Adverse Selection in Distributive Politics*

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Abstract

Many policy reforms involve gains for some voters at a cost borne by others, and voters may not be well-informed about who gains and loses. This paper shows that the interaction of distributive politics and asymmetric information generates an *adverse selection effect*: when an uninformed voter contemplates many other voters supporting a policy, she concludes that she is unlikely to benefit from it. This force of suspicion may induce voters to reject policies that would be selected with high probability were all information public. We identify a form of "negative correlation" that is necessary and sufficient for this electoral failure.

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Contents

1 Introduction

Many economic policies affect both aggregate welfare and its distribution. Trade policy is a salient example, where the degree of trade liberalization impacts economic growth and the distribution of income so that the gains of some are inextricably intertwined with the losses of others.[1] Similarly, healthcare, immigration, and environmental policies have important aggregate and distributional consequences. Our paper studies how private information about the distributional consequences of economic policies can generate an adverse selection effect that induces voters to select policies that do not reflect their collective preferences and information. We interpret this strategic effect as formalizing "suspicion" whereby voters fear that support for a policy from others diminishes their chances of benefiting from it. We believe that adverse selection may be germane to distributive politics: voter suspicion has been a recurring theme of recent political discourse,[2] manifesting as distrust of the political and technocratic elite, conflicts between urban and rural voters, and a growing divide between the interests of low and high skill workers.

A starting point for our analysis is that information is often scarce. For example, most voters are unlikely to know how their real wages are affected by trade reforms.[3] Analogously, voters may find it difficult to obtain accurate information about how they are affected by immigration or healthcare reforms. This information problem is exacerbated by a contentious political debate on these polarizing issues and by voters' lack of exposure to different information sources.[4] In effect, voters often rely upon information that is biased, originates from interested parties, and is generally of poor quality. Only a small fraction of voters may be relatively well-informed about the outcomes being decided.

To illustrate how distributive politics generates an adverse selection effect when information is scarce, consider an electorate voting on a trade reform. This reform may open

[1] See Feenstra and Hanson (1999), Goldberg and Pavcnik (2007), Antras, de Gortari, and Itskhoki (2016), and Autor, Dorn, and Hanson (2016).

[2] For instance, President Obama connected inequality to suspicion in his farewell address: "But stark inequality is also corrosive to our democratic idea..... a recipe for more cynicism and polarization in our politics." (January 10, 2017).

[3] Trade economists themselves lack a consensus, as illustrated by the rich heterogeneity of predictions that come from trade models on how trade influences wages and employment. Some mechanisms focus on factor abundance and heterogeneity across industries whereas others highlight both within industry heterogeneity, and the differential impact across low and high wage workers. We thank Gordon Hanson for a helpful discussion on this point.

[4] The study of media markets highlights how the media may have a motive to bias information (e.g. Gentzkow and Shapiro, 2006), how such information provision may influence voting behavior (DellaVigna and Kaplan, 2007; Martin and Yurukoglu, 2017), and how each voter may consult only a limited number of information sources, thereby concentrating media power (Prat, 2017; Kennedy and Prat, 2017).

new markets for exporting firms but also lead to the outsourcing of economic activities in other sectors. Some voters may be privately informed about whether they gain or lose from this reform whereas others remain uninformed. Consider an uninformed voter, Alice, who ex ante perceives this reform to be beneficial. Given the collective choice procedure, Alice knows that the policy has a chance of passing only when others support it, and that others might support this reform only when they have received information that is good news for them. Plausibly, Alice may reflect upon the support of others for the reform as being bad news for her: since all regions and sectors do not benefit from trade, she may fear that their gains increase the likelihood that she may incur a loss of wages or work. Analogously, she may worry that support comes from a political and economic elite that is better positioned to capture the gains from trade. Contingencies such as these—when good news for others is bad news for Alice—are those that we model and define as *negative correlation*.

Our main results describe the implications of negative correlation. We study an election between a status quo and a policy reform where voters are uncertain about the number and identity of voters who gain and lose from the reform. Some voters obtain private information that pertains to this aggregate and distributional uncertainty whereas other voters remain uninformed. We study settings with a fixed electorate size as well as those in which the population size is uncertain. Our two main results are:

> Theorem 1. If payoffs are *negatively* correlated, there is a strict equilibrium that selects the ex ante inferior policy with high probability, which would be rejected if all information were public.
>
> Theorem 2. If payoffs are *not negatively* correlated, all equilibria select the ex ante efficient policy with high probability, which would be accepted if all information were public.

Negative correlation is therefore necessary and sufficient for electoral failures in this environment. Of particular interest to us is the voter reasoning that results in electoral failure when payoffs are negatively correlated. When a voter considers supporting a policy, she realizes that it is chosen only when it commands sufficiently high support from others. Knowing that others are self-interested, she fears that if she supports that policy when she is uninformed, she would find herself on the losing end of this deal. Thus, parallel to the logic of adverse selection in markets, the support of others "contaminates" a policy from a voter's perspective and induces her to reject it.

We illustrate our results with three examples in Section 1.1. Example 1 conveys the key intuitions with five voters whose votes are aggregated according to simple majority-rule. While we use a pivotal-voter model to frame our results, we believe that the logic of adverse selection applies for other forms of voter rationality. Example 2 offers a proof of concept that shows how similar behavior emerges in an ethical voter model (Coate and Conlin, 2004; Feddersen and Sandroni, 2006) with a continuum of voters. Broadly, we view our stylized model as formalizing how adverse selection undermines electoral behavior in the context of distributive politics. Example 3 uses a two-voter example with a unanimity rule to illustrate the connection to adverse selection forces and the no-trade theorem.

The condition of negative correlation pins down when this adverse-selection effect dominates voting behavior. Unlike the classic market settings, many policies are not zero-sum and so there is a countervailing force to adverse selection. For instance, when Alice learns (or infers) that others are winners, she may update positively because she infers that the policy creates many winners. Were there to be ex post transfers from winners to losers, this countervailing force would mitigate adverse selection from her perspective. Our condition of negative correlation formalizes when distributional uncertainty overtakes this countervailing force so that on net, Alice feels crowded out by the success of others.

We use the negative-correlation condition to compare policies by their susceptibility to adverse selection in a tractable sub-class of our model, where uncertainty is only about the number and identity of winners. First, we compare policies that involve greater polarization between winners and losers—measured as the ratio of the loss incurred by losers relative to the gains that accrue to winners—and show that such policies have greater potential to generate negatively correlated payoffs. We then formalize a measure of the degree to which a voter feels "crowded out" when she learns that others are winners. Formally, we order probability distributions over the number of winners by their potential to generate negatively correlated payoffs. We show that this order is complete and transitive and is represented using the ex interim expectation of the number of winners. Moreover, we show that this order is implied by the familiar likelihood ratio dominance order.

Finally, we show that the *kind* of information received by voters mitigates or exacerbates the issue of adverse selection. If all the information that is provided is of aggregate outcomes—say GDP or economic growth—without identifying distributional consequences, then payoffs are never negatively correlated. By contrast, if the information that is provided is purely distributional, then payoffs may be negatively correlated. This result complements analyses of information-providers and media outlets. For example, Perego and Yuksel (2017) find that competing news organizations may have a motive to provide polarizing information to voters rather than about common valence terms. Moreover, Prat (2017) and Kennedy and Prat (2017) describe how voters are exposed to only a narrow range of information sources. It is probable then that voter-groups are turning to information providers that describe how a policy change affects them rather than the electorate as a whole. Moreover, if different voter-groups are turning to different information providers, it is likely that voters are asymmetrically informed. Thus, our analysis indicates how the market for news may have detrimental consequences for subsequent voting behavior.

To summarize these comparative statics predictions, payoffs are more likely to be negatively correlated in a trade reform if (i) free trade exacerbates economic inequality; (ii) it is likely, from an ex ante perspective, that there will be a large number of both winners and losers; and (iii) the political debate in the build-up to the election focuses on the distributional rather than aggregate consequences of free trade. It appears to us that these ingredients are present both in trade policies and in other policy choices that have been at the forefront of recent elections.

1.1 Examples

We use three examples to illustrate the logic of adverse selection in distributive politics. For these examples, we abstract from *aggregate uncertainty* (about the number of policy beneficiaries) and assume that all uncertainty is *distributional*, pertaining to the identity of those beneficiaries. Our general framework exposited in Section 2 permits both aggregate and distributional uncertainty, allows for uncertainty about the size of the electorate, and encompasses a broader class of information structures.

Example 1 (Negative correlation)**.** Five voters choose between autarky and free trade using simple-majority rule. Each voter's payoff from autarky is normalized to 0. Relative to autarky, three voters are "winners" from free trade, each obtaining a gain of 1, and the other two are "losers," each obtaining a payoff of -1. Voters are exchangeable and so each permutation of winners and losers is ex ante equally likely.

If the identity of the winners were commonly known, every equilibrium in weakly undominated strategies would select free trade because each of the three winners would vote for it. At the other extreme, if it were commonly known that every voter is uninformed, free trade wins again: each voter expects to be a winner with probability $\frac{3}{5}$, yielding an ex ante expected gain from free trade of $\frac{1}{5}$. Thus, both with complete and no resolution of uncertainty, free trade defeats autarky in a majority-rule election.

Our interest is in settings where voters may privately learn how they fare under free trade. Suppose that a voter learns her payoff from free trade (becomes "informed") with probability $\lambda > 0$ and otherwise remains uninformed, and that this random process is independent across voters. Adhering to our motivation that information is scarce, we study equilibrium behavior when λ is small.

We construct a strict equilibrium in which autarky wins with high probability when voters have private information.[5] In this equilibrium, each voter who is informed votes for her preferred outcome—free trade for a winner and autarky for a loser—and all voters who remain uninformed vote for autarky. To see why this is a strict equilibrium, consider the incentives of an uninformed voter, Alice. Her vote influences her payoff only when it breaks a tie: of the other voters, exactly two vote for free trade. Because all uninformed voters are voting for autarky, Alice infers that the two voting for free trade must be informed winners. Since there can be only three winners, the odds that Alice is a winner drops in this contingency from the ex ante probability of $\frac{3}{5}$ to $\frac{1}{3-2\lambda}$, which is below $\frac{1}{2}$ so long as $\lambda < \frac{1}{2}$.[6] Thus, if information is scarce, Alice recognizes that a vote for free trade influences the outcome only when free trade is unfavorable to her, and consequently, votes in favor of autarky. As a result, autarky wins the election with a probability of at least $(1-\lambda)^3$, which is significant when λ is small.

The reason that an uninformed Alice votes for autarky, despite viewing free trade to be superior ex ante, is that she infers that her vote matters when others are supporting trade. In this contingency, she ascribes sufficiently high probability to being "crowded out" from the benefits of free trade herself that it is no longer attractive for her. It is this kind of adverse-selection reasoning that our model formalizes.

Another equilibrium also exists for this example, in which all uninformed voters vote for free trade. Conditional on being pivotal, a voter has an even stronger motive to support free trade. While this "good equilibrium" exists in this example, it does not always do so in our general model.[7] Even when it exists, we see here a potential instability introduced by distributional considerations—elections may succeed or fail depending on how voters expect others to behave—that contrasts with successful information aggregation results that apply across all equilibria (Feddersen and Pesendorfer, 1996, 1997).

This distinction is the core of our results: we show that when preferences across outcomes are *negatively correlated* across voters, and information is scarce, then there always exists an equilibrium in which this perverse outcome materializes (Theorem 1); if preferences are not negatively correlated, every equilibrium is guaranteed to succeed (Theorem 2).

[5]An equilibrium is strict if each voter has strict incentives to follow the prescribed strategies. Accordingly such equilibria are in weakly undominated strategies.

[6]It follows from a direct calculation that the probability that Alice is a winner conditional on being pivotal is $\frac{3(1-\lambda)^2}{3-2\lambda}\left(\frac{1}{3}\right) + \frac{4\lambda(1-\lambda)}{3-2\lambda}\left(\frac{1}{2}\right) + \frac{\lambda^2}{3-2\lambda}(1)$, where she is considering the conditional likelihood that that both, one, or neither of the other voters voting for autarky are uninformed.

[7]For certain cases, the perverse equilibrium is the unique strict equilibrium in symmetric strategies.

Example 2 (Ethical Voters)**.** The above example, and our general model, involves contingent reasoning based on being pivotal. We now consider an example, where the same force can influence behavior in an ethical voter framework where no voter anticipates being pivotal, but groups of voters nevertheless condition on this prospect.

A continuum of voters is divided into three equally sized groups— agriculture (A), manufacturing (M), and services (S)—and votes are aggregated using simple-majority rule. Members of each group obtain a payoff from autarky that is normalized to 0, but trade liberalization has differential effects depending on which group faces the threat of import substitution. There are three ex ante equally likely states of the world $\{\omega_A, \omega_M, \omega_S\}$, where ω_G denotes the state in which group G is threatened and the other groups benefit from trade. In that state, members of group G each obtain a payoff of $-3/2$, and members of the other groups obtain a payoff of 1. Each voter votes ethically in the sense of Coate and Conlin (2004) and Feddersen and Sandroni (2006): holding fixed the behavior of members of the other group, members of each group follow the rule that maximizes the payoffs of that group.

As in Example 1, free trade wins both under complete and no resolution of uncertainty (the ex ante payoff is 1/6). Now suppose that the groups are asymmetrically and privately informed. We assume that each group privately learns the true state of the world with probability λ and remains uninformed otherwise.[8] Now there exists a consistent ethical profile in which each group benefits from voting for autarky whenever uninformed: holding fixed this behavior, the likelihood that one's group is a winner conditioning on another group being in favor of the policy is $\frac{\lambda}{2\lambda-\lambda^2}$, which is less than $\frac{3}{5}$ whenever $\lambda < \frac{1}{3}$. At those interim odds, an uninformed group is better off with autarky, and hence, autarky wins with probability $1-\lambda^2$.

As in Example 1, each group supports autarky because it fears that if the group were to support free trade when uninformed, free trade wins in those contingencies where they are more likely to face import substitution. We see here that the optimal ethical rule involves contingent reasoning that generates an adverse selection effect, even though no individual voter conditions on her vote being pivotal.

Example 3 (Connection to No-Trade Theorems)**.** Our final example connects the elementary logic of the voting examples above to adverse selection in markets. Consider Alice and Bob who are voting on whether to adopt a safe action or a risky action. Each obtains a payoff of 0 from the safe action, but from the risky action, one voter obtains a payoff of -1 and the other obtains a payoff of 2; ex ante, each is equally likely to be

[8] Since voters in each group share common interests, we implicitly assume that information gained by a voter in a group G is freely shared with other members of her group.

the winner. The risky action is adopted if and only if Alice and Bob unanimously vote in favor of it.

