# INVESTMENTS IN SOCIAL TIES, RISK SHARING AND INEQUALITY

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Abstract. This paper provides a framework to study the formation of risk-sharing networks through costly social investments, in particular the inefficiencies and resulting inequality associated with such processes. First, individuals invest in relationships to form a network. Next, neighboring agents negotiate risk-sharing arrangements. There is never underinvestment, but overinvestment is possible and we find a novel trade-off between efficiency and equality. The most stable efficient network also generates the most inequality. When the income correlation structure is generalized by splitting individuals into groups, such that incomes across groups are less correlated but these relationships are more costly, there can be underinvestment across group but not within group. We find that more central agents have better incentives to form across-group links, reaffirming the efficiency inequality trade-off. In general, endogenous network formation in the risk sharing context tends to result in highly asymmetric networks and stark inequalities in consumption levels. Evidence from 185 Indian village networks is congruent with our model.

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## 1. Introduction

In the context of missing formal insurance markets and limited access to lending and borrowing, incomes may be smoothed through informal risk-sharing agreements that utilize social connections. A large theoretical and empirical literature studies how well informal arrangements replace the missing markets.[1] However, the existing literature does not investigate a potential downside to these agreements: if people's network position affects the share of surplus generated by risk sharing they appropriate, social investments may be distorted and inequality may endogenously arise.[2]

Our starting premise is that social networks are endogenous and that their structure affects how the surplus from risk sharing is split. There is growing empirical evidence that risk-sharing networks respond to financial incentives, and that in general risk-sharing networks form endogenously, in a way that depends on the economic environment: see for example recent work by Binzel et al. (2017) and Banerjee et al. (2014b,c), which in different contexts look at how social networks respond to the introduction of financial instruments such as savings vehicles or microfinance. Our main goal is to develop a theoretical framework that can be used to think about the endogeneity of risk sharing networks, and to interpret how these networks change after certain economic interventions, or more generally after changes in the economic environment.

In this paper we provide an examination of these issues, by considering a simple two stage model. In the first stage villagers invest in costly bilateral relationships, knowing that in the second stage they will reach informal risk-sharing agreements. These agreements determine how the surplus generated by risk sharing is distributed, and they depend on the endogenous structure of the social network from the first stage. In this way we elucidate new costs associated with informal risk-sharing. Once incomes have been realized, risk sharing typically reduces inequality by smoothing incomes. Nevertheless, asymmetric equilibrium networks generate inequality in expected utilities terms. Agents occupying more advantageous positions in the social network appropriate considerably more of the benefits generated by risk sharing. Indeed, seeking to occupy such positions in the network might lead villagers to spend too much time building social capital. Alternatively, if risk sharing with one neighbor generates positive spillovers for other neighbors, there can be too little investment in forming relationships undermining the effectiveness of informal risk-sharing.

Empirical work suggests that both underinvestment and overinvestment in social capital are possible, in different contexts. Austen-Smith and Fryer (2005) cites numerous references from sociology and anthropology, suggesting that members of poor communities allocate

[1]An incomplete list of papers includes Rosenzweig (1988), Fafchamps (1992), Coate and Ravallion (1993), Townsend (1994), Udry (1994), Ligon, Thomas and Worrall (2002), Fafchamps and Gubert (2007), Bloch, Genicot, and Ray (2008), Angelucci and di Giorgi (2009), Jackson, Rodriguez-Barraquer and Tan (2012), Ambrus, Mobius and Szeidl (2014).

[2]Previous works that do consider the network formation problem include Bramoullé and Kranton (2007a,b) in the theoretical literature and Attanasio et al. (2012) in the experimental literature. For a related paper outside the networks framework, see Glaeser et al. (2002).

inefficiently large amounts of time to activities maintaining social ties, instead of productive activities. In contrast, Feigenberg et al. (2013) find evidence in a microfinance setting that it is relatively easy to experimentally intervene and create social ties among people that yield substantial benefits. One explanation for this finding is that there is underinvestment in social relationships.

It is important to study the above aspects of informal risk sharing, both to put related academic work (which often takes social connections to be exogenously given) into context and to guide policy choices. Consider, for example, microfinance. Two central aims of such interventions are to increase the efficiency of investment decisions by providing better access to capital and to reduce inequality. Clearly the value of microfinance then depends on whether informal risk sharing promotes equality or inequality and whether there is underinvestment, overinvestment or efficient investment in social connections. If there is overinvestment, microfinance has a greater scope for efficiency savings in terms of reducing people's allocation of time into social investments. With underinvestment, however, it has more scope for smoothing incomes. If there is neither under- nor overinvestment, it tells us that informal risk sharing is working relatively well as a second-best solution. Understanding which regime applies can help anticipate policy implications and evaluate welfare impacts of interventions.

For analytical tractability and expositional purposes, in the main text we impose several specific assumptions: agents have CARA utilities, their income realizations are jointly normal, and that surplus is negotiated according to a particular bargaining process, split-the-difference negotiations (Stole and Zwiebel (1996)). In Appendix I we extend our main results to much more general settings, dropping all of the specific assumptions above.

