = 0$ and $\frac{\partial^2 \hat{\pi}}{\partial \underline{u} \partial \hat{\sigma}} = -h''(\hat{\sigma} + \underline{u}) < 0$. Then, using Topkis' theorem and the supermodularity of the function $\hat{\pi}(A, b, \psi, \hat{\sigma}, \underline{u})$ in ψ, $\hat{\sigma}$ and $-b$, we obtain the results in the statement of Proposition 3. ■

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