Suppose that each voter learns the identity of the winner from the risky action with probability λ and is uninformed otherwise. Then, for every $\lambda > 0$, there exists a strict equilibrium in which each uninformed voter votes for the safe action.[9] In this strategy profile, Alice realizes that her choice to vote for the risky action changes the outcome only when Bob is voting for the risky action. Given this strategy profile, Bob is doing so only when he is the winner, and so an uninformed Alice would be supporting the policy only on those occasions where she is guaranteed to lose. Thus, her strict best-response is to vote for the safe action. The reasoning parallels that of adverse selection and the no-trade theorem (Akerlof, 1970; Milgrom and Stokey, 1982): the choice to trade with someone else matters when that person finds it in her interests to do so, which indicates that such trades may not be beneficial.

1.2 Related Literature

A vast literature studies distributive politics and special-interest politics from a range of different perspectives, and emphasizes how polarization and inequality may lead to political and economic failures.[10] Our central contribution to this literature is to illustrate how asymmetric information generates a powerful adverse selection effect on voting behavior when policies affect both aggregate welfare and its distribution.

An important precedent for our work is Fernandez and Rodrik (1991)'s argument for why electorates resist reform. They show that a policy that would win an election ex post—if all voters knew their payoffs from the policy—might fail ex ante. The wedge that they describe is decision-theoretic: the median voter is unwilling to bear the risks of policy reform even if she knows that a strict majority of voters benefit from such reform. Our result shows that asymmetric information amplifies and strengthens this effect: even if the median voter prefers the policy reform ex ante and is willing to bear the risk, she may not do so ex interim when she recognizes that the support of others for this policy diminishes her chance to be a beneficiary.

Our findings also resonate with the idea that voters fear losing control when the gains of others are not correlated with their own gains, and therefore insufficiently experiment with reforms. Strulovici (2010) illustrates this logic in a dynamic environment with public information. Our condition of negative correlation is reminiscent of his notion of

[9] For $\lambda > \frac{1}{2}$, this is the unique equilibrium.

[10] See Grossman and Helpman (2001), Acemoglu and Robinson (2005), Londregan (2006), Roemer (2006), and Ray and Esteban (2017) for recent surveys.

"adversity," where a voter anticipates that reforms are more likely to be chosen when she is a loser than when she is a winner. The conditions lack a precise connection because negative-correlation is a condition on primitives, not equilibria, whereas adversity is a condition on endogenous objects in an equilibrium. His and our results illustrate complementary mechanisms by which misaligned interim preferences generate electoral failures. In his model, the misalignment emerges dynamically when information is public whereas in ours, it emerges in a static environment with asymmetric information. Accordingly, his analysis speaks to status quo biases, and our analysis speaks to adverse selection when voters are affected unequally by policies. The adverse selection that we model generates status quo biases in some contexts and excessive demand for political reforms in others.[11]

Also related to our work is the literature dating back to the work of Olson (1965), Tullock (1983), and the "Virginia school" of political economy that connects inefficient redistributive schemes to asymmetric information. Some of the work in this vein highlights how small groups that have high stakes from the policy choice may be best informed and able to lobby policymakers. Esteban and Ray (2006) illustrate how asymmetric information introduces the potential for signal-jamming and informational inefficiency in this context. Other work in this literature, as exemplified by Coate and Morris (1995), studies issues of political agency where reputational considerations lead to inefficient decision making. We view our paper as complementing this literature in illustrating a different way in which asymmetric information impedes the performance of elections in a distributive context.

We compare equilibrium outcomes of elections in which voters are privately informed with a benchmark in which all information is public, as in the work pioneered by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996, 1997). Our main difference is that voters neither share common values nor have aligned ex interim preferences, and that payoffs may be negatively correlated. The negative result in Theorem 1 contrasts with the positive results obtained in settings where "good news" for some voters is necessarily "good news" for others (Feddersen and Pesendorfer, 1996, 1997).

Our paper is not the first to investigate how misalignments might cause informational inefficiency; see Kim and Fey (2007), Gul and Pesendorfer (2009), Bhattacharya (2013), and Acharya (2016). But our results emphasize a different set of intuitions than this prior literature. The logic of our paper is that of adverse selection: analogous to a no-trade

[11] In both cases, an uninformed voter may infer from others' support for a policy—independent of whether that policy is a reform or a status quo—that she should vote for the other policy. Accordingly, adverse selection provides a potential contributory explanation for both status quo biases and the demand for excessive reforms in highly unequal democracies, and highlights a thread that connects these apparently disparate behaviors.

theorem, an uninformed voter draws strong negative conclusions about the policy reform when she conditions on being pivotal and views the support of others as "contaminating" the policy. By contrast, the prior literature studies the confounded learning that occurs when types have opposing ordinal preferences, which makes the mapping from payoff-relevant states to vote-shares non-invertible.[12] We view confounded learning and adverse selection to be distinct patterns of behavior and bearing on different applications. In studying distributive politics, our analysis emphasizes the race between aggregate and distributional uncertainty, as well as aggregate and distributional information, and derives comparative statics on the degree to which voters feel "crowded out" by the support of others. Modeling adverse selection and all possible payoff correlations requires a departure from the standard model in which payoffs are conditionally i.i.d., and we capture these effects in a model that differs from those studied by the prior literature.

Our results apply both in cases where the size of the electorate is fixed or is uncertain. In the spirit of Myerson (1998, 2000), we consider a setting where the size of the electorate is the sum of a positive constant and a Poisson random variable. When the population size is uncertain, we find that a voter attributes high probability to the election being small when she conditions on being pivotal. Because of this property, our results involve assessing whether payoffs are negatively correlated only for the smallest possible electorate size. This property of believing that the election is small, conditional on being pivotal, is similar to the "participation curse" that Ekmekci and Lauermann (2016a,b) derive in pure common-value elections.

The literature has also revealed other reasons for informational inefficiency that are complementary to those we study, e.g., because voters wish to influence subsequent policy choices (Razin, 2003), information is costly (Martinelli, 2006), there is aggregate uncertainty about the distribution of preferences (Feddersen and Pesendorfer, 1997) or the precision of information (Mandler, 2012), or the policymaker cannot commit to a voting rule (Morgan and Stocken, 2008; Levit and Malenko, 2011; Battaglini, 2016).

2 Model

2.1 The Environment

Each of a finite population of voters, $\mathcal{N} = \{1, \ldots, n\}$, votes for one of two policies—a status quo policy Q and a reform R—and these votes are aggregated by a threshold-voting rule. The number of voters, n, is potentially uncertain and not necessarily commonly

[12] The conclusion is similar to confounded learning that occurs in herding models (Smith and Sørensen, 2000) when there are opposed types.

known by the voters. The reform R is implemented if at least τ fraction of the votes are cast for R, where $0 \leq \tau \leq 1$, and thus, in a population of size n, R is implemented if it receives at least $\lceil \tau n \rceil$ votes (where $\lceil \cdot \rceil$ is the ceiling function).

Each voter's (ex post) payoff from Q being selected is normalized to 0. The (ex post) payoffs from R being selected are uncertain: nature chooses a payoff profile v from $\mathcal{V}^n$, where v_i specifies voter i's payoff from R being chosen, and $\mathcal{V} \subseteq \mathbb{R}$ is a finite set of possible ex post payoffs. Before casting a vote, each voter i obtains private signal s_i that can convey information about the payoff profile, and is drawn from $\mathcal{S} \equiv \{s^0, s^1, ..., s^K\}$. The voting environment is therefore described by a probability distribution P on $\Omega = \{(n, v, s) : n \in \mathbb{N}, v \in \mathcal{V}^n, s \in \mathcal{S}^n\}$. We use capital letters to denote random variables on Ω, and lower-case letters to denote their realizations. For a state $\omega = (n, v, s) \in \Omega$, let $N(\omega) = n$, $S(\omega) = s$, $S_i(\omega) = s_i$, $S_{-i}(\omega) = s_{-i}$, $V(\omega) = v$, and $V_i(\omega) = v_i$ denote the random variables describing, respectively, the population size, signal profile, voter i's signal, the signal profile of voters other than i, the payoff profile, and voter i's payoff from R being chosen.[13] For a non-null event $E \subseteq \Omega$, $V_i(E) \equiv \sum_{\omega \in \Omega} V_i(\omega) P(\omega|E)$ denotes voter i's conditional expected payoff from R being chosen when she knows event E.

We impose four assumptions on the primitives of the model (Ω, P, τ). The first assumption is that voters are ex ante symmetric.

Assumption 1. *Voters are exchangeable:* $P(n, v, s) = P(n, \tilde{v}, \tilde{s})$ *for every permutation* $(\tilde{v}, \tilde{s})$ *of* (v, s).[14]

Our second assumption distinguishes signal s^0, which we describe as an *uninformative signal*, from the remaining signals $\mathcal{M} \equiv \mathcal{S} \backslash \{s^0\}$, which we describe as *informative*.

Assumption 2. *There is an uninformative signal, and informative signals are sufficient:*

(a) **Uninformative signal:** *For all* $(n, v, s) \in \Omega$ *with* $s_i = s^0$, $P(s_i) > 0$ *and* $P(n, v, s) = P(n, v, s_{-i}) P(s_i)$.

(b) **Informative signals:** *For all* $(n, v, s) \in \Omega$ *with* $s_i \neq s^0$, $V_i(n, s) > 0$ *if and only if* $V_i(s_i) > 0$.

Assumption 2(a) asserts that there is a strictly positive probability that each voter receives the "null" signal s^0, and that signal conveys no information about the population size, the payoff profile, and the signals received by other voters. Assumption 2(b) speaks

[13] Given any random variable X on Ω, we denote by $\{x\} \equiv \{\omega : X(\omega) = x\}$ the event where x is realized for X, omitting the brackets when it is clear that x is an event. In particular, s_i^k is the event $\{\omega : S_i(\omega) = s^k\}$.

[14] We say that $(\tilde{v}, \tilde{s})$ is a permutation of (v, s) if there is a one-to-one mapping $\psi : \mathcal{N} \to \mathcal{N}$ such that $(v_i, s_i) = (\tilde{v}_{\psi(i)}, \tilde{s}_{\psi(i)})$ for all $i \in \mathcal{N}$.

to the informativeness of the other signals, $\mathcal{M} \equiv \left\{s^1, ..., s^K\right\}$: if a voter obtains an informative signal, then her own information is a sufficient statistic for the entire signal profile in determining her ordinal ranking between Q and R. A special case of Assumption 2 is where each informed voter observes directly her payoff from R, as considered in the examples in Section 1.1 and some of the prior literature (Feddersen and Pesendorfer, 1996). More generally, Assumption 2(b) does not imply that informed voters observe or are all that well-informed (in an objective sense) about their payoffs from R, but simply that such individuals are well-informed *relative* to the electorate, insofar as learning others' signals does not flip their (interim) ordinal rankings of Q and R.[15]

We partition the set of informative signals, $\mathcal{M}$, into "good" and "bad" news. Signals $\mathcal{G} \equiv \left\{s^k \in \mathcal{M} : V_i\left(s^k\right) \geq 0\right\}$ convey *good-news* about the reform: a voter who receives signal $s_i \in \mathcal{G}$ expects that she will benefit from R being chosen. Likewise, signals $\mathcal{B} \equiv \left\{s^k \in \mathcal{M} : V_i\left(s^k\right) < 0\right\}$ convey *bad-news* about R being chosen and that the voter is better off with Q. For a signal profile s, $M(s)$ is the number of informed voters, and $G(s)$ is the number of voters who received good news.

Assumption 3. *Ex interim payoffs satisfy non-redundancy and no ties:*

(a) **Non-redundancy**: *If $P(n) > 0$, then $P\left(\frac{G}{n} \geq \tau | n\right) > 0$.*

(b) **No ties:** *If $P(E) > 0$, then $V_i(E) \neq 0$.*

Assumption 3(a) and (b) are bookkeeping assumptions that simplify our exposition without playing a substantive role. Assumption 3(a) guarantees that under public information it would always be possible for R to win; this assumption is not necessary for our results, but the environment would be uninteresting if it fails. Assumption 3(b) holds generically and allows us to avoid tie-breaking rules.

Our fourth assumption describes the uncertainty about the population size. For concreteness, we adopt a Poisson distribution, but all of our results apply to any distribution over the population size that has thinner tails than a Poisson distribution, including the notable case of distributions with finite support.[16] Recall that N denotes the random variable for population size.

Assumption 4. *We assume the following about the distribution of population size:*

[15] Indeed, our analysis in Proposition 7 corresponds to a case where no voter is objectively all that well-informed about her ex post payoffs.

[16] The Poisson-population model for voting was proposed by Myerson (1998, 2000), and features in a number of recent papers (Krishna and Morgan, 2011; Battaglini, 2016; Ekmekci and Lauermann, 2016a). Whenever there is uncertainty about the population size, a voter's *subjective perception* of the number of voters differs from the objective distribution. Appendix A.1.2 describes the formal properties of the marginal distribution over population size and the appropriate subjective perspective adopted by each voter that we use to establish our results.

(a) **Lowest Population Size**: *The random variable* N *takes a minimum value of* $n_0 \geq 2$, *where* $\lceil \tau n_0 \rceil < \lceil \tau(n_0 + 1) \rceil$.

(b) **Poisson Distribution**: $N - n_0$ *is distributed Poisson with mean* $\mu \in \mathbb{R}_+$.[17]

Assumption 4(a) guarantees that the minimal possible population size involves at least $n_0 \geq 2$ voters, which is necessary for our notion of payoffs being "negatively correlated" (Definition 1 on p. 15) to be well-defined. Furthermore, it assumes that the level of support needed for R to pass with n_0 voters is not the same as that needed to pass with n_0+1 voters, which simplifies exposition. Assumption 4(b) assumes that the population-size is the sum of n_0 and a Poisson random variable.

A Decomposition: We study electoral outcomes when information is scarce, i.e., each voter is uninformed with high probability. Assumptions 1 and 2(a) permit a decomposition of P into the probability that a voter obtains the uninformative signal s^0 and a probability distribution over all other primitives. This decomposition permits us to meaningfully vary the probability of being informed while holding everything else fixed.

We use λ to denote the probability that a voter obtains an informative signal.[18] The primitive probability distribution P can then be viewed as a member of a family indexed by $(\tilde{P}, \lambda)_{\lambda \in (0,1)}$, where, for every event $\{n, s\} \subseteq \Omega$, $\tilde{P}(s) = P(s)\lambda^{-n}$ if $s \in \mathcal{M}^n$, and $\tilde{P}(s) = 0$ otherwise. While λ parameterizes the probability that a voter is informed, $\tilde{P}$ is a joint distribution over $\tilde{\Omega} \equiv \{(n, v, s) \in \Omega : s \in \mathcal{M}^n\}$. The distribution P corresponds to the unique element of the family $(\tilde{P}, \lambda)_{\lambda \in (0,1)}$ where $\lambda = 1 - P(s_i^0)$. This decomposition is described and established in Appendix A.1.3.

2.2 Strategies and Equilibrium

We consider symmetric Bayes-Nash equilibria in which each voter plays weakly undominated strategies. Henceforth, we refer to these weakly undominated symmetric equilibria simply as *equilibria*.[19]

We contrast this setting with a *public information environment*, where the population size and the entire signal profile s is observed by each voter. The following result summarizes equilibrium existence in these two environments, and additional properties that hold in the public information environment.