In the first stage of interactions, agents choose with whom to form connections. Link formation is costly, as in Myerson (1991) and Jackson and Wolinsky (1996). In the second stage, pairs of agents who have formed a connection *commit* to a bilateral risk-sharing agreement (transfers contingent on income realizations). We assume that these agreements can be perfectly enforced. We investigate agreements satisfying two simple properties. First we require agreements to be pairwise efficient, in that no pair of directly connected agents leave gains from trade on the table.[3] Second, following Stole and Zwiebel (1996), we require the agreements to be robust to "split-the-difference" renegotiations.[4] We show that this leads to the surplus being divided by the Myerson value,[5] a network-specific version of the Shapley

[3]Although we consider a model in which there is perfect bilateral risk sharing, we could easily extend the model so that some income is perfectly observed, some income is private, and there is perfect risk sharing of observable income and no risk sharing of unobservable income. This would be consistent with the theoretical predictions of Cole and Kocherlakota (2001) and the empirical findings of Kinnan (2011). In the CARA utilities setting, such unobserved income outside the scope of the risk-sharing arrangement does not affect our results.

[4]Stole and Zwiebel (1996) model bargaining between many employees and an employer. This scenario can be represented by a star network with the employer at the center.

[5]For related noncooperative foundations for the Myerson value, see Fontenay and Gans (2014) and Navarro and Perea (2013). Slikker (2007) also provides noncooperative foundations, although the game analyzed is not decentralized: offers are made at the coalitional level.

value.[6] The transfers required to implement the agreements we identify are particularly simple. Each agent receives an equal share of aggregate realized income (as in Bramoullé and Kranton, 2007a) and on top of that state independent transfers are made.[7]

A key implication of the Myerson value determining the division of surplus is that agents who are more centrally located, in a certain sense, receive a higher share of the surplus. Moreover, in our risk-sharing context it implies that agents receive larger payoffs from providing "bridging links" to otherwise socially distant agents than from providing local connections.[8] Empirical evidence supports this feature of our model—see Goyal and Vega-Redondo (2007), and references therein from the organizational literature: Burt (1992), Podolny and Baron (1997), Ahuja (2000), and Mehra et al. (2001).

In the network formation stage, we study the set of pairwise-stable networks (Jackson and Wolinsky, 1996).[9]

Our analysis considers a community comprising of different groups where all agents within each group are ex-ante identical, and establishing links within groups is cheaper than across groups. We also assume that the income realizations of agents within groups are more positively correlated than across groups. Groups can represent different ethnic groups or castes in a given village, or different villages. We find that there can be overinvestment within groups but not underinvestment, whereas across groups underinvestment is likely to be the main concern.

To see the intuition about overinvestment within groups, we first consider the case of homogeneous agents, that is, when there is only one group. Using the inclusion–exclusion principle from combinatorics,[10] we develop a new metric to describe how far apart two agents located in a network are, which we call the Myerson distance. Using this distance we provide a complete characterization of stable networks. We show that for homogenous agents there can never be underinvestment in social connections, as agents establishing an essential link (connecting two otherwise unconnected components of the network) always receive a benefit exactly equal to the social value of the link. However, overinvestment, in the form of redundant links, is possible, and becomes widespread as the cost of link formation decreases.

Our main finding is that even though agents are ex-ante identical, if stable networks are asymmetric, inequality will result. We identify a novel trade-off between efficiency and inequality. Among all possible efficient network structures, we find that the most stable (in

[6]The Myerson value is also often assumed in social networks contexts on normative grounds, as a fair allocation: see a related discussion on pp. 422–425 of Jackson (2010).

[7]For investigations of the division of surplus in social networks in other contexts, see Calvo-Armengol (2001, 2003), Corominas-Bosch (2004), Manea (2011), Kets et al. (2011) and Elliott and Nava (2016).

[8]More precisely, in Section 4 we introduce the concept of Myerson distance to capture the social distance between agents in the network, and show that a pair of agents' payoffs from forming a relationship are increasing in this measure.

[9]Results from Calvo-Armengol and Ilkilic (2009) imply that under some parameter restrictions—for example when agents are ex ante identical—the set of pairwise-stable outcomes is equivalent to the (in general more restrictive) set of pairwise Nash equilibrium outcomes.

[10]See Chapter 10 in van Lint and Wilson (2001).

the sense of being stable for the largest set of parameter values) results in the most unequal division of surplus (for any inequality measure in the Atkinson class). Conversely, the least stable efficient network entails the most equal division of surplus among all efficient networks. Although agents are ex-ante identical, efficiency considerations push the structure of social connections towards asymmetric outcomes that elevate certain individuals. Socially central individuals emerge endogenously from risk-sharing considerations alone.

Turning attention to the case of multiple groups, we find that across-group underinvestment becomes an issue when the cost of maintaining links across groups is sufficiently high.[11] The reason is that the agents who establish the first connection across groups receive less than the social surplus generated by the link, providing positive externalities for peers in their groups. To consider which agents are best incentivised to provide across group links we introduce a new measure of network centrality which we term Myerson centrality. Agents more central in this sense have better incentives to provide across group links. This provides a second force pushing some agents within a group to be more central than others. For example, with two groups, we show that the most stable efficient network structure involves stars within groups, connected by their centers. This reinforces the trade-off between efficiency and equality in the many-groups context. Our model also predicts that more central agents within groups should play a particularly highlighted role, relative to peripheral agents, in maintaining across group links when the value of informal risk sharing is smaller, as in this case maintaining such links does not provide enough individual benefits for peripheral individuals.