[17] Hence, $P(n = n_0 + z) = \frac{\mu^z e^{-\mu}}{z!}$ for all $z \in \mathbb{N} \cup \{0\}$. We follow the convention that $0^0 \equiv 1$. Hence, when $\mu = 0$, the population size is n_0 with probability 1.

[18] This parameter is identical across voters (because of Assumption 1) and population sizes (because of Assumption 2(a))

[19] In such an equilibrium, the voting behavior of informed voters is straightforward: if voter i obtains good news ($s_i \in \mathcal{G}$), she votes for R; if she obtains bad news ($s_i \in \mathcal{B}$), she votes for Q.

Proposition 1. *The private information environment has an equilibrium. The public information environment has a unique equilibrium, which is strict and symmetric.*

2.3 Discussion of Modeling Assumptions

Below, we clarify some of the details of our model.

Status Quo and Reforms: The sense in which Q is a status quo is that the payoffs from that policy are normalized to 0, and it is relative to this normalization that the payoff from the reform R is assessed. While we find it convenient to frame our notation and analysis in these terms, our results themselves have no bearing on status quo or pro-reform biases (and all of our results apply *mutatis mutandi* with the notation reversed).

Ex Ante Symmetry: To develop a simple framework that illustrates how an adverse selection effect can generate electoral failures in the context of distributive politics, we have assumed that voters are ex ante symmetric. Because our results pertain to strict equilibria, slight asymmetries do not affect our results. A natural extension is to allow voters to belong to different groups, and vary both the likelihoods of benefiting from R and the likelihood of obtaining information across groups. While we expect that the exact conclusions shall differ, we anticipate that the qualitative insight of adverse selection persists so that uninformed members of one group vote against policies because they recognize that the support of others for that policy is bad news for them.

Information Structure: We study a stylized information structure where voters are either entirely uninformed or sufficiently well-informed (relative to others) that learning the entire signal profile does not change one's ordinal ranking. We comment on this information structure below.

Because we view information to be scarce in distributive politics, our goal is to focus on the behavior of relatively uninformed voters. It simplifies our analysis to model uninformed voters as being completely uninformed but it is not crucial to our results; because the equilibrium we study in Theorem 1 is strict, perturbing the model so that uninformed voters are slightly informed would not change our results.

We assume in Assumption 2(b) that informed voters learn the ordinal ranking from their individual signal that they would have from the entire signal profile. Such an assumption implies that informed voters have a weakly dominant action and need not consider contingencies when assessing how to vote. As such, the assumption permits us to maintain our focus on the behavior of uninformed voters. The literature (e.g.

Feddersen and Pesendorfer, 1996) often makes a similar (but stronger) assumption—namely that informed voters learn their cardinal payoffs from policies perfectly—for the same purpose; relative to that stronger assumption, the additional generality afforded by Assumption 2(b) is useful for our analysis of the differential impact of aggregate and distributional information.

3 Distributive Politics and Negative Correlation

This section describes our main results, which identify a form of negative correlation that is necessary and sufficient for privately informed voters to choose an outcome that differs from that which would be chosen if all information were public. The outcome also differs from one that voters prefer ex ante, when it is commonly known that everyone is uninformed. We begin by establishing these benchmarks, and then compare collective choice with asymmetric information against these benchmarks.

3.1 Benchmarks: Ex Ante Optimal and Public Information

To compare behavior against a simple public information benchmark, we assume throughout that R is the ex ante superior policy for every population size.

Assumption 5. *When a voter learns only the population size, then R is the superior policy: $V_i(n) > 0$ for all n.*

We now consider the public information benchmark. When information is scarce, the public information benchmark selects R with high probability in the unique equilibrium: whenever it is publicly observed that each voter is uninformed, voting R is a weakly dominant action.

Proposition 2. *For every $\varepsilon > 0$, there exists $\tilde{\lambda} > 0$ such that, for all $\lambda < \tilde{\lambda}$, R wins with probability exceeding $1-\varepsilon$ in the unique equilibrium of the public information environment.*

3.2 Negative Correlation Fosters Electoral Failures

We begin by describing an (interim) expected payoff that is important for our definition of negative correlation. Instead of defining negative correlation for each possible size of the electorate, it suffices for our results to do so only for the smallest possible size, n_0. For every κ in $[0, 1]$, let

$$V^G(\kappa) \equiv V_i(N = n_0, S_i = s^0, M = G = \lceil \kappa n_0 \rceil - 1). \tag{1}$$

This term is the expected payoff for voter i when she conditions on the population size being n_0, she receives the uninformative signal s^0, $\lceil \kappa n_0 \rceil - 1$ other voters receive good news, and all others receive no information. Because voters are exchangeable, a voter subscript is unnecessary for $V^G(\kappa)$. Assumption 3(a) guarantees that the conditioning event on which $V^G(\tau)$ is based has positive probability, and so $V^G(\kappa)$ is well-defined.

The threshold $\lceil \kappa n_0 \rceil - 1$ puts the proportion of voters who obtain good information to be just one vote shy of a κ fraction. Learning that so many voters obtain information does not change voter i's belief about R (because of Assumption 2(a)) but learning that they all observed good news is potentially informative. In particular, it conveys the good news that there are at least $\lceil \kappa n_0 \rceil - 1$ winners but also the bad news that of the realized number of winners, $\lceil \kappa n_0 \rceil - 1$ of those "slots" have been occupied by others. These two different features push in opposite directions—one bearing on aggregate considerations and the other bearing on distributional considerations—and evaluating whether the bad news effect dominates at the voting rule τ characterizes negative correlation.

Definition 1. *Payoffs are τ**-negatively correlated** if $V^G(\tau) < 0$.*

Being τ-negatively correlated implies that when an uninformed voter considers the prospect that the number of voters who have received good news is just one voter shy of a τ proportion, and only those voters receive information at all, her expected payoff from R would be strictly negative if the population size were n_0. In this contingency, learning that $\lceil \tau n_0 \rceil - 1$ voters are winners contaminates the reform R sufficiently from an uninformed voter's perspective that she would prefer Q in this contingency.

Formally, negative correlation is a property of the (interim) expected payoff generated by the joint distribution P but is independent of λ, the probability with which a voter obtains information. Thus, holding fixed a probability distribution on ex post profiles, whether expected payoffs are negatively correlated depends on the information structure, but not on the likelihood that a voter becomes informed.

Using this definition, we describe our main result.

Theorem 1. *Suppose payoffs are τ-negatively correlated. Then, for every $\varepsilon > 0$, there exists $\tilde{\lambda} > 0$ such that for all $\lambda < \tilde{\lambda}$, there exists a strict equilibrium in which Q wins with probability exceeding $1 - \varepsilon$.*

The result above highlights that when payoffs are τ-negatively correlated, Q wins with high probability if the probability with which an individual voter obtains information is low. This outcome contrasts with collective choice when information is public (where R is selected with high probability, as described in Proposition 2). The source of this electoral failure is that asymmetric information generates an adverse selection effect in distributive politics, and that effect can induce a voter to reject policies that she dislikes when supported by others.

We make a few notes about this equilibrium and result. Because the equilibrium is strict, it is robust to perturbations of the environment. For example, furnishing uninformed voters with a small amount of additional information—so that their ex interim beliefs diverge (slightly) from their prior beliefs —would not preclude the behavior described in Theorem 1. Moreover, it is straightforward to see that augmenting the voting game with the possibility for abstention does not eliminate this equilibrium; conditional on all others voting, it remains a strict equilibrium to also do so. Furthermore, since the size of the electorate may be arbitrarily large, this electoral failure is compatible with scenarios in which collectively, there are many informed voters.

Our result leaves room for the possibility of equilibria that are better for uninformed voters. We do not know whether such equilibria always exist: we discuss in Section 5 conditions under which they may exist, as well as conditions under which the equilibrium that we describe in Theorem 1 is the unique strict symmetric equilibrium. Regardless, the negative result contrasts with the positive result that we derive in Theorem 2 when payoffs are not negatively correlated; in that case, all equilibria are guaranteed to select R with high probability.[20]

The remainder of this subsection sketches the argument for Theorem 1 in words, with the formal proof in the Appendix. The argument is direct and it illustrates the role of adverse selection: an uninformed voter votes against R because she recognizes that it passes when it has the support of others. Moreover, the sketch illustrates why the result applies both to a fixed population size n_0 and a population size with uncertainty.

Sketch of the Proof: Our proof is constructive in that we specify a strategy profile, establish that it is a strict equilibrium, and show that it selects Q with high probability.

Step 1. Consider the strategy profile where each uninformed voter votes for Q, and each informed voter chooses her weakly dominant action. We claim that this strategy profile is a strict equilibrium when λ is sufficiently small.

[20]Theorem 1 also contrasts with the positive information aggregation results that apply across equilibria in Feddersen and Pesendorfer (1996, 1997). However, our choice of limits differs from this prior work. They fix a conditionally i.i.d. information structure and take the expected population size to infinity so that collectively, the electorate is almost perfectly informed. By contrast, we study environments in which information is scarce: we consider a finite expected population-size and consider how even slight degrees of asymmetric information can undermine elections in distributive politics.

To see why, suppose that an uninformed voter conditions on being pivotal. She recognizes that relative to the realized population size n, the fraction of votes for R is $\lceil \tau n \rceil - 1$. All of these votes must be from voters who are informed winners, whereas all remaining votes for Q may be from either uninformed voters or informed losers. An uninformed voter's best-response is unclear at this stage because we have not assumed that the voter knows the realized population size and Definition 1 references only conditional payoffs when the population size is n_0. So we tackle the issues introduced by population-uncertainty in two sub-steps.

(i) Suppose that it is commonly known that the population size is n_0. When information is scarce, an uninformed voter anticipates that most votes for Q are being cast by voters who are uninformed rather than informed losers. Conditioning on this event, her ex interim payoff from R approximates $V^G(\tau)$, which is strictly negative by Definition 1. Therefore, her strict best response is to vote for Q.

(ii) This sub-step proves that conditioning on being pivotal, an uninformed voter places high probability on the population size being n_0: given the strategy profile and scarcity of information, it is more likely that a voter is pivotal in a smaller election than in a larger election.[21] Therefore, the strict incentives to vote for Q described in (i) apply even when the voter is uncertain about the size of the electorate.

Step 2. When the probability of being informed is low, the fraction of voters who are uninformed is high. Because each uninformed voter votes for Q, it wins with high probability.

3.3 A Converse Result: Necessity of Negative Correlation

We show that τ-negative correlation is also necessary for the existence of an equilibrium that selects Q when information is scarce. Specifically, we establish that if payoffs are not τ-negatively correlated, then all equilibria select R with high probability.

Our argument requires an additional assumption on interim payoffs stated below.

Assumption 6. *$V^G(\cdot)$ satisfies positively connectedness: if $V^G(\kappa) > 0$ and $V^G(\kappa') > 0$ for $\kappa' > \kappa$, then $V^G(\kappa'') > 0$ for every κ'' such that $\kappa \leq \kappa'' \leq \kappa'$.*

This assumption equates to "single-crossing from above," where $V^G(\kappa)$ can cross 0 from above only once, and in combination with Assumption 5 (R is ex ante optimal),

[21] Albeit in a different model, Ekmekci and Lauermann (2016a) highlight how this is a general feature of a responsive equilibrium.

the above assumption implies that $V^G(\cdot)$ crosses 0 at most once, and from above. We view this condition to be intuitive: if the conditional likelihood favors R when there is a proportion κ of winners other than voter i, and when there is a proportion $\kappa' > \kappa$ of winners other than voter i, then voter i must also prefer R when there is an intermediate proportion of winners. Indeed, all of the examples in the paper satisfy this condition; however, Assumption 6 is not implied by our other assumptions.[22]

Using Assumption 6, we show that when payoffs are not τ-negatively correlated, every equilibrium of the private information environment coincides with the unique equilibrium of the public information environment with high probability.

Theorem 2. *Suppose Assumption 6 is satisfied. If payoffs are not τ-negatively correlated, then for all $\varepsilon > 0$, there exists $\tilde{\lambda} > 0$ such that for all $\lambda < \tilde{\lambda}$, R wins with probability at least $1 - \varepsilon$ in every equilibrium.*

The argument for Theorem 2 is involved because it applies across all equilibria, but its intuition connects once more to adverse selection. When payoffs are not τ-negatively correlated, we establish that the strategy profile considered in Theorem 1 is no longer an equilibrium: even if all of the votes for R are from those who obtain good news, that is not enough to make an uninformed voter prefer Q. Thus, the strategy profile where every uninformed voter votes for Q is not an equilibrium. The single-crossing condition (Assumption 6) ensures that when one has uninformed voters, in addition to informed winners, voting for R, these additional votes do not contaminate R and deter an uninformed voter from supporting R. This is then sufficient to ensure that there are no equilibria in mixed strategies where the status quo wins with probability exceeding ε when information is scarce.

In combining Theorems 1 and 2, we see that electoral outcomes depend starkly on whether payoffs are negatively correlated: in the former case, there exists an equilibrium in which Q is selected with high probability, whereas in the latter case, all equilibria select R with high probability. Therefore, it is important to investigate features of an economic environment that exacerbate or mitigate τ-negative correlation. We turn to this question in the next section.

4 When are Payoffs Negatively Correlated?

Our analysis so far has focused on identifying a condition in a distributional environment under which adverse selection causes collective choice to respond differently when infor-

[22]An example is available upon request.

mation is private than when it is public. The goal of this section is to understand when payoffs are negatively correlated.

We perform this exercise using a tractable sub-class of the model that decouples aggregate and distributional uncertainty. This sub-class allow us to identify three features of the environment that determine whether payoffs are negatively correlated: (i) *polarization ratios*, (ii) *crowding out*, and (iii) the *nature of information*.

This tractable sub-class has a simplified structure where the only two dimensions of a policy that are relevant are the number of winners and their identity. Suppose that the reform R generates *winners* and *losers*; each winner obtains $v_w > 0$ and each loser obtains $-v_l < 0$.[23] Uncertainty is about the number and identity of winners: the number of winners is denoted by the random variable η (*aggregate uncertainty*), and the identity of winners is determined by a random vector ρ (*distributional uncertainty*) where ρ_i denotes the priority of voter i in being a winner (a lower ρ_i indicating a higher priority). Voter i is a winner if and only if $\rho_i \leq \eta$, and so her payoff from the alternative depends on the realization of both aggregate and distribution uncertainty. We denote by W_i the event that voter i is a winner, and by L_i its complement. The information structure remains as in Section 2.

This model is a special case of Section 2, where P is now a joint distribution on the number of winners, the priority ranking, and voters' information, and we assume that Assumptions 1–3 are satisfied.[24] For expositional clarity, we assume that the population size is fixed at n_0 and commonly known (although this is unnecessary for the analysis).

4.1 Polarization Ratios

In this setting, the reform R is ex ante optimal when $P(W_i)v_w - (1 - P(W_i))v_l > 0$, where $P(W_i)$ is the ex ante probability that voter i is a winner.[25] This inequality can be re-written as

$$\frac{P\left(W_i\right)}{1 - P(W_i)} > \frac{v_l}{v_w}. \tag{2}$$

We describe the RHS of the above inequality as the *polarization ratio*, which specifies the cost incurred by each loser relative to the gain that accrues to each winner. The LHS is the ex ante likelihood ratio of being a winner.

[23]We treat each as an ex post payoff, but it may be an expected payoff conditional on being a winner or loser drawn independently from everything else in the game.

[24]Note that Assumption 1 does not imply that ρ and η are independent, and so we do not impose it for our general analysis of this setting.

[25]Necessarily, $P(W_i) = \sum_{\eta=1}^{n_0} \frac{P(\eta)\eta}{n}$.

The condition of τ-negative correlation compares the polarization ratio, not with the ex ante likelihood ratio, but with a conditional likelihood ratio. Let $P(W_i|\lceil \tau n_0 \rceil - 1)$ denote the probability that i is a winner conditional on herself being uninformed and knowing that there are exactly $\lceil \tau n_0 \rceil - 1$ informed voters, all of whom have good news. Payoffs are then τ-negatively correlated if and only if

$$\frac{P(W_i|\lceil \tau n_0 \rceil - 1)}{1 - P(W_i|\lceil \tau n_0 \rceil - 1)} < \frac{v_l}{v_w}. \tag{3}$$

Comparing (2) and (3), we can find payoffs such that (i) the reform is ex ante optimal, and (ii) payoffs are τ-negatively correlated if and only if

$$P(W_i|\lceil \tau n_0 \rceil - 1) < P(W_i). \tag{4}$$

In other words, for a voter i who receives no information, the prospect that exactly $\lceil \tau n_0 \rceil - 1$ other voters are informed and received good news diminishes her chances of being a winner relative to her prior belief.

It follows directly from (3) that increasing the polarization ratio makes a policy reform more susceptible to negative correlation (and hence a proof is omitted).

Proposition 3. *If (P, v_w, v_l) generates τ-negatively correlated payoffs, then for every (v_w', v_l') such that $\frac{v_l'}{v_w'} > \frac{v_l}{v_w}$, (P, v_w', v_l') also generates τ-negatively correlated payoffs.*

Figure 1 illustrates this perspective on τ-negative correlation. For a policy reform R with ex post payoffs of (v_w, v_l), its polarization ratio is the slope of the ray that connects the origin to (v_w, v_l). We compare this ray with two others, one of which has the slope corresponding to the LHS of (2) (the steeper ray) and the other has the slope corresponding to the LHS of (3) (the flatter ray). If the ray reflecting the ex post payoffs of the reform is sandwiched between these two rays, then R is both ex ante optimal and (expected) payoffs are τ-negatively correlated; this is the region of interest for our analysis in Section 3. If the polarization ratio is sufficiently low, then R is ex ante optimal, and payoffs are not τ-negatively correlated. If the polarization ratio is sufficiently high, then R is ex ante inferior and payoffs are τ-negatively correlated. As seen in the picture, taking policy reforms and increasing the gains of winners but with a higher disproportionate cost being borne by losers may induce payoffs that are τ-negatively correlated.

Were redistribution from winners to losers possible, such redistribution would necessarily lower the polarization ratio, and thereby make the policy choice less prone to negatively correlated payoffs. Of course, committing to redistribution is difficult for a political process, as illustrated by Acemoglu (2003), Jain and Mukand (2003), and Roessler,

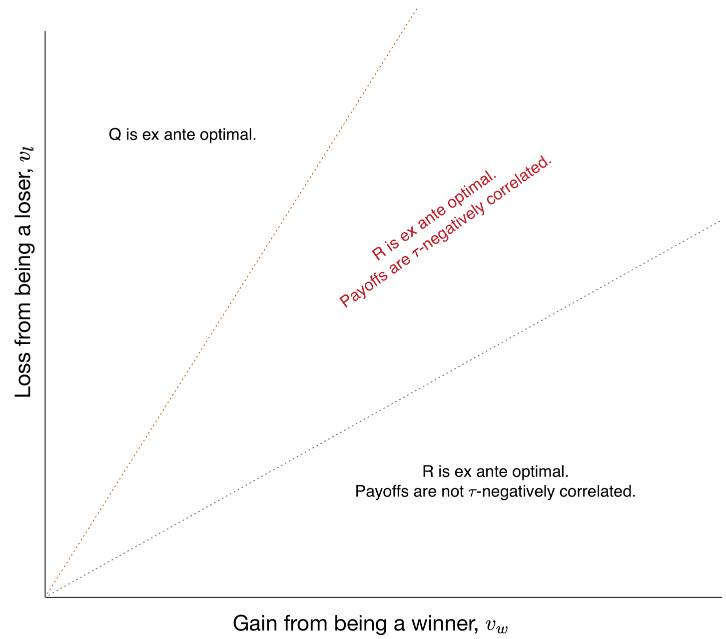

Figure 1: Polarization Ratios and Negative Correlation.

Shelegia, and Strulovici (2018).

4.2 Crowding Out: A Stochastic Order

The preceding discussion fixed the distribution P and compared polarization ratios. Here, we compare distributions to measure the degree of "crowding out" that a voter anticipates from learning that others are winners. Recall that the crowding-out effect relates to the degree to which the bad news of learning that other "winner-slots" have been occupied counteracts the good news that there are many winners. Below, we show that this intuition is precise: conditional expectations over the number of winners represents the appropriate measure of crowding out (from the perspective of τ-negative correlation) and probability distributions with a higher conditional expectation are less prone to τ-negative correlation. We also show that a stronger condition involves shifting the probability distribution on the number of winners that dominate in the sense of the likelihood-ratio dominance order; such shifts strengthen the good-news component of learning that others are winners.

In this analysis, we assume that an informed voter directly observes whether she is a winner or loser. We say that $P' \succeq_{\tau\text{-}nc} P$ if for every (v_w, v_l), whenever (P', v_w, v_l) generates τ-negatively correlated payoffs, so does (P, v_w, v_l). Accordingly, $\succeq_{\tau\text{-}nc}$ reflects "resistance to τ-negative correlation."

We show that $\succeq_{\tau\text{-}nc}$ is complete and transitive and obtain a representation. Recall that η is the random variable that denotes the number of winners from R.

Proposition 4. *The binary relation $\succeq_{\tau\text{-}nc}$ is complete and transitive. Moreover, $P' \succeq_{\tau\text{-}nc} P$ if and only if $E_{P'}(\eta | M = G = \lceil \tau n_0 \rceil - 1) \geq E_P(\eta | M = G = \lceil \tau n_0 \rceil - 1)$.*

Thus, distribution P is more susceptible to τ-negatively correlated payoffs than P' if upon conditioning on $\lceil \tau n_0 \rceil - 1$ voters obtaining good news and all others remaining uninformed, one expects fewer winners with a prior P than with a prior P'. We view this representation to intuitively capture the intuition that with P', learning that others are winners is better news than in P.

Our second result proves that $\succeq_{\tau\text{-}nc}$ is implied by the familiar likelihood ratio dominance order: say that $P' \succeq_{LR} P$ if for every z and z' in $\{0, \ldots, n_0\}$ where $z' > z$,

$$\frac{P'(\eta = z')}{P'(\eta = z)} \geq \frac{P(\eta = z')}{P(\eta = z)}. \tag{5}$$

We establish the following claim.

Proposition 5. *If $P' \succeq_{LR} P$, then $P' \succeq_{\tau\text{-}nc} P$ for every τ.*

Hence, shifts in the distribution over the number of winners that improve it in the sense of likelihood-ratio dominance are guaranteed to make collective choice more resistant to negative correlation (regardless of the voting rule).[26] The channel is straightforward: shifting mass to a higher number of winners (in an MLRP-way) reduces the crowding-out effect from learning that others are winners.

4.3 Aggregate Versus Distributional Information

Finally, we compare collective choice when voters obtain only aggregate information about the policy choices to collective choice when only distributional information is obtained. For example, contrast the following messages:

- *Free trade makes people on average better off.*
- *Corporate America gains from free trade before "ordinary Americans" see any gains.*

We view the former as describing implications for the aggregate number of winners, η, without offering information about distributional consequences, whereas the latter

[26] As our proof makes clear, all that is necessary is that the inequality in (5) holds for all values that exceed $\lceil \tau n_0 \rceil - 1$.

conveys information primarily about the priority ranking, ρ. Below, we show that the first kind of information does not foster negative correlation but the second does.

We first consider the case in which all information that voters obtain is potentially informative about aggregate ex post payoffs, but is completely uninformative about its distribution. Formally, for every signal profile s, the expected payoffs of any two voters conditioning on s are identical; in other words, information does not discriminate between voters. We show that in this case, (4) cannot be satisfied, leading to the following result.

Proposition 6. *If voters' signals are informative only about aggregate consequences, payoffs are never τ-negatively correlated.*

In this case, information that is good news for others—e.g., there are many winners—is also good news for an uninformed voter. Thus, even if opening trade barriers is polarizing, when voters do not obtain any information about the identities of winners and losers, then payoffs cannot be τ-negatively correlated for any voting rule τ.[27]

When all information is distributional and not about aggregate consequences, the opposite holds.

Proposition 7. *If informative signals reveal a voter's priority but are uninformative about the number of winners, payoffs are τ-negatively correlated for some (v_w, v_l).*

When information is about distributional and not aggregate consequences, then information that is good news for other voters—them having a higher priority—is bad news for oneself. In this case, the inequality in (4) is always satisfied, and so there are polarization ratios that ensure that payoffs are τ-negatively correlated.

We view these conclusions to be germane to discussions of politics and the media, particularly given the existing analysis of media providers and their motives to provide polarizing information (Gentzkow and Shapiro, 2006; Perego and Yuksel, 2017), and the degree to which news is segregated (Prat, 2017; Kennedy and Prat, 2017). Our analysis indicates that such media biases, which lead voters to focus on distributional information, can have important implications for collective choice.

5 Extensions

We briefly describe some additional results and intuitions.

[27]As seen in the proof, Proposition 6 applies to the general framework described in Section 2.

Other Equilibria: Our negative result proves the existence of a symmetric strict equilibrium in which the electorate selects the wrong policy when payoffs are negatively correlated; the contrast between this result and Theorem 2 (where no such equilibrium exists) illustrates the implications of adverse selection for distributive politics. In some cases, an alternative equilibrium exists (e.g., in Example 1), but in general, it may not. For instance, in Example 3, there are parameter ranges where the equilibrium that we study is unique.[28]

Continuity properties of stable equilibria indicate that a better equilibrium for uninformed voters exists when $\lambda \approx 0$. Consider the Bayesian game defined when $\lambda = 0$ in strategic form. There exists an equilibrium of this game in the hyperstable set (as defined by Kohlberg and Mertens 1986) in which all uninformed voters vote for R. Their results indicate then that a nearby equilibrium exists when $\lambda \to 0$.[29] However, we do not know whether this equilibrium always exists alongside the equilibrium that we construct in Theorem 1; it is possible that there are values of λ for which the equilibrium that we construct in Theorem 1 exists and this alternative equilibrium does not.

Under certain conditions, we show that the equilibrium that we construct in Theorem 1 is unique within the class of symmetric strict equilibria. Fix the population size of the electorate to be n_0, and analogous to our definition of $V^G(\kappa)$ in (1), define

$$V^B(\kappa) \equiv V_i(N = n_0, S_i = s^0, M = B = \lceil (1-\kappa)n_0 \rceil). \tag{6}$$

The expression describes an uninformed voter's expected payoff when she is uninformed, $n_0 - \lceil \kappa n_0 \rceil$ receive bad news, and all others are uninformed. We establish the following claim.

Proposition 8. *Suppose $V^B(\tau) < 0$. Then, for every $\varepsilon > 0$, there exists $\tilde{\lambda} > 0$ such that for all $\lambda < \tilde{\lambda}$, Q wins with probability exceeding $1 - \varepsilon$ in every strict equilibrium.*

In conjunction with Theorem 1, Proposition 8 implies that when payoffs are both τ-negatively correlated and $V^B(\tau) < 0$, not only is there a strict equilibrium in which Q wins, but this is the unique strict equilibrium. The condition that $V^B(\tau) < 0$ is

[28] The setting that we model is not a common interest problem, even for uninformed voters, and therefore, the arguments of McLennan (1998) do not immediately apply. Because voters have different ex post payoffs, symmetric strategy profiles that maximize ex ante payoffs need not be equilibria. For instance, in Example 3, the symmetric strategy profile that maximizes ex ante welfare is that in which all uninformed voters choose the risky action. However, this is not an equilibrium when λ exceeds $\frac{1}{2}$. It may be possible to extend the approach of McLennan (1998) to construct other equilibria in some cases, using the population-uncertainty extension of McLennan (1998) in Ekmekci and Lauermann (2016a). We thank Mehmet Ekmekci for a useful discussion on this point.

[29] The sequence of games generated as $\lambda \to 0$ can be defined as payoff perturbations of the game where $\lambda = 0$. We thank Navin Kartik for this argument.

compatible with payoffs being τ-negatively correlated; jointly, this uniqueness result holds when payoffs are τ-negatively correlated but not too negatively correlated.

Comparison of voting rules: We do not undertake a formal comparison of voting rules because under Assumption 6, it is trivial: if payoffs are τ-negatively correlated for voting rule τ, then they are also τ'-negatively correlated under voting rule $\tau' > \tau$. Thus, leaning the voting rule in favor of the ex ante optimal policy, R, mitigates the issues identified in Theorem 1.

Correlation of Information and Interests: We have assumed that a voter's information is independent of her interests. But voters often share the concerns that the political and economic elites support reforms because they are privy to their details. One may extend the results to a setting where, violating Assumption 2(a), only people who gain from R may obtain an informative signal and losers never obtain information at all. In this case, "no news" is actually "bad news" because it raises the likelihood that one is better off with Q. This form of polarization amplifies the potential for electoral failures because uninformed voters directly obtain a negative signal in addition to inferring that they may lose from policies that informed voters are supporting.

6 Conclusion

This paper develops a framework to investigate the impact of asymmetric information on distributive politics. Voters often have influence on distributive issues, either in the context of direct democracy or when selecting candidates who adopt different stances on these policies. We find that asymmetric information coupled with distributive politics generates an adverse selection effect that leads voters to reject policies that are ex ante preferable, and would have been preferred if all information were public.

This perverse outcome is driven by two considerations that we view to be endemic to distributive conflict: first, information is scarce, and so few voters are informed about how they are affected by policy reforms; second, payoffs are negatively correlated, so that conditioning on others obtaining good news about the policy reform, one becomes more pessimistic about one's own fate. The strategic force is that of "suspicion": an uninformed voter understands that other voters are self-interested and thus, policies that are likely to command near-majority support may not be in her interest. Our analysis shows that information about the distribution of welfare exacerbates suspicion, whereas information about purely aggregate welfare mitigates it.