We also provide empirical evidence to support our theoretical findings, using unique network data from 185 Indian villages. The data were collected in the context of a large field experiment in which a randomly chosen half of the villages gained access to local banking services, providing exogenous variation in access to formal loans. Both in the treated and non-treated villages, near complete network data were collected on within-village actual and potential financial transactions, as well as on financial links to households outside the village. Using these data, we test our model's theoretical prediction that the association between Myerson centrality within the village network and having financial links outside the village becomes more positive when villagers have access to formal banking and therefore the value of informal financial links is smaller. This prediction bears out in the data. In particular, the relationship between Myerson centrality and outside links is significantly more positive in villages that were randomly chosen to receive formal banking services. This prediction can confidently be interpreted as a causal influence of a reduction in the value of outside links because it is based on truly exogenous variation in formal credit access, which gives rise to a significant reduction in the number of network links, consistent with a decline in the relative importance of informal risk-sharing.

[11]While across-group overinvestment remains possible, the main concern when across-group link costs are relatively high is underinvestment.

Among the theoretical studies on social networks and informal risk sharing that are most related to ours include Bramoullé and Kranton (2007a,b), Bloch et al. (2008), Jackson et al. (2012), Billand et al. (2012), Ali and Miller (2013, 2016), and Ambrus et al. (2014). Many of these papers focus on the enforcement issues we abstract from, and investigate how social capital can be used to sustain cooperation for lower discount factors than would otherwise be possible. We take a complementary approach and instead focus on the distribution of surplus and the incentives this creates for social investments. One way of viewing our approach is an assumption on the discount factor in a dynamic version of our model. As long as the discount factor is high enough, our equilibrium agreements satisfy the necessary incentive compatibility constraints to be able to be enforced in equilibrium of the dynamic game.

Among the aforementioned papers, Bramoullé and Kranton (2007a,b) and Billand et al. (2012) investigate costly network formation. Bramoullé and Kranton's (2007a,b) model assumes that the surplus on a connected income component is equally distributed, independently of the network structure. This rules out the possibility of overinvestment or inequality, and leads to different types of stable networks than in our model. Instead of assuming optimal risk-sharing arrangements, Billand et al. (2012) assume an exogenously given social norm, which prescribes that high-income agents transfer a fixed amount of resources to all low-income neighbors. This again leads to very different predictions regarding the types of networks that form in equilibrium.

More generally, network formation problems are important. Establishing and maintaining social connections (relationships) is costly, in terms of time and other resources. However, on top of direct consumption utility, such links can yield many economic benefits. Papers studying formation in different contexts include Jackson and Wolinsky (1996), Bala and Goyal (2000), Kranton and Minehart (2001), Hojman and Szeidl (2008), and Elliott (2015). Notable in this literature is a lack of empirical work, which can be attributed to a number of innate difficulties that taking these models to data presents. Our many observations of social networks that are relatively independent of each other, coupled with our approach to circumventing data limitations, allow us to provide a first step towards testing predictions based on the overall network structure. And although we study a specific network formation problem tailored to risk sharing in villages, the general structure of our problem is relevant to other applications.[12]

The remainder of the paper is organized as follows. Section 2 describes risk sharing on a fixed network. In Section 3 we introduce a game of network formation with costly link formation. We focus on network formation within a single group in Section 4 and then turn to the formation of across-group links in Section 5. We then generate comparative static

[12]For a different and more specific application, suppose researchers can collaborate on a project. Each researcher brings something heterogenous and positive to the value of the collaboration, so that the value of the collaboration is increasing in the set of agents involved. Collaboration is possible only when it takes place among agents who are directly connected to another collaborator and surplus is split according to the Myerson value (as in our work, motivated by robustness to renegotiations). Such a setting fits into our framework.

results in Section 6, which we use to take our model to the data in Section 7. Section 8 concludes.

## 2. Preliminaries and Risk Sharing on a Fixed Network

To study social investments and the network formation problem, first we need to specify what risk-sharing arrangements take place once the network is formed. Below we introduce an economy in which agents face random income realizations, introduce some basic network terminology, and discuss risk-sharing arrangements for a given network.

2.1. **The socio-economic environment.** We denote the set of agents in our model by $\mathbf{N}$, and assume that they are partitioned into a set of groups $\mathbf{M}$. We let $G : \mathbf{N} \to \mathbf{M}$ be a function that assigns each agent to a group; i.e., if $G(i) = g$ then agent $i$ is in group $g$. One interpretation of the group partitioning is that $\mathbf{N}$ represents individuals in a region (such as a district or subdistrict), and groups correspond to different villages in the region. Another possible interpretation is that $\mathbf{N}$ represents individuals in a village, and the groups correspond to different castes.