Our work suggests a number of avenues for future investigation. It would be useful to understand both in the lab and field the degree to which voters update negatively based on the support that others express for a political policy. We have also abstracted from the source and diffusion of information as well as the design of policies. Since interested parties—politicians, lobbyists, activists, and elites—are often involved both in designing policies and promoting (or criticizing) them, the scope for adverse selection appears rife in democracies with highly unequal balances of political and economic power.

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A Appendix

Our appendix is divided into three parts. Appendix A.1 develops some preliminary notation and results. Appendix A.2 proves our two main results. Appendix A.3 proves our additional results on comparative statics and uniqueness.

A.1 Preliminaries

In this section, we develop some preliminary notation and results. Appendix A.1.1 develops some necessary notation. Appendix A.1.2 introduces notation for population uncertainty and proves a result that we use in Theorems 1 and 2. Appendix A.1.3 describes details of the decomposition result discussed at the end of Section 2.1. Appendices A.1.4–A.1.6 describe notation for strategies and equilibria, and prove our preliminary results on existence and the public information benchmark.

A.1.1 Some Notation

Let $\bar{v} \equiv \max\{|v_i| : v \in \mathcal{V}\})$ be the absolute value of the largest loss/gain from R. We use g and m to denote typical realizations of the random variables G and M, respectively, and Z (with typical realization z) to denote the random part of the population size. For any realization z, denote

$$\tau_z \equiv \lceil \tau(n_0 + z) \rceil - 1. \tag{7}$$

Note that if the electorate's size is $n_0 + z$, R wins if and only if it receives weakly more than $\tau_z + 1$ votes. Many of our results use the term τ_0, which is 1 vote fewer than the minimum necessary for R to pass when n_0 is the size of the population.

A.1.2 Population Uncertainty

The population size is $n_0 + Z$ where Z is distributed Poisson with mean $\mu \geq 0$. Since we establish Theorems 1 and 2 directly for the case where $\mu = 0$, suppose that $\mu > 0$.

We first derive an uninformed voter's subjective perception about the size of the population based on the event that she is recruited. Our approach follows Myerson (1998) where we suppose that there is a finite population of $\bar{N} \geq n_0$ potential voters, each voter is equally likely to be recruited, and we take $\bar{N} \to \infty$.

Denote by R_i^k the event that voter i is recruited and she has signal s_i^k. Our analysis focuses on the event R_i^0, namely where voter i obtains signal s_i^0 (i.e., is uninformed), and conditioning on this event, a voter's belief that there are m informed voters in a population of size $n_0 + z$. It follows from standard calculations[30] that taking the limit $\bar{N} \to \infty$ yields

$$\begin{aligned} P(M = m, Z = z|R_i^0) &= \left(\frac{n_0+z}{n_0+\mu}\right) P(m|N=n_0+z-1)P(z) \\ &= \left(\frac{n_0 + z}{n_0 + \mu}\right) \binom{n_0+z-1}{m} \lambda^m (1-\lambda)^{n_0+z-1-m} P(z), \end{aligned} \tag{8}$$

where the second equality substitutes the definition of $P(m|N = n_0 + z - 1)$. Abusing notation, we write the LHS as $P(m, z|R_i^0)$, and we prove a result that bounds a series of sums that we use in the proofs of Theorems 1 and 2.

Lemma 1. *Let n_0 be a strictly positive integer and Z be a random variable that has full support on the non-negative integers, mean $\mu \geq 0$, and satisfies $\lim_{z\to\infty} \frac{P(z+1)}{P(z)} = k$ for some $k \in [0, \frac{1}{4})$. Then there is $\bar{L}$ such that, for all $\lambda \in (0,1)$ and positive integer $\bar{m}$, $\sum_{z=0}^{\infty} \sum_{m=\bar{m}}^{n_0+z} P(m, z|R_i^0) \leq \sum_{z=0}^{\infty} \sum_{m=\bar{m}}^{n_0+z} \binom{n_0+z}{\lceil 0.5(n_0+z)\rceil} P(m, z|R_i^0) \leq \lambda^{\bar{m}} \bar{L}$.*

Proof. Since $\binom{n_0+z}{\lceil 0.5(n_0+z)\rceil} \geq 1$ for all z, it follows by (8) that

$$\begin{aligned} \sum_{z=0}^{\infty} \sum_{m=\bar{m}}^{n_0+z} P(m, z|R_i^0) &\leq \sum_{z=0}^{\infty} \sum_{m=\bar{m}}^{n_0+z} \binom{n_0 + z}{\lceil 0.5(n_0 + z)\rceil} P(m, z|R_i^0) \\ &= \sum_{z=0}^{\infty} \sum_{m=\bar{m}}^{n_0+z-1} \binom{n_0+z}{\lceil 0.5(n_0+z)\rceil} \frac{n_0+z}{n_0+\mu} \binom{n_0+z-1}{m} \lambda^m (1-\lambda)^{n_0+z-1-m} P(z) \\ &\leq \lambda^{\bar{m}} \sum_{z=0}^{\infty} \sum_{m=\bar{m}}^{n_0+z} \frac{n_0+z}{n_0+\mu} \binom{n_0 + z}{\lceil 0.5(n_0 + z)\rceil}^2 P(z) \\ &\leq \lambda^{\bar{m}} \sum_{z=0}^{\infty} \frac{(n_0+z)^2}{n_0+\mu} \binom{n_0 + z}{\lceil 0.5(n_0 + z)\rceil}^2 P(z), \end{aligned}$$

where the second inequality follows because all terms are non-negative, $\lambda^m \leq \lambda^{\bar{m}}$ when $m \geq \bar{m}$, $(1-\lambda) \leq 1$, and $\binom{n_0+z-1}{m} \leq \binom{n_0+z}{\lceil 0.5(n_0+z)\rceil}$ for all $m \in \{\bar{m}, ..., n_0 + z - 1\}$.

[30] We suppress the calculations for expositional brevity. Details are available upon request.

Let $a(z) \equiv \frac{(n_0+z)^2}{n_0+\mu}\binom{n_0+z}{\lceil 0.5(n_0+z)\rceil}^2$. By the ratio test for series, and the tail property of Z, we establish the result if we show that $\lim_{z\to\infty}\frac{a(z+1)}{a(z)} < 4$. For this, we consider two cases. First, suppose n_0+z is odd. Then, $\frac{a(z+1)}{a(z)} = \left(\frac{n_0+z+1}{n_0+z}\right)^2\left(\frac{n_0+z+1}{0.5(n_0+z+1)}\right)^2$, which converges to 4. Now, suppose n_0+z is even. Then, $\frac{a(z+1)}{a(z)} = \left(\frac{n_0+z+1}{n_0+z}\right)^2\left(\frac{n_0+z+1}{0.5(n_0+z+2)}\right)^2$, which also converges to 4. Hence, by the tail property of Z, $\frac{a(z+1)P(z+1)}{a(z)P(z)}$ converges to a limit strictly less than 1, and so the the series $\sum_{z=0}^{\infty} a(z)P(z)$ converges to some $\bar{L}$. □

Remark 1. Because $\lim_{z\to\infty}\frac{P(z+1)}{P(z)} = 0$ when Z follows a Poisson distribution, Lemma 1 ensures our results hold under Assumption 4. Our proofs in Theorems 1 and 2 use only the result established in Lemma 1, and so the theorems apply to distributions with thinner tails as well as fatter tails so long as the series $\sum_{z=0}^{\infty} a(z)P(z)$ in Lemma 1 converges. Since Lemma 1 holds for any distribution with a finite support, our theorems also apply in those cases.

A.1.3 Decomposition

We prove the decomposition result discussed at the end of Section 2.1. Consider a distribution P satisfying Assumptions 1 and 2(a). Let $\lambda = 1 - P(s_i^0)$ and, for all $\{n, s\} \subset \Omega$, let $\tilde{P}(s) = P(s)\lambda^{-n}$ if $s \in \mathcal{M}^n$ for some n and $\tilde{P}(s) = 0$ otherwise. The following claims show that $\tilde{P}$ is a probability distribution on $\tilde{\Omega}$, and that for every $\lambda' \in (0,1)$ there exists a unique P' in the family $(\tilde{P}, \lambda)$ satisfying Assumptions 1 and 2.

Claim 1. *$\tilde{P}$ is a probability distribution.*

Proof. By definition, $\tilde{P}$ assigns non-negative weight to each element in $\tilde{\Omega}$. We show that $\sum_{\omega\in\tilde{\Omega}} \tilde{P}(\omega) = 1$. It suffices to show that $\sum_{\{\omega\in\tilde{\Omega}:N(\omega)=n\}} \tilde{P}(\omega|n) = 1$ for every n. Fix some n. It follows by integration on $\mathcal{S}^n$ that $\sum_{s\in\mathcal{M}^n}\tilde{P}(s|n) = \lambda^{-n}\sum_{s\in\mathcal{M}^n} P(s|n)$, and so it suffices to establish that $\sum_{s\in\mathcal{M}^n} P(s|n) = \lambda^n$.

Fix a positive integer q that is strictly less than n, and consider an event $\{n, s_1, \ldots, s_q\} \equiv \{n, s^q\}$ where $s_i \in \mathcal{M}$ for every $i = 1, \ldots, q$. Then, by Assumption 3(a),

$$\begin{aligned} P(s^q|n) &= P(s^q|n)P(s_{q+1}^0|n) + \sum_{s_{q+1}\in\mathcal{M}} P(s^q, s_{q+1}|n) \\ &= P(s^q|n)(1-\lambda) + \sum_{s_{q+1}\in\mathcal{M}} P(s^q, s_{q+1}|n) = \frac{1}{\lambda}\sum_{s_{q+1}\in\mathcal{M}} P(s^q, s_{q+1}|n). \end{aligned} \tag{9}$$

Proceeding by induction, $P(s^q|n) = \left(\frac{1}{\lambda}\right)^{n-q}\sum_{j=q+1}^{n}\ \sum_{s_j\in\mathcal{M}} P(s_1, \ldots, s_n|n)$. Substituting $q = 1$, and adding across all $s_1 \in \mathcal{M}$ yields

$$\sum_{s_1 \in \mathcal{M}} P(s_1|n) = \sum_{s_1 \in \mathcal{M}} \left(\frac{1}{\lambda}\right)^{n-1} \sum_{j=2}^{n} \sum_{s_j \in \mathcal{M}} P(s_1, \dots, s_n|n) = \left(\frac{1}{\lambda}\right)^{n-1} \sum_{s \in \mathcal{M}^n} P(s|n). \tag{10}$$

By the same reasoning leading to (9), $P(s_2, \dots, s_n|n) = \frac{1}{\lambda} \sum_{s_1 \in \mathcal{M}} P(s_1, s_2, \dots, s_n|n)$ for each $(s_2, \dots, s_n) \in \mathcal{S}^{n-1}$. As $\sum_{(s_2,\dots,s_n) \in \mathcal{S}^{n-1}} P(s_2, \dots, s_n|n) = 1$, it follows that

$$1 = \sum_{(s_2,\dots,s_n) \in \mathcal{S}^{n-1}} \frac{1}{\lambda} \sum_{s_1 \in \mathcal{M}} P(s_1, s_2, \dots, s_n|n) = \frac{1}{\lambda} \sum_{s_1 \in \mathcal{M}} P(s_1|n),$$

implying that $\sum_{s_1 \in \mathcal{M}} P(s_1|n) = \lambda$. Using (10), we conclude that $\sum_{s \in \mathcal{M}^n} P(s|n) = \lambda^n$. □

The following claim shows that, for each $\lambda' \in (0, 1)$, there exists a unique distribution P' in the family $(\tilde{P}, \lambda)_{\lambda \in (0,1)}$ that satisfies Assumptions 1–4. For each n and $s \in \mathcal{S}^n$, let $\tilde{E}(n, s) = \{\tilde{s} \in \mathcal{M}^n : \tilde{s}_j = s_j \text{ whenever } s_j \neq s^0\}$.

Claim 2. *For each $\lambda' \in (0, 1)$, there exists a unique distribution P' in the family $(\tilde{P}, \lambda)_{\lambda \in (0,1)}$ that satisfies Assumptions 1 and 2. In particular, for any $\omega = (n, v, s) \in \Omega$,*

$$P'(\omega) = \lambda^{M(\omega)} (1 - \lambda)^{n - M(\omega)} \sum_{s' \in \tilde{E}(n,s)} \tilde{P}(n, v, s'). \tag{11}$$

Proof. We fix n and consider $\omega \in \Omega$ such that $N(\omega) = n$, proceeding by induction. First, suppose $M(\omega) = n$. Then (11) follows from our construction in the text. Now suppose (11) is true for any ω where $M(\omega) = m + 1$ for some $m < n$. We establish below that this is true for any ω' where $M(\omega') = m$.

Consider any $\omega' = (n, v, s)$ where $M(\omega') = m$, and suppose $s_i = s^0$. Observe that,

$$\begin{aligned} P(v, s_{-i}|n) &= \sum_{s'_i \in \mathcal{S}} P(v, s_{-i}, s'_i|n) = P(v, s_{-i}|n) P(s'_i = s^0|n) + \sum_{s'_i \in \mathcal{M}} P(v, s_{-i}, s'_i|n) \\ &= P(v, s_{-i}|n)(1 - \lambda) + \sum_{s'_i \in \mathcal{M}} P(v, s_{-i}, s'_i|n) = \frac{1}{\lambda} \sum_{s'_i \in \mathcal{M}} P(v, s_{-i}, s'_i|n). \end{aligned} \tag{12}$$

where the first equality is by definition, the second equality follows from Assumption 3(a), the third equality follows from $P(s'_i = s^0|n) = (1 - \lambda)$, and the fourth equality follows from simplification. Using Assumption 3(a), $s_i = s^0$, and $P(s_i|n) = (1 - \lambda)$, it follows that $P(\omega'|n) = (1-\lambda)P(v, s_{-i}|n)$. Substituting (12) for $P(v, s_{-i}|n)$ yields

$$P(v, s|n) = \left(\frac{1-\lambda}{\lambda}\right) \sum_{s_i' \in \mathcal{M}} P(v, s_{-i}, s_i'|n) = \lambda^{M(\omega')} (1-\lambda)^{n-M(\omega')} \sum_{s' \in \tilde{E}(n,s)} \tilde{P}(v, s'|n),$$

where the second equality follows from the induction hypothesis and simplification. □

A.1.4 Strategies and equilibrium

Let $\Sigma \equiv \{\sigma : \mathcal{S} \to [0,1]\}$ be the set of mappings from signals to $[0,1]$. In the private information environment, a strategy for voter i can be described by $\sigma_i \in \Sigma$, where $\sigma_i(s_i)$ is the probability that i votes for R when receiving signal s_i. With some abuse of notation, a symmetric strategy profile can also be described in terms of an element $\sigma \in \Sigma$; in particular, we denote by (σ_i, σ_{-i}) a strategy profile where voter i follows strategy $\sigma_i \in \Sigma$, and other voters follow the symmetric strategy $\sigma_{-i} \in \Sigma$.

Let $P_\sigma(\tau_{N(\omega)} + 1|\omega)$ denote the probability that at least $\lceil \tau N(\omega) \rceil$ voters vote for R when voters follow the symmetric strategy profile σ and state ω is realized. Then voter i's expected payoff function for the symmetric strategy profile σ and signal s_i^k is defined by $\pi_i(\sigma|s_i^k) \equiv \sum_{\omega \in \Omega} P_\sigma(\tau_{N(\omega)} + 1|\omega) V_i(\omega) P(\omega|R_i^k)$.

Definition 2 (Equilibrium)**.** *A symmetric strategy profile σ is an equilibrium if the following conditions are satisfied for all n and $i \in \{1, ..., n\}$:*

(i) $\pi_i(\sigma|.) \geq \pi_i(\sigma_i', \sigma_{-i}|.)$ for all $\sigma_i' \in \Sigma$,

(ii) for each $s_i \in \mathcal{S}$: if there exists σ_i' such that $\pi_i(\sigma_i', \tilde{\sigma}_{-i}|s_i) \geq \pi_i(\tilde{\sigma}|s_i)$ for all $\tilde{\sigma}$, then $\pi_i(\sigma_i, \tilde{\sigma}_{-i}|s_i) \geq \pi_i(\sigma_i', \tilde{\sigma}_{-i}|s_i)$ for all $\tilde{\sigma}$.

The equilibrium is strict *if the inequality in part (i) is strict for all $s_i \in \mathcal{S}$.*

Condition (i) is the standard requirement of a Bayes Nash equilibrium (BNE): for all signals, voters play a best response to the strategies of other voters. Condition (ii) states that voters play a weakly undominated strategy. In the public information environment, strategies, payoffs, and equilibrium are defined analogously, except that voters condition on the population size n and signal profile $s \in \mathcal{S}^n$.

A.1.5 Equilibrium characterization

We first provide a simple characterization of equilibrium in the private information environment. For $\alpha \in [0,1]$, define the symmetric strategy profile σ^α as follows:

$$\sigma_i^\alpha(s_i) \equiv \begin{cases} \alpha & \text{if } s_i = s^0 \\ 1 & \text{if } s_i \in \mathcal{G} \\ 0 & \text{if } s_i \in \mathcal{B} \end{cases} .$$

Define the function $\Pi : [0,1] \to \mathbb{R}$ as follows:

$$\begin{aligned} \Pi(\alpha) &\equiv \pi_i(\sigma_i^1, \sigma_{-i}^\alpha | s_i^0) - \pi_i(\sigma_i^0, \sigma_{-i}^\alpha | s_i^0) \\ &= \sum_{z=0}^{\infty} \sum_{m=0}^{n_0+z-1} \sum_{g=0}^{m} V(g,m,z) P_\alpha(piv|g,m,z) P(g|s_i^0, m, z) P(m, z|R_i^0), \end{aligned}$$

where $V(g,m,z) \equiv V_i(s_i^0, g, m, n_0+z)$ is a voter's expected payoff conditional on receiving the uninformative signal, g voters receiving good news, m voters being informed in a population of size $n_0 + z$ (by Assumption 2, this does not need a voter subscript); $P_\alpha(piv|g,m,z)$ is the probability that the voter is pivotal given $(G,M,Z) = (g,m,z)$ and that other voters are following the strategy profile σ_{-i}^α, which is

$$P_\alpha(piv|g,m,z) \equiv \begin{cases} f(\alpha,g,m,z) & \text{if } \alpha \in (0,1)\ ,\ 0 \le \tau_z - g \le n_0 + z - 1 - m \\ 1 & \text{if } \alpha = 0\ ,\ \tau_z - g = 0 \\ & \text{or } \alpha = 1\ ,\ \tau_z - g = n_0 + z - 1 - m \\ 0 & \text{otherwise} \end{cases} ,$$

where, for $\alpha \in (0,1)$ and $0 \le \tau_z - g \le n_0 + z - 1 - m$,

$$f(\alpha,g,m,z) = \binom{n_0 + z - 1 - m}{\tau_z - g} \alpha^{\tau_z - g} (1-\alpha)^{n_0+z-1-(m-g)-\tau_z};$$

$P(g|s_i^0, m, z)$ is the probability for an uninformed voter that g voters receive good news conditional on m voters being informed in a population of size $n_0 + z$; $P(m, z|R_i^0)$ is the probability that there are $n_0 + z - 1$ other voters, $m \le n_0 + z - 1$ of whom are informed, conditional on voter i being recruited as an uninformed type (see Equation 8), and $P(z) \equiv P(Z = z)$ is the probability that the population is of size $n_0 + z$. Hence, $\Pi(\alpha)$ is the difference in the expected payoff for an uninformed voter when they vote for R versus voting for Q, conditional on other voters following the strategy profile σ_{-i}^α, because the payoff-difference is non-zero only when the voter is pivotal.

Lemma 2. *The function* Π *is well-defined and continuous on* $[0,1]$.

Proof. Define the sequence of functions $\{w_z : [0,1] \to \mathbb{R}\}$ by

$$w_z(\alpha) = \sum_{m=0}^{n_0+z-1} \sum_{g=0}^{m} V(g,m,z) P_\alpha(piv|g,m,z) P(g|s_i^0, m, z) P(m,z|R_i^0),$$

for all $\alpha \in [0,1]$, so that $\Pi(\alpha) = \sum_{z=0}^{\infty} w_z(\alpha)$. Fix some $z \in \mathbb{N} \cup \{0\}$. The argument α enters w_z only in the term $P_\alpha(piv|g,m,z)$. The function $f(\alpha|m,g)$, used to define $P_\alpha(piv|g,m,z)$, is continuous on $(0,1)$ for all (g,m,z) such that $0 \leq \tau_z - g \leq n_0 + z - 1 - m$. In addition,

$$\lim_{\alpha \to 0} f(\alpha|g,m,z) = \begin{cases} 1 & \text{if } \tau_z - g = 0 \\ 0 & \text{otherwise} \end{cases}, \quad \text{and}$$

$$\lim_{\alpha \to 1} f(\alpha|g,m,z) = \begin{cases} 1 & \text{if } \tau_z - g = n_0 + z - 1 - m \\ 0 & \text{otherwise} \end{cases}.$$

Hence, $P_\alpha(piv|g,m,z)$ is continuous in α on $[0,1]$, and so $w_z : [0,1] \to \mathbb{R}$ is continuous. Moreover, for all (g,m,z) and α, $|w_z(\alpha)| \leq \bar{v} P(z)$. Since $\sum_{z=0}^{\infty} \bar{v} P(z) = \bar{v} \sum_{z=0}^{\infty} P(z) = \bar{v}$, it follows by the Weierstrass M-test that the series $\sum_{z=0}^{\infty} w_z(\alpha)$ converges absolutely and uniformly, and so $\Pi(\alpha)$ is well-defined. Since each of the functions w_z are continuous on $[0,1]$, it then follows by the uniform limit theorem that $\Pi(\alpha)$ is continuous on $[0,1]$. $\square$

Lemma 3. *Strategy profile* σ^* *is an equilibrium if and only if* $\sigma^* = \sigma^\alpha$ *for some* $\alpha \in [0,1]$ *and one of the following three conditions is satisfied: (i)* $\alpha = 1$ *and* $\Pi(\alpha) \geq 0$*, (ii)* $\alpha = 0$ *and* $\Pi(\alpha) \leq 0$*, or (iii)* $\alpha \in (0,1)$ *and* $\Pi(\alpha) = 0$*. Moreover,* σ^α *is a strict equilibrium if and only if either (i')* $\alpha = 1$ *and* $\Pi(\alpha) > 0$*, or (ii')* $\alpha = 0$ *and* $\Pi(\alpha) < 0$.

Proof. By standard arguments, the strategy profile σ^α is a BNE if and only if one of the conditions (i)–(iii) is satisfied, and is a strict BNE if and only if either condition (i') or (ii') is satisfied. Moreover, by Assumption 2(b), when σ^α is a BNE, then it is an equilibrium. It therefore remains to show that if σ^* is an equilibrium, then there must be some $\alpha \in [0,1]$ such that $\sigma^* = \sigma^\alpha$.

Suppose σ^* is an equilibrium but $\sigma_i^*(s_i^k) = \beta \neq 1$ for some $s^k \in \mathcal{G}$. Let σ' be the strategy where $\sigma'(s) = \frac{1}{2}$ for all $s \in \mathcal{S}$. When voters other than i follow strategy profile σ'_{-i}, voter i is pivotal in a population of size $n_0 + z$ with probability $P(piv|n_0+z, \sigma'_{-i}) = \binom{n_0+z-1}{\tau_z} \left(\frac{1}{2}\right)^{n_0+z-1} > 0$. Moreover, conditioning on being pivotal conveys no information about the information received by other voters. By Lemma 1 and the Weierstrass M-test, $\sum_{z=0}^{\infty} P(piv|n_0+z,\sigma'_{-i})V_i(s_i^k,n_0+z)P(z)$ converges absolutely to some $\tilde{c}>0$ because, for all z, $\binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} \le (n_0+z)\binom{n_0+z}{\lceil 0.5(n_0+z)\rceil}^2$, and $|V_i(s_i^k,n_0+z)| \le \bar{v}$.

As a result, $\pi_i(\sigma_i^*,\sigma'_{-i}|s_i^k) = \beta\tilde{c} < \tilde{c} = \pi_i(\sigma_i^1,\sigma_{-i}|s_i^k)$. On the other hand, by Assumption 2(b), $\pi_1(\sigma_i^1,\tilde{\sigma}_{-1}|s_i^k) \ge \pi_1(\tilde{\sigma}|s_i^k)$ for all $\tilde{\sigma}$. Hence, σ^* is not an equilibrium. An analogous argument shows that, in an equilibrium, it must be the case that $\sigma_i^*(s_i^k)=0$ whenever $s^k \in \mathcal{B}$. Since σ^* is a symmetric strategy profile, it follows that $\sigma^* = \sigma^\alpha$ for some $\alpha \in [0,1]$. □

A.1.6 Proofs of Preliminary Results (Propositions 1 and 2)

Proof of Proposition 1 on p. 13. First, consider the private information environment. By Lemma 3, if $\Pi(0) \le 0$ then σ^0 is an equilibrium, and if $\Pi(1) \ge 0$ then σ^1 is an equilibrium. It remains to show that there is an equilibrium when $\Pi(0) > 0$ and $\Pi(1) < 0$. In that case, since Π is continuous on $[0,1]$ by Lemma 1, it follows by the intermediate value theorem that there exists some α^* such that $\Pi(\alpha^*) = 0$ and so σ^{α^*} is an equilibrium.

For public information, define the symmetric strategy profile σ^{pub} as follows:

$$\sigma^{pub}(n,s) = \begin{cases} 1 & \text{if } V_i(n,s) > 0 \\ 0 & \text{otherwise} \end{cases}.$$

By standard arguments, σ^{pub} is an equilibrium. To show that this is the unique equilibrium, suppose for contradiction that $\sigma^* \ne \sigma^{pub}$ is also an equilibrium. For some (z,s), $\sigma(z,s) \ne \sigma^{pub}(z,s)$ and, by Assumption 3(a), $V_i(s) \ne 0$. Define the strategy σ' as in the proof of Lemma 3. Now consider the case $V_i(s) > 0$, and $\sigma_i(s) < 1$. Then

$$\begin{aligned}\pi_i(\sigma_i^{pub},\sigma'_{-i}|s,n_0+z) &= V_i(s)\binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} \\ &> \sigma_i^*(s)V_i(s)\binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} = \pi_i(\sigma_i^*,\sigma_{-i}|s,n_0+z).\end{aligned}$$

The case where $V_i(s) < 0$ is symmetric. Therefore, σ^* violates condition (ii) in the definition of an equilibrium. □

Proof of Proposition 2 on p. 14. In the public information environment, the unique equilibrium is σ^{pub} from the proof of Proposition 1. Given this strategy profile, if every voter receives the uninformative signal, then $V(G=M=0,Z=z) > 0$ for all z implies that R wins the election in the unique equilibrium. The probability that all voters receive the uninformative signal is $(1-\lambda)^{n_0+z}$ in a population of size n_0+z. Hence, the probability that R wins the election is at least $\sum_{z=0}^{\infty}(1-\lambda)^{n_0+z}P(z)$. For any fixed $\bar{z}$, this is greater than $(1-\lambda)^{n_0+\bar{z}}P(Z \leq \bar{z})$. Since $P(Z \leq \bar{z})$ converges to 1, we can choose $\bar{z}$ so that $P(Z \leq \bar{z}) \geq \sqrt{(1-\varepsilon)}$. Now fix $\bar{\lambda}' \in (0,1)$ such that $(1-\bar{\lambda}')^{n_0+\bar{z}} \geq \sqrt{1-\varepsilon}$. Then, for all $\lambda \in (0,\bar{\lambda}')$, $\sum_{z=0}^{\infty}(1-\lambda)^{n_0+z}P(z) \geq (\sqrt{1-\varepsilon})^2 = 1-\varepsilon$, and so R wins with probability exceeding $1-\varepsilon$. □

A.2 Proof of Main Results (Theorems 1 and 2)

A.2.1 Proof of Theorem 1 on p. 15

Suppose payoffs are τ-negatively correlated (i.e., $V^G(\tau) < 0$). Recall that σ^0 is the strategy profile where uninformed voters choose Q, and informed voters choose R when they receive good news and Q when they receive bad news. We first show that there exists $\bar{\lambda} > 0$ such that for all $\lambda \in (0,\bar{\lambda})$, σ^0 is a strict equilibrium. By Lemma 3, the strategy profile σ^0 is a strict equilibrium if and only if the following is strictly negative

$$\Pi(0)=\pi_i\left(\sigma_i^1,\sigma_{-i}^0|s_i^0\right)-\pi_i\left(\sigma_i^0,\sigma_{-i}^0|s_i^0\right)=\sum_{z=0}^{\infty}V(\tau_z,m,z)P(G=\tau_z|m,z)P(m,z|R_i^0), \quad (13)$$

where $V(\tau_z,m,z)=V(G=\tau_z,M=m,Z=z)$.

We first consider the summand in (13) where $z=0$:

$$\sum_{m=\tau_0}^{n_0-1}\binom{n_0-1}{m}\lambda^{m-\tau_0}(1-\lambda)^{n_0-1-m}P(G=\tau_z|M=m,Z=0)V(\tau_z,m,0)P(Z=0).$$

Since $\lambda<1$, $V(\tau_z,m,0)\leq\bar{v}$, and $P(G=\tau_z|m,z)\leq 1$, the term is bounded above by

$$\binom{n_0-1}{\tau_0}P(G=\tau_0|M=\tau_0,Z=0)P(Z=0)V^G(\tau)+\lambda\bar{v}P(Z=0)\sum_{m=\tau_0+1}^{n_0-1}\binom{n_0-1}{m}.$$

We now consider the remaining terms in the series (13). Since $V(\tau_z,m,z)\leq\bar{v}$, it follows by Lemma 1 that the remaining series is bounded above by $\lambda\bar{L}\bar{v}$, where $\bar{L}$ does not depend on λ. As a result, $\Pi_i(0)$ is bounded above by

$$\binom{n_0-1}{\tau_0}P(G=\tau_0|M=\tau_0,Z=0)P(Z=0)V^G(\tau)+\lambda\bar{v}\left(\bar{L}+\sum_{m=\tau_0+1}^{n_0-1}\binom{n_0-1}{m}P(Zz=0)\right).$$

The first term does not depend on λ and is strictly negative because payoffs are τ-negatively correlated (i.e., $P(G=\tau_0|M=\tau_0,Z=0)V^G(\tau)<0$) and $P(Z=0)>0$. In the second term, the bracket is a finite positive number that does not depend on λ.