Agents in $\mathbf{N}$ face uncertain income realizations. For tractability, we assume that incomes are jointly normally distributed, with expected value $\mu$ and variance $\sigma^2$ for each agent.[13] We assume that the correlation coefficient between the incomes of any two agents within the same group is $\rho_w$, while between the incomes of any two agents not in the same group it is $\rho_a < \rho_w$.[14] That is, we assume that incomes are more positively correlated within groups than across groups, so that all else equal, social connections across groups have a higher potential for risk sharing.

Although we introduce the possibility of correlated incomes in a fairly stylized way, our paper is one of the first to permit differently correlated incomes between different pairs of agents. Such correlations are central to the effectiveness of risk-sharing arrangements, as shown below.

We refer to possible realizations of the vector of incomes as *states*, and denote a generic state by $\omega$. We let $y_i(\omega)$ denote the income realization of agent $i$ in state $\omega$.

Agents can redistribute realized incomes; hence their consumption levels can differ from their realized incomes. We assume that all agents have constant absolute risk aversion (CARA) utility functions:

$$v(c_i) = -\frac{1}{\lambda} e^{-\lambda c_i}, \tag{1}$$

where $c_i$ is agent $i$'s consumption and $\lambda > 0$ is the coefficient of absolute risk aversion. The assumption of CARA utilities, together with jointly normally distributed incomes, greatly

[13]This specification implies that we cannot impose a lower bound on the set of feasible consumption levels. As we show below, our framework readily generalizes to arbitrary income distributions, but the assumption of normally distributed shocks simplifies the analysis considerably.

[14]It is well-known that for a vector of random variables, not all combinations of correlations are possible. We implicitly assume that our parameters are such that the resulting correlation matrix is positive semidefinite.

enhances the tractability of our model: as we show below it leads to a transferable utility environment in which the implemented risk-sharing arrangements are relatively simple. This utility formulation can also be considered a theoretical benchmark case with no income effects. The empirical relevance of predictions from this benchmark model is examined in Section 6. We generalize the theory in Appendix I.

2.2. **Basic network terminology.** Before proceeding, we introduce some standard terminology from network theory. A social network $L$ is an undirected graph, with nodes $\mathbf{N}$ corresponding to the different agents, and links representing social connections. Abusing notation we also let $L$ denote the set of links in the network. We will refer to the agents linked to agent $i$, $\mathbf{N}(i;L) := \{j : l_{ij} \in L\} \subset \mathbf{N}$, as $i$'s *neighbors*. Where there should be no confusion we abuse notation by writing $\mathbf{N}(i)$ instead of $\mathbf{N}(i;L)$. The *degree centrality* of an agent is simply the number of neighbors she has (i.e., the cardinality of $\mathbf{N}(i;L)$). An agent's neighbors can be partitioned according to the groups they belong to. Let $\mathbf{N}_g(i;L)$ be $i$'s neighbors on network $L$ from group $g$. A *walk* is a sequence of different agents $\{i, k, k', \ldots, k'', j\}$ such that every pair of adjacent agents in the sequence is linked. A *path* is a walk in which all agents are different. The *path length* of a path is the number of agents in the path.

We will sometimes refer to subsets of agents $\mathbf{S} \subseteq \mathbf{N}$ and denote the subgraphs they generate by $L(\mathbf{S}) := \{l_{ij} \in L : i, j \in \mathbf{S}\}$. A subset of agents $\mathbf{S} \subseteq \mathbf{N}$ is *path-connected* on $L$ if, for each $i \in \mathbf{S}$ and each $j \in \mathbf{S}$, there exists a path connecting $i$ and $j$. For any network there is a unique partition of $\mathbf{N}$ such that there are no links between agents in different partition elements but all agents within a partition element are path-connected. We refer to these partition elements as *network components*. A *shortest path* between two path-connected agents $i$ and $j$ is a path connecting $i$ and $j$ with a lower path length than any other. The *diameter* of a network component $C \subset L$ is $d(C)$, the maximum value—taken over all pairs of agents in $C$—of the length of a shortest path. A network component is a *tree* when there is a unique path between any two agents in the component. A *line network* is the unique (tree) network, up to a relabeling of agents, in which there is a path from one (end) agent to the other (end) agent that passes through all other agents. A *star network* is the unique tree network, up to a relabeling of agents, in which one (center) agent is connected to all other agents.

2.3. **Risk-Sharing Agreements.** We assume that income cannot be directly shared between agents $i, j \in \mathbf{N}$ unless they are connected, i.e., $l_{ij} \in L$. However, we let agents' income realizations be publicly observed within their network component, so agents can make transfer arrangements contingent on it. We consider this environment with perfectly observable incomes within a component as a benchmark model, which is a relatively good description of village societies in which people closely monitor each other. It is also straightforward to extend the model so that some income is publicly observed (and shared) while the remaining income is privately observed (and never shared). Results are very similar for this more general setting.[15]