Hence, there is $\bar{\lambda} \in (0,1)$ such that, for all $\lambda < \bar{\lambda}$, $\Pi_i(\lambda) < 0$ and so σ^0 is an equilibrium.

Now suppose that the voters follow the strategy profile σ^0. If all voters are uninformed, Q wins the election. Hence, following the argument in the proof of Proposition 2, there exists $\bar{\lambda}'$ such that for all $\lambda \in (0, \bar{\lambda}')$, Q wins with probability exceeding $(1-\varepsilon)$ in the equilibrium strategy σ^0.

A.2.2 Proof of Theorem 2 on p. 18

Suppose payoffs are not τ-negatively correlated and, without loss of generality, let $\lambda \leq \frac{1}{2}$. Let $v^* = \min_{\kappa \in [0,\tau]} V^G(\kappa) P(G = \lceil \kappa n_0 \rceil - 1 | M = \lceil \kappa n_0 \rceil - 1, Z = 0)$. By Assumption 5, $v^* > 0$ because payoffs are not τ-negatively correlated and $V^G(0) > 0$. Recall that, for $\alpha \in [0,1]$, σ^α is the (private) strategy profile where uninformed voters chose R with probability α and choose Q with probability $(1-\alpha)$, and informed voters choose R when they receive good news and Q when they receive bad news. The following Lemma establishes that there is a sufficiently small λ so that σ^α is not an equilibrium.

Lemma 4. *For every $\bar{\alpha} \in (0,1)$, there exists $\lambda_{\bar{\alpha}} \in (0,1)$ such that, if $\alpha \in (0, \bar{\alpha})$ and $\lambda \in (0, \lambda_{\bar{\alpha}})$, then $\Pi(\alpha) > 0$.*

Proof. Fix some $\bar{\alpha} \in (0,1)$ and let $\alpha \in (0, \bar{\alpha})$. For $\omega = (n, v, s) \in \Omega$, an uninformed voter i is pivotal if and only if τ_z vote for R. If $G(n, v, s) = g$ this requires $q \equiv \tau_z - g$ uninformed voters to choose R. Define $\Theta \equiv \{(q, m, z) \in \mathbb{N}^3 : 0 \leq \tau_z - q \leq m \leq n_0 + z - 1\}$. We want to show that, for λ sufficiently small,

$$\Pi(\alpha) = \sum_{z=0}^{\infty} \sum_{m=0}^{n_0+z-1} \sum_{g=0}^{m} V(g, m, z) P(piv|g, m, z) P(g|m, z) P(m, z|R_i^0) \equiv \sum_{\theta \in \Theta} c(\theta) > 0,$$

where for $\theta = (q, m, z) \in \Theta$, $c(\theta) \equiv \alpha^q \lambda^m \tilde{V}(q, m, z) \tilde{P}(q|m, z) B(\theta) A(\theta) P(z)$, $\tilde{V}(q, m, z) \equiv V(g = \tau_z - q, m, z)$, $\tilde{P}(q|m, z) \equiv P(g = \tau_z - q|m, z)$, $B(\theta) \equiv \binom{n_0+z-1-m}{q}\binom{n_0+z-1}{m}$, and $A(\theta) \equiv (1-\alpha)^{n_0+z-1-m-q}(1-\lambda)^{n_0+z-1-m}$.

We first provide a lower bound for $\Pi(\alpha)$ by giving a lower bound $\underline{c}(\theta)$ of $c(\theta)$ for each $\theta \in \Theta$. We partition Θ into four sets.

(1) $\Theta_1 = \{(q, m, z) \in \Theta : z = 0, m = \tau_0 - q\}$. Then, $\tilde{V}(q, m, z)\tilde{P}(q|m, z) \geq v^* > 0$ and $P(z) > 0$. Hence, for $(q, m, z) \in \Theta_1$, $c(q, m, z) \geq \underline{c}(q, m, z) \equiv \alpha^q \lambda^{\tau_0 - q} v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0}$ $P(Z = 0)$, because $B(q, m, z) \geq 1$, $A(q, m, z) \geq \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0}$. As a result, $\sum_{\theta \in \Theta_1} \underline{c}(\theta) = \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(Z = 0)$.

(2) $\Theta_2 = \{(q, m, z) \in \Theta : z > 0, m = 0\}$. Then, $\tilde{V}(q, m, z) = \tilde{V}(q, 0, z) \equiv \tilde{V}(0|z) > 0$. Hence, for $(q, m, z) \in \Theta_2$, $c(q, m, z) \geq \underline{c}(q, m, z) \equiv 0$. As a result, $\sum_{\theta \in \Theta_2} \underline{c}(\theta) = 0$.

(3) $\Theta_3 = \{(q,m,z) \in \Theta : z = 0, m > \tau_0 - q\}$. In that case, $\tilde{V}(q,m,z) \geq -\bar{v}$ which is negative. Hence, for $(q,m,z) \in \Theta_3$, $c(q,m,z) \geq \underline{c}(q,m,z) \equiv -\alpha^q \lambda^{\tau_0 - q+1} \bar{v} \binom{n_0}{\lceil 0.5 n_0 \rceil}^2$, because $m \geq \tau_0 - q + 1$, and so $\lambda^m \leq \lambda^{\tau_0 - q+1}$, $\tilde{P}(q|m,z) \leq 1$, $B(q,m,z) \leq \binom{n_0}{\lceil 0.5 n_0 \rceil}^2$, $A(q,m,z) \leq 1$, and $P(Z=0) \leq 1$. As a result,

$$\sum_{\theta \in \Theta_2} \underline{c}(\theta) = -\sum_{q=0}^{\tau_0} \sum_{m=\tau_0-q+1}^{n_0-q-1} \alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0}{\lceil 0.5 n_0 \rceil}^2 \geq -\sum_{q=0}^{\tau_0} n_0 \alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0}{\lceil 0.5 n_0 \rceil}^2.$$

(4) $\Theta_4 = \{(q,m,z) \in \Theta : z > 0, m > 0\}$. In that case, $\tilde{V}(q,m,z) \geq -\bar{v}$ which is negative. Hence, for all $(q,m,z) \in \Theta_4$,

$$c(q,m,z) \geq \underline{c}(q,m,z) \equiv \begin{cases} -\alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z) & \text{if } q \leq \tau_0, \\ -\alpha^q \lambda \bar{v} \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z) & \text{if } q > \tau_0, \end{cases}$$

because $m \geq \tau_z - q$, and so $\lambda^m \leq \lambda^{\tau_0-q+1}$ when $q \leq \tau_0$, and $\lambda^m \leq \lambda$ when $q > \tau_0$, $\tilde{P}(q|m,z) \leq 1$, $B(q,m,z) \leq \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2$, and $A(q,m,z) \leq 1$. Since $\underline{c}(\theta) < 0$ for every $\theta \in \Theta_4$,

$$\begin{aligned}
\sum_{\theta \in \Theta_4} \underline{c}(\theta) \geq & -\sum_{q=0}^{\tau_0} \sum_{z=0}^{\infty} \sum_{m=\tau_z-q}^{n_0+z-1-q} \alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z) \\
& - \sum_{q=\tau_0+1}^{\infty} \sum_{z=0}^{\infty} \sum_{m=\max\{0,\tau_z-q\}}^{\max\{0,n_0+z-1-q\}} \alpha^q \lambda \bar{v} \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z) \\
\geq & -\sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0-q+1} \bar{v} \sum_{z=0}^{\infty} (n_0+z) \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z) \\
& - \sum_{q=\tau_0+1}^{\infty} \alpha^q \lambda \bar{v} \sum_{z=0}^{\infty} (n_0+z) \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z)
\end{aligned}$$

By Lemma 1, the series $\sum_{z=0}^{\infty} (n_0+z) \binom{n_0+z}{\lceil 0.5(n_0+z) \rceil}^2 P(z)$ converges absolutely to some $\bar{c}$. As a result, $\sum_{\theta \in \Theta_4} \underline{c}(\theta) \geq -\lambda \bar{v} \bar{c} \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0-q} - \lambda \bar{v} \bar{c} \sum_{q=\tau_0+1}^{\infty} \alpha^q$.

Since $(\Theta_1, \Theta_2, \Theta_3, \Theta_4)$ is a partition of Θ, and $n_0 \binom{n_0}{\lceil 0.5 n_0 \rceil}^2 P(Z=0) \leq \bar{c}$,

$$\begin{aligned}
\Pi(\alpha) &\geq \sum_{\theta \in \Theta_1} \underline{c}(\theta) + \sum_{\theta \in \Theta_2} \underline{c}(\theta) + \sum_{\theta \in \Theta_3} \underline{c}(\theta) + \sum_{\theta \in \Theta_4} \underline{c}(\theta) \\
&\geq \left(v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(Z=0) - 2\lambda \bar{v} \bar{c} \right) \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0-q} - \lambda \bar{v} \bar{c} \sum_{q=\tau_0+1}^{\infty} \alpha^q
\end{aligned}$$

$$= \left(v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(Z=0) - 2\lambda\bar{v}\bar{c}\right) \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} - \lambda\bar{v}\bar{c}\alpha^{\tau_0+1}\frac{1}{1-\alpha}$$

$$\geq \left(v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(Z=0) - 2\lambda\bar{v}\bar{c}\right) \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} - \lambda\bar{v}\bar{c}\alpha^{\tau_0}\frac{1}{1-\bar{\alpha}}$$

$$= \alpha^{\tau_0}\left(v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(Z=0) - \lambda\bar{v}\bar{c}\left(2 + \frac{1}{1-\bar{\alpha}}\right)\right)$$

$$+ \left(v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(Z=0) - 2\lambda\bar{v}\bar{c}\right) \sum_{q=0}^{\tau_0 - 1} \alpha^q \lambda^{\tau_0 - q}.$$

There exists some $\bar{\lambda} \in (0,1)$ such that, for all $\lambda < \bar{\lambda}$, $v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(Z = 0) > \lambda\bar{v}\bar{c}\left(2 + \frac{1}{1-\bar{\alpha}}\right)$. Hence, for $\lambda < \bar{\lambda}$, $\Pi_i(\alpha) > 0$. □

We now use Lemma 4 to complete the proof. Fix $\varepsilon \in (0,1)$. We want to show that there exists λ_ε such that in every equilibrium of the private information environment, R wins with probability exceeding $1-\varepsilon$. (The proof for the public information environment follows from Proposition 1).

First, fix $\bar{z}_\varepsilon$ such that $P(Z \leq \bar{z}_\varepsilon) \geq (1-\varepsilon)^{\frac{1}{3}}$. Such $\bar{z}_\varepsilon$ exists because $P(z)$ is countably additive, and so $\lim_{z' \to \infty} P(Z \leq z') = 1$.

For a given λ, if voters follow a strategy profile σ^α, and a population size $n_0 + z$ is realized, the probability that R wins exceeds $(1-\lambda)^{n_0+z}\alpha^{n_0+z}$, which describes the probability that all voters are uninformed and vote for R in a population $n_0 + z$ given the strategy profile σ^α. Hence, the ex ante probability that R wins exceeds $(1-\lambda)^{n_0+\bar{z}_\varepsilon}\alpha^{n_0+\bar{z}_\varepsilon}P(Z \leq \bar{z}) \geq (1-\lambda)^{n_0+\bar{z}_\varepsilon}\alpha^{n_0+\bar{z}_\varepsilon}(1-\varepsilon)^{\frac{1}{3}}$.

Now let $\bar{\lambda}_\varepsilon = 1 - (1-\varepsilon)^{\frac{1}{3(n_0+\bar{z}_\varepsilon)}}$, and let $\bar{\alpha}_\varepsilon = (1-\varepsilon)^{\frac{1}{3(n_0+\bar{z}_\varepsilon)}}$. Then, $\bar{\lambda}_\varepsilon \in (0,1)$ and $\bar{\alpha}_\varepsilon \in (0,1)$. Moreover, if $\lambda < \bar{\lambda}_\varepsilon$ and $\alpha > \bar{\alpha}_\varepsilon$, then the probability that R wins when voters follow the strategy profile σ^α exceeds $(1-\varepsilon)^{\frac{n_0+\bar{z}}{3(n_0+\bar{z})}}(1-\varepsilon)^{\frac{n_0+\bar{z}}{3(n_0+\bar{z})}}(1-\varepsilon)^{\frac{1}{3}} = (1-\varepsilon)$.

Finally, by Lemma 4, there exists $\bar{\lambda}'_\varepsilon \in (0, \frac{1}{2})$ such that, if $\lambda < \bar{\lambda}'_\varepsilon$ and $\alpha \leq \bar{\alpha}_\varepsilon$, then σ^α is not an equilibrium in the private information environment. Let $\lambda_\varepsilon = \min\{\bar{\lambda}'_\varepsilon, \bar{\lambda}_\varepsilon\}$. Then for all $\lambda < \lambda_\varepsilon$, σ^α is an equilibrium only if R wins with probability exceeding $(1-\varepsilon)$.

To complete the proof, it only remains to show that, when $\lambda < \lambda_\varepsilon$, then σ^0 is not an equilibrium, but this argument follows closely from Theorem 1 (using the assumption that payoffs are not τ-negatively correlated to show that a lower bound on $\Pi(0)$ is strictly positive for sufficiently small λ).

A.3 Proofs for Additional Results (Propositions 4–8)

Proof of Proposition 4 on p. 22. Note that $P \succeq_{\tau\text{-}nc} P'$ if and only if $\frac{P(W_i|\lceil \tau n_o \rceil - 1)}{1-P(W_i|\lceil \tau n_o \rceil -1)} \geq \frac{P'(W_i|\lceil \tau n_o \rceil - 1)}{1-P'(W_i|\lceil \tau n_o \rceil -1)}$. Hence, $\succeq_{\tau\text{-}nc}$ is complete and transitive. The representation follows directly from the subsequent calculations:[31]

$$\begin{aligned} P(W_i|\lceil \tau n_0 \rceil - 1) &= \sum_{\hat{\eta}=0}^{n_0} P(W_i|S_i = s^0, M = G = \tau_0, \eta = \hat{\eta}) P(\hat{\eta}|S_i = s^0, M = G = \tau_0) \\ &= \sum_{\hat{\eta}=\tau_0}^{n_0} \left(\frac{\hat{\eta} - \tau_0}{n_0 - \tau_0} \right) P(\hat{\eta}|M = G = \tau_0) \\ &= \left(\frac{1}{n_0 - \tau_0} \right) \left(-\tau_0 + \sum_{\hat{\eta}=\tau_0}^{n_0} \hat{\eta} P(\hat{\eta}|M = G = \tau_0) \right) \\ &= \left(\frac{1}{n_0 - \tau_0} \right) \Big(-\tau_0 + E_P(\eta|M = G = \tau_0) \Big), \end{aligned}$$

where the second equality uses Assumptions 1 and 2(a).

□

Proof of Proposition 5 on p. 22. We use the following lemma:

Lemma 5. *For every $m \geq 1$, and vectors $a, q, r \in \mathbb{R}^m_+$ such that $a \cdot q \neq 0$, $q_1 \leq ... \leq q_m$, and $r_1 \leq ... \leq r_m$,*

$$\frac{\sum_{i=1}^m r_i a_i q_i}{\sum_{i=1}^m a_i q_i} \geq \frac{\sum_{i=1}^m r_i a_i}{\sum_{i=1}^m a_i}. \tag{14}$$

Proof. Because all the denominators are non-negative, (14) is equivalent to $\sum_{i,j=1}^m r_i q_i a_i a_j \geq \sum_{i,j=1}^m r_i q_j a_i a_j$, which is obtained by cross-multiplying and re-grouping terms. Each $a_i a_j$, which is non-negative, is multiplied by $r_i q_i + r_j q_j$ on the LHS and $r_i q_j + r_j q_i$ on the RHS. Therefore, this inequality is satisfied if for each i and j

$$r_i q_i + r_j q_j \geq r_i q_j + r_j q_i. \tag{15}$$

For $i \geq j$, (15) is equivalent to $(r_i - r_j) q_i \geq (r_i - r_j) q_j$, which is true since $r_i - r_j \geq 0$ and $q_i \geq q_j$. Therefore (14) is satisfied. □

We use Lemma 5 to prove our result. To distinguish random variables from their realizations, we use η to denote the random variable representing the number of winners

[31] Recall that $\tau_0 \equiv \lceil \tau n_0 \rceil - 1$, as defined in (7).

in each state, and $\hat{\eta}$ as a particular realization of η. Observe that

$$P(\hat{\eta}|M = G = \tau_0) = \frac{P(G = \tau_0|\hat{\eta}, M = \tau_0)P(\hat{\eta})}{\sum_{\tilde{\eta}=\tau_0}^{n_0} P(G = \tau_0|\tilde{\eta}, M = \tau_0)P(\tilde{\eta})} = \frac{\frac{\hat{\eta}!}{(\hat{\eta}-\tau_0)!}P(\hat{\eta})}{\sum_{\tilde{\eta}=\tau_0}^{n_0} \frac{\tilde{\eta}!}{(\tilde{\eta}-\tau_0)!}P(\tilde{\eta})},$$

where we use Assumption 2(a) to derive that $P(\tilde{\eta}|M = \tau_0) = P(\tilde{\eta})$, and

$$P(G = \tau_0|\tilde{\eta}, M = \tau_0) = \frac{\binom{\tilde{\eta}}{\tau_0}\binom{n_0-\tilde{\eta}}{0}}{\binom{n_0}{\tau_0}} = \frac{\tilde{\eta}!}{(\tilde{\eta}-\tau_0)!}\frac{(n_0-\tau_0)!}{n_0!}.$$

So, by the proof of Proposition 4, $P \succeq_{\tau\text{-}nc} P'$ if and only if

$$\frac{\sum_{\hat{\eta}=\tau_0}^{n_0}(\hat{\eta}-\tau_0)\frac{\eta!}{(\eta-\tau_0)!}P(\hat{\eta})}{\sum_{\hat{\eta}=\tau_0}^{n_0}\frac{\hat{\eta}!}{(\hat{\eta}-\tau_0)!}P(\hat{\eta})} \geq \frac{\sum_{\hat{\eta}=\tau_0}^{n_0}(\hat{\eta}-\tau_0)\frac{\eta!}{(\eta-\tau_0)!}P'(\hat{\eta})}{\sum_{\hat{\eta}=\tau_0}^{n_0}\frac{\hat{\eta}!}{(\hat{\eta}-\tau_0)!}P'(\hat{\eta})}. \tag{16}$$

Consider a transformation of variables from $\hat{\eta}$ to i such that $i = \hat{\eta} - (\tau_0 - 1)$. With this new index, define the vectors r, a, q such that $r_i = i - 1$, $a_i = \frac{(i+\tau_0-1)!}{(i-1)!}P'(i + \tau_0 - 1)$ and $q_i = \frac{P(i+\tau_0-1)}{P'(i+\tau_0-1)}$. Inequality (16) is then re-written as $\frac{\sum_{i=1}^{n_0-(\tau_0-1)} r_i a_i q_i}{\sum_{i=1}^{n_0-(\tau_0-1)} a_i q_i} \geq \frac{\sum_{i=1}^{n_0-(\tau_0-1)} r_i a_i}{\sum_{i=1}^{n_0-(\tau_0-1)} a_i}$. Because r_i is non-decreasing in i and $a_i \geq 0$, it follows from Lemma 5 that the above inequality is satisfied if q_i is non-decreasing in i. Therefore, a sufficient condition for (16) is that for every $\hat{\eta} \geq \tau_0$, $\frac{P(\hat{\eta})}{P'(\hat{\eta})}$ is non-decreasing in $\hat{\eta}$, implied by condition (5). □

Proof of Proposition 6 on p. 23. Signals convey no distributional information when for every signal profile $s \in \mathcal{S}^{n_0}$ and for every pair of voters i and j, $V_i(s) = V_j(s)$. We show that $V^G(\tau) > 0$ in two steps.

Step 1: We show that $P(s_i \in \mathcal{G}, s_j \in \mathcal{B}) = 0$. Towards a contradiction, consider a non-null signal profile $s \in \mathcal{S}^{n_0}$ where $s_i \in \mathcal{G}$ and $s_j \in \mathcal{B}$. Then, $V_i(s_i) > 0$ and $V_j(s_j) < 0$, and by Assumption 2(b), the sign of $V_i(s)$ is that of $V_i(s_i)$ and the sign of $V_j(s)$ is that of $V_j(s_j)$. But since signals convey no distributional information, $V_i(s) = V_j(s)$.

Step 2: Consider the event $E \equiv \{s \in \mathcal{S}^{n_0} : S_i = s^0, M = G = \tau_0\}$. Observe that $V^G(\tau) = V_i(E) = \sum_{s\in E} V_i(s)P(s|E)$. Consider a particular $s \in E$. Because $s_i = s^0$, by Assumption 2(a), $V_i(s) = V_i(s_{-i})$. By Bayes Rule,

$$\begin{aligned} V_i(s_{-i}) &= (1-\lambda)V_i(s_{-i}) + \sum_{s'\in\mathcal{G}} V_i(s', s_{-i})P(s'|s_{-i}) + \sum_{s'\in\mathcal{B}} V_i(s', s_{-i})P(s'|s_{-i}) \\ &= (1-\lambda)V_i(s_{-i}) + \sum_{s'\in\mathcal{G}} V_i(s', s_{-i})P(s'|s_{-i}) = \frac{1}{\lambda}\sum_{s'\in\mathcal{G}} V_i(s', s_{-i})P(s'|s_{-i}), \end{aligned}$$

where the second equality follows from Step 1, and the third equality follows from rearranging terms. By definition of $\mathcal{G}$, $V_i(S_i = s') > 0$ for every $s' \in \mathcal{G}$, and therefore, it follows from Assumption 2(b) that $V_i(s', s_{-i}) > 0$. Therefore, the above expression confirms that for every $s \in E$, $V_i(s) > 0$, and therefore, $V^G(\tau) = V_i(E) > 0$. □

Proof of Proposition 7 on p. 23. We assume that signals convey no aggregate information (i.e., $P(\eta|s) = P(\eta)$ for all η and s) and a voter with an informative signal learns her priority. This implies that priorities (ρ) and the number of winners (η) must be independent and, by Assumption 1, that the prior distribution over priorities is uniform. As a result,

$$P(W_i) = P(\rho_i \leq \eta) = \sum_{\hat{\rho}=1}^{n_0} P(\rho_i = \hat{\rho}, \eta \geq \rho) = \sum_{\hat{\rho}=1}^{n_0} P(\rho_i = \hat{\rho}) P(\eta \geq \hat{\rho}) = \sum_{\hat{\rho}=1}^{n_0} \frac{1}{n_0} P(\eta \geq \hat{\rho}).$$

We want to show that $P(W_i|\tau_0) < P(W_i)$, where $P(W_i|\tau_0)$ is the probability that an uninformed voter i is a winner conditional on $M = G = \tau_0$. First note that there is a priority $\rho^* \in \{0, ..., n\}$ such that $V_j(\rho_j) > 0$ if and only if $\rho_j \leq \rho^*$ (i.e., all priorities less than or equal to ρ^* are good news, and higher priorities are bad news). Now consider an uninformed voter i in the pivotal event where $M = G = \tau_0$ (for simplicity we denote this event by τ_0). Observe that

$$\begin{aligned}
P(W_i|\tau_0) =& P(\rho_i \leq \eta|\tau_0) = \sum_{\hat{\rho}=1}^{n_0} P(\rho_i = \hat{\rho}, \eta \geq \hat{\rho}|\tau_0) = \sum_{\hat{\rho}=1}^{n_0} P(\rho_i = \hat{\rho}|\tau_0) P(\eta \geq \hat{\rho}) \\
=& \sum_{\hat{\rho}=1}^{\rho^*} P(\rho_i = \hat{\rho}|\tau_0) P(\eta \geq \hat{\rho}) + \sum_{\hat{\rho}=\rho^*+1}^{n_0} P(\rho_i = \hat{\rho}|\tau_0) P(\eta \geq \hat{\rho}) \\
=& \sum_{\hat{\rho}=1}^{\rho^*} P(\rho_i \leq \rho^*|\tau_0) P(\rho_i = \hat{\rho}|\rho_i \leq \rho^*, \tau_0) P(\eta \geq \hat{\rho}) \\
&+ \sum_{\hat{\rho}=\rho^*+1}^{n_0} P(\rho_i > \rho^*|\tau_0) P(\rho_i = \hat{\rho}|\rho_i > \rho^*, \tau_0) P(\eta \geq \hat{\rho}),
\end{aligned}$$

where the first and second equalities follow by definition; the third equality follows because η is independent of ρ; the fourth equality follows by definition; and the fifth equality follows by Bayes rule. Then, because the marginal distribution over ρ is uniform,

$$P(W_i|\tau_0) = \sum_{\hat{\rho}=1}^{\rho^*} \left(\frac{\rho^* - \tau_0}{n_0 - \tau_0}\right) \left(\frac{1}{\rho^*}\right) P(\eta \geq \hat{\rho}) + \sum_{\hat{\rho}=\rho^*+1}^{n_0} \left(\frac{n_0 - \rho^*}{n_0 - \tau_0}\right) \left(\frac{1}{n_0 - \rho^*}\right) P(\eta \geq \hat{\rho}),$$

As a result, $P(W_i|\tau_0) - P(W_i) = \frac{\tau_0(\rho^* - n_0)}{n_0 \rho^* (n_0 - \tau_0)} \sum_{\hat{\rho}=1}^{\rho^*} P(\eta \geq \hat{\rho}) + \frac{\tau_0}{n_0(n_0 - \tau_0)} \sum_{\hat{\rho}=\rho^*+1}^{n} P(\eta \geq$

$\hat{\rho})$. The sign of the right hand side in the above equation is negative if and only if $\frac{\rho^*-n_0}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*}P(\eta\geq\hat{\rho})+\sum_{\hat{\rho}=\rho^*+1}^{n_0}P(\eta\geq\hat{\rho})<0$. The preceding inequality is equivalent to $\frac{1}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*}P(\eta\geq\hat{\rho})>\frac{1}{n_0}\sum_{\hat{\rho}=1}^{n_0}P(\eta\geq\hat{\rho})$, which, by further manipulation, is equivalent to $\frac{1}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*}P(\eta<\hat{\rho})<\frac{1}{n_0}\sum_{\hat{\rho}=1}^{n_0}P(\eta<\hat{\rho})$. As $P(\eta<\hat{\rho})\geq P(\eta<\rho^*)$ for all $\hat{\rho}>\rho^*$ (with a strict inequality for some $\hat{\rho}$), it follows that the right hand side of the above inequality is strictly greater than $\frac{1}{n_0}\sum_{\hat{\rho}=1}^{\rho^*}P(\eta<\hat{\rho})+\left(\frac{n_0-\rho^*}{n_0}\right)P(\eta<\rho^*)$. It is therefore sufficient to show that $\frac{1}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*}P(\eta<\hat{\rho})\leq\frac{1}{n_0}\sum_{\hat{\rho}=1}^{\rho^*}P(\eta<\hat{\rho})+\left(\frac{n_0-\rho^*}{n_0}\right)P(\eta<\rho^*)$, which is equivalent to showing that $\left(\frac{n_0-\rho^*}{n_0\rho^*}\right)\sum_{\hat{\rho}=1}^{\rho^*}P(\eta<\hat{\rho})\leq\left(\frac{n_0-\rho^*}{n_0}\right)P(\eta<\rho^*)$. Hence, the result follows because $P(\eta<\hat{\rho})\leq P(\eta<\rho^*)$ for all $\hat{\rho}=1,...,\rho^*$. □

Proof of Proposition 8 on p. 24. If σ is a strict equilibrium, it must be that case that $\sigma\in\{\sigma^0,\sigma^1\}$ by Lemma 3. When λ is sufficiently small, Q wins with probability exceeding $1-\varepsilon$ in the strategy profile σ^0. Thus, we need to show that, for λ sufficiently small, the strategy profile σ^1 is not an equilibrium when $V^B(\tau)<0$. For the strategy profile σ^1, an uninformed voter i is pivotal if and only if $B=n_0-\tau_0$. Hence, σ^1 is an equilibrium if and only if

$$\sum_{m=n_0-\tau_0}^{n_0-1}V_i\left(S_i=s^0,M=m,B=n_0-\tau_0\right)P\left(B=n_0-\tau_0|M=m\right)P\left(M=m\right)\geq 0.$$

Using arguments similar to those in the proof of Theorem 1, σ^1 is not an equilibrium if

$$\left(\frac{\lambda}{1-\lambda}\right)\bar{v}\sum_{m=n_0-\tau_0+1}^{n_0-1}\binom{n_0-1}{m}<-V^B(\tau)P(B=n_0-\tau_0|M=n_0-\tau_0)\binom{n_0-1}{n_0-\tau_0}.$$

Since the RHS is strictly positive and does not depend on λ, there is a $\tilde{\lambda}$ such that, for all $\lambda<\tilde{\lambda}$, the strict inequality holds and σ^1 is not an equilibrium. □