Formally, a risk-sharing agreement on a network $L$ specifies transfer $t_{ij}(\omega, L) = -t_{ji}(\omega, L)$ between neighboring agents $i$ and $j$ for every possible state $\omega$. Abusing notation where there should be no confusion we sometimes drop the second argument and write $t_{ij}(\omega)$ instead of $t_{ij}(\omega, L)$. The interpretation is that in state $\omega$ agent $i$ is supposed to transfer $t_{ij}(\omega)$ units of consumption to agent $j$ if $t_{ij}(\omega) > 0$, and receives this amount from agent $j$ if $t_{ij}(\omega) < 0$. Given a transfer arrangement between neighboring agents, agent $i$'s consumption in state $\omega$ is $c_i(\omega) = y_i(\omega) - \sum_{j \in \mathbf{N}(i)} t_{ij}(\omega)$. It is straightforward to show that state-contingent consumption plans $(c_i(\cdot))_{i \in \mathbf{N}}$ are feasible, that is they can be achieved by bilateral transfers between neighboring agents, if and only if for each component $C$, contained agents $S$, $\sum_{i \in S} c_i(\omega) = \sum_{i \in S} y_i(\omega)$ for every state $\omega$.

A basic assumption we make in our model is that given all other risk-sharing arrangements, an agreement reached by linked agents $i$ and $j$ must leave no gains from trade on the table. There must be no other agreement that can make both $i$ and $j$ strictly better off holding fixed the agreements of other players. We call such transfers *pairwise efficient*.[16]

By the well-known Borch rule (see Borch (1962), Wilson (1968)) a necessary and sufficient condition for this property is that for all neighboring agents $i$ and $j$,

$$\left(\frac{\partial v_i(c_i(\omega))}{\partial c_i(\omega)}\right) \Big/ \left(\frac{\partial v_j(c_j(\omega))}{\partial c_j(\omega)}\right) = \left(\frac{\partial v_i(c_i(\omega'))}{\partial c_i(\omega')}\right) \Big/ \left(\frac{\partial v_j(c_j(\omega'))}{\partial c_j(\omega')}\right) \tag{2}$$

for every pair of states $\omega$ and $\omega'$. But if this holds for all neighboring agents $i$ and $j$ then the same condition must hold for all pairs of agents on a component of $L$, independently of whether they are directly or indirectly connected. Hence, pairwise-efficient risk-sharing arrangements are equivalent to Pareto-efficient agreements at the component level. For this reason, below we establish some important properties of Pareto-efficient risk-sharing arrangements on components.

Proposition 1 shows that the CARA utilities framework has the convenient property that expected utilities are transferable, in the sense defined by Bergstrom and Varian (1985). This can be used to show that ex-ante Pareto efficiency is equivalent to minimizing the sum of the variances, and it is achieved by agreements that in every state split the sum of the incomes on each network component equally among the members and then adjust these shares by state-independent transfers. The latter determine the division of the surplus created by the

[15]Kinnan (2011) finds evidence that hidden income can explain imperfect risk sharing in Thai villages relative to the enforceability and moral hazard problems we are abstracting from. Cole and Kocherlakota (2001) show that when individuals can privately store income, state-contingent transfers are not possible and risk sharing is limited to borrowing and lending.

[16]More formally, transfers $\{t_{ij}(\omega, L)\}_{\omega \in \Omega,\, ij: l_{ij} \in L}$ are pairwise efficient for a network $L$ if there is no pair of agents $ij : l_{ij} \in L$ and no alternative transfers $\{t'_{ij}(\omega, L)\}_{\omega \in \Omega,\, ij: l_{ij} \in L}$ such that $t'_{kl}(\omega, L) = t_{kl}(\omega, L)$ for all $kl \neq ij$ and all $\omega \in \Omega$, that gives both $i$ and $j$ strictly higher expected utility.

risk sharing agreement. We emphasize that this result does not require any assumption on the distribution of incomes, only that agents have CARA utilities.

**Proposition 1.** *For CARA utility functions certainty-equivalent units of consumption are transferable across agents, and if $L(\mathbf{S})$ is a network component, the Pareto frontier of ex-ante risk-sharing agreements among agents in $\mathbf{S}$ is represented by a simplex in the space of certainty-equivalent consumption. The ex-ante Pareto-efficient risk-sharing agreements for agents in $\mathbf{S}$ are those that satisfy*

$$\min \sum_{i \in \mathbf{S}} \operatorname{Var}(c_i) \quad \text{subject to} \quad \sum_{i \in \mathbf{S}} c_i(\omega) = \sum_{i \in \mathbf{S}} y_i(\omega) \quad \text{for every state } \omega,$$

*and they consist of agreements of the form*

$$c_i(\omega) = \frac{1}{|\mathbf{S}|} \sum_{k \in \mathbf{S}} y_k(\omega) + \tau_i \quad \text{for every } i \in \mathbf{S} \text{ and state } \omega,$$

*where $\tau_i \in \mathbb{R}$ is a state-independent transfer made to $i$ and $\sum_{k \in \mathbf{S}} \tau_k = 0$.*

The proof of Proposition 1 is in Section A of the Supplementary Appendix. Proposition 1 implies that the total surplus generated by efficient risk-sharing arrangements is an increasing function of the reduction in aggregate consumption variance (the sum of consumption variances). For a general distribution of shocks, this function can be complicated. However, if shocks are jointly normally distributed then $c_i = \frac{1}{|\mathbf{S}|} \sum_{k \in \mathbf{S}} y_k + \tau_i$ is also normally distributed, and $\mathrm{E}(v(c_i)) = \mathrm{E}(c_i) - \frac{\lambda}{2} \operatorname{Var}(c_i)$.[17] Hence in this case the total social surplus generated by efficient risk-sharing agreements is proportional to the aggregate consumption variance reduction. This greatly simplifies the computation of surpluses in the analysis below.

We use $TS(L)$ to denote the expected total surplus generated by an ex-ante Pareto-efficient risk-sharing agreement on network $L$, relative to agents consuming in autarky:

$$TS(L) := CE\Big(\Delta \operatorname{Var}(L, \emptyset)\Big), \tag{3}$$

where, for $L' \subset L$, $\Delta \operatorname{Var}(L, L')$ is the additional variance reduction obtained by efficient risk-sharing on network $L$ instead of $L'$, and $CE(\cdot)$ denotes the certainty-equivalent value of a variance reduction.

For a network $L$, consisting of a single component, if all agents are from the same group then as there are CARA utility functions and normally distributed incomes

$$TS(L) = CE\Big(\Delta \operatorname{Var}(L, \emptyset)\Big) = \frac{\lambda}{2}\Big(\Delta \operatorname{Var}(L, \emptyset)\Big) = \frac{\lambda}{2}(n-1)\sigma^2(1-\rho_w) = (n-1)V, \tag{4}$$

where $V := \frac{\lambda}{2}\sigma^2(1-\rho_w)$.

2.4. **Division of Surplus.** The assumption that neighboring agents make pairwise-efficient risk-sharing agreements pins down agreements up to state-independent transfers between neighboring agents, but does not constrain the latter transfers (hence the division of surplus)

[17]See, for example, Arrow (1965).

in any way. To determine these transfers, we follow the approach in Stole and Zwiebel (1996) and require that agreements are robust to split-the-difference renegotiations. This implies that the transfer is set in a way such that the incremental benefit that the link provides to the two agents is split equally between them. This can be interpreted as a social norm. For a detailed motivation of this assumption, and for noncooperative microfoundations, see Stole and Zwiebel (1996).

Splitting the incremental benefits of a risk sharing link equally between two agents requires calculating the expected payoffs $i$ and $j$ would receive if they did not have an agreement. We therefore have to consider what agreements would prevail on the network without $l_{ij}$ to find the risk sharing agreements $i$ and $j$ can reach on $L$, and so on. This results in a recursive system of conditions.

More formally, for a network $L$ a contingent transfer scheme

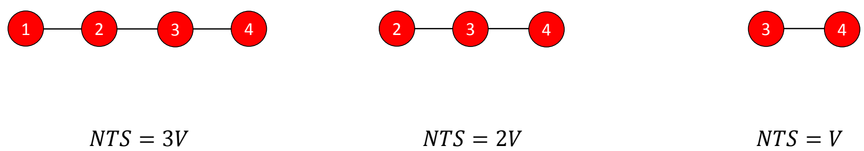
$$\mathcal{T}(L) := \{t_{ij}(\omega, L')\}_{\omega\in\Omega,\, L'\subseteq L,\, ij:l_{ij}\in L}, \tag{5}$$

specifies all transfers made in all subnetworks of $L$ in all states of the world. The expected utility of agent $i$ on a network $L' \subseteq L$ given a contingent transfer scheme $\mathcal{T}(L)$ is denoted $u_i(L', \mathcal{T}(L))$. Where there should be no confusion, we will abuse notation and drop the second argument.

For any network $L$, the expected utility vector $(u_1, ..., u_{|\mathbf{N}|})$ is *robust to split-the-difference renegotiation* if there is a contingent transfer scheme $\mathcal{T}(L)$ such that $u_i = u_i(L, \mathcal{T}(L))$ for every $i \in \mathbf{N}$ and the following conditions hold:

(i) $u_i(L') - u_i(L' \setminus \{l_{ij}\}) = u_j(L') - u_j(L' \setminus \{l_{ij}\})$ for every $l_{ij} \in L'$ and $L' \subseteq L$;
(ii) transfers $\{t_{ij}(\omega, L')\}_{\omega\in\Omega,\, ij:l_{ij}\in L'}$ are pairwise efficient for all $L' \subseteq L$.

Suppose all agents are from the same group, we have CARA utilities, incomes are normally distributed and we want to find payoffs robust to split-the-difference renegotiation for the line network shown in Figure 1a. A first necessary condition is that agents 1 and 2 benefit equally from their link so that $u_1(L) - u_1(L \setminus \{l_{12}\}) = u_2(L) - u_2(L \setminus \{l_{12}\})$. But in order to ensure this condition is satisfied, we need to know $u_1(L \setminus \{l_{12}\})$ and $u_2(L \setminus \{l_{12}\})$. Normalizing the autarky utility of all agents to 0, without the link $l_{12}$ agent 1 is isolated so $u_1(L \setminus \{l_{12}\}) = 0$. However, to find $u_2(L \setminus \{l_{12}\})$ we need to find payoffs for the three node network in Figure 1b. For this network robustness to split-the-difference renegotiation requires that $u_2(L\setminus\{l_{12}\}) - u_2(L\setminus\{l_{12}, l_{23}\}) = u_3(L\setminus\{l_{12}\}) - u_3(L\setminus\{l_{12}, l_{23}\})$. While $u_2(L\setminus\{l_{23}, l_{23},\}) = 0$, we need to consider the two node network shown in Figure 1c to find $u_3(L \setminus \{l_{12}, l_{23}\})$. For this network, payoffs must satisfy $u_3(L\setminus\{l_{12}, l_{23}\}) - u_3(L\setminus\{l_{12}, l_{23}, l_{34}\}) = u_4(L\setminus\{l_{12}, l_{23}\}) - u_4(L \setminus \{l_{12}, l_{23}, l_{34}\})$. As $u_3(L \setminus \{l_{12}, l_{23}, l_{34}\}) = u_4(L \setminus \{l_{12}, l_{23}, l_{34}\}) = 0$, the above condition simplifies to $u_3(L \setminus \{l_{12}, l_{23}\}) = u_4(L \setminus \{l_{12}, l_{23}\}) = V/2$, where the last equality follows from pairwise efficiency. Considering the three node network again, we now have the condition $u_2(L \setminus \{l_{12}\}) = u_3(L \setminus \{l_{12}\}) - V/2$. As the link $l_{23}$ generates an incremental surplus of $V$ to be split between agents 2 and 3, pairwise efficiency implies that $u_2(L \setminus \{l_{12}\}) = V/2$ and




(A) Line network (B) Three nodes (C) Two nodes

FIGURE 1. To find (gross) expected utilities that are robust to split-the-difference renegotiations on the (formed) line network shown we need to consider the expected utilities that would be obtained on all subnetworks.

$u_3(L \setminus \{l_{12}\}) = V$. Finally, returning to the line network, we now have $u_1(L) = u_2(L) - V/2$. As the link $l_{12}$ generates incremental surplus of $V$, $u_1(L) = V/2$ and $u_2(L) = V$.[18]

Below we show that the requirement of robustness to split-the-difference renegotiation implies that the total surplus created by the risk-sharing agreement is divided among agents according to the Myerson value (Myerson 1977, 1980). The Myerson value is a cooperative solution concept defined in transferable utility environments that is a network-specific version of the Shapley value. The basic idea behind it is the same as for the Shapley value.[19] For any order of arrivals of the players, the incremental contribution of an agent $i$ to the total surplus can be derived as the difference between the total surpluses generated by subgraph $L(\mathbf{S})$ and subgraph $L(\mathbf{S} \setminus \{i\})$ if agents $\mathbf{S} \setminus \{i\}$ arrive before $i$. It is easy to see that, for any arrival order, the total surplus generated by $L$ gets exactly allocated to the set of all agents. The Myerson value then allocates the average incremental contribution of a player to the total surplus, taken over all possible orders of arrivals (permutations) of the players, as the player's share of the total surplus. Thus, agent $i$'s Myerson value is[20]

$$MV_i(L) := \sum_{\mathbf{S} \subseteq \mathbf{N}} \frac{(|\mathbf{S}| - 1)!(|\mathbf{N}| - |\mathbf{S}|)!}{|\mathbf{N}|!} \Big( TS(L(\mathbf{S})) - TS(L(\mathbf{S} \setminus \{i\})) \Big). \tag{6}$$

[18]This argument only outlines why the payoffs $u_1(L) = V/2$ and $u_2(L) = V$ are necessary for robustness to split-the-difference renegotiations. By considering all other subnetworks, it can be shown that the payoffs $u_1(L) = u_4(L) = V/2$ and $u_2(L) = u_3(L) = V$ are the unique payoffs that are robust to split-the-difference renegotiations.

[19]We therefore follow Hart and Moore (1990), among others, in using the Shapley value to study investment decisions.

[20]Our assumption that there is perfect risk sharing among path-connected agents ensures that a coalition of path-connected agents generates the same surplus regardless of the exact network structure connecting them. This means that we are in the communication game world originally envisaged by Myerson. We do not require the generalization of the Myerson value to network games proposed in Jackson and Wolinsky (1996), which somewhat confusingly is also commonly referred to as the Myerson value. See Ambrus, Gao and Milan (2016) for a model of informal risk-sharing in which the exact shape of the network matters in terms of the surplus that agents can attain.

**Proposition 2.** *For any network $L$, any risk-sharing agreement that is robust to split-the-difference renegotiation yields expected payoffs to agents equal to their Myerson values: $u_i(L) = MV_i(L)$.*

*Proof.* Theorem 1 of Myerson (1980) states that there is a unique rule for allocating surplus for all subnetworks of $L$ that satisfies the requirements of efficiency at the component level (note that this is an implicit requirement in Myerson's definition of an allocation rule) and, what Myerson (1980) defines as the equal-gains principle. Moreover, the expected payoff the above rule allocates to any player $i$ is $MV_i$. Requirement (i) in our definition of robustness to split-the-difference renegotiation is equivalent to the equal-gains principle as defined in Myerson (1980). Theorem 1 of Wilson (1968) implies that efficiency at the component level is equivalent to pairwise efficiency between neighboring agents, which is requirement (ii) in our definition of robustness to split-the-difference renegotiation. The result then follows immediately from Theorem 1 of Myerson (1980). □

Proposition 2 is a direct implication of Myerson's axiomatization of the value. A special case of Proposition 2 is Theorem 1 of Stole and Zwiebel (1996), which in effect restricts attention to a star network.[21] Our contribution is to point out that their connection between robustness to split-the-difference renegotiations and the Shapley value can be extended to apply to all networks.

The above result shows that any decentralized negotiation procedure between neighboring agents that satisfies two natural properties (not leaving surplus on the table, and robustness to split-the-difference negotiations) leads to the total surplus created by risk-sharing divided according to the Myerson value, and to state-independent transfers between neighboring agents that implement this surplus division. Hence, from now on we assume that in the network formation process, all agents expect the surplus to be divided according to the Myerson value implied by the network that eventually forms.

Although we followed a decentralized approach to get to the implication that surplus is divided by the Myerson value, we note that on normative grounds such a division is also cogent in contexts in which there is a centralized community level negotiation over the division of surplus. This is because the Myerson value is a formal way of defining the fair share of an individual from the social surplus, as his average incremental contribution to the total social surplus (where the average is taken across all possible orders of arrival of different players, in the spirit of the Shapley value).

In Appendix I, we generalize our model by relaxing the CARA utility assumption, relaxing the assumption that incomes are normally distributed and considering a broad class of allocation rules.

[21] Relative to Myerson's axiomatization, Stole and Zwiebel (1996) generate the key system of equations through considering robustness to renegotiations as we describe above, while Myerson wrote down the system of equations based only on fairness considerations. Stole and Zwiebel (1996) also provide non-cooperative bargaining foundations that underpin this system.

## 3. Investing in Social Relationships

Having defined how formed networks map into risk-sharing arrangements, we can now consider agents' incentives to make investments into social capital, which we think of as the set of relationships that enable risk sharing. We begin by providing the overall framework for the analysis. Then we look at a special case of our model, in which there is a single group. Building on these results we then consider the multiple group case.

In this section we formalize a game of network formation in which establishing links is costly, define efficient networks and identify different types of investment inefficiency.

We consider a two-period model in which in period 1 all agents simultaneously choose which other agents they would like to form links with, and in period 2 agents agree upon the ex-ante Pareto-efficient risk-sharing agreement specified in the previous section (i.e., the total surplus from risk sharing is distributed according to the Myerson value), for the network formed in the first period.[22]

Implicit in our formulation of the timing of the game is the view that relationships are formed over a longer time horizon than that in which agreements are reached about risk sharing. By the time such agreements are being negotiated, the network structure is fixed, and investments into forming social relationships are sunk. In addition, as mentioned in the introduction, the second stage agreements can be viewed as a reduced form treatment of a dynamic game with many state realizations—as long as the discount factor is high enough, our agreements will satisfy the required incentive compatibility constraints for an equilibrium.

In period 1 the solution concept we apply to identify which networks form is pairwise stability. The collection of links formed is social network $L$, and agent $i$ pays a cost $\kappa_w > 0$ for each link $i$ has to someone in the same group, and $\kappa_a > \kappa_w$ for each links $i$ has to someone from a different group. Normalizing the utility from autarky to 0, we abuse notation[23] and let agent $i$'s *net* expected utility if network $L$ forms be

$$u_i(L) = MV_i(L) - |\mathbf{N}_{G(i)}(i;L)|\kappa_w - \Big(|\mathbf{N}(i;L)| - |\mathbf{N}_{G(i)}(i;L)|\Big)\kappa_a. \tag{7}$$

A network $L$ is *pairwise stable* with respect to expected utilities $\{u_i(L)\}_{i\in\mathbf{N}}$ if and only if for all $i, j \in \mathbf{N}$, (i) if $l_{ij} \in L$ then $u_i(L) - u_i(L \setminus \{l_{ij}\}) \geq 0$ and $u_j(L) - u_j(L \setminus \{l_{ij}\}) \geq 0$; and (ii) if $l_{ij} \notin L$ then $u_i(L \cup l_{ij}) - u_i(L) > 0$ implies $u_j(L \cup l_{ij}) - u_j(L) < 0$. In words, pairwise stability requires that no two players can both strictly benefit by establishing an extra link with each other, and no player can benefit by unilaterally deleting one of his links. From now on we will use the terms pairwise-stable and stable interchangeably.

[22]For a complementary treatment of network formation when surplus is split according to the Myerson value, see Pin (2011).

[23]In the previous section when investments had already been sunk we used $u_i(L)$ to denote $i$'s expected payoff before link formation costs.

Existence of a pairwise-stable network in our model follows from a result in Jackson (2003), stating that whenever payoffs in a simultaneous-move network formation game are determined based on the Myerson value, there exists a pairwise-stable network.

Our specification assumes that two agents forming a link have to pay the same cost for establishing the link. However, the set of stable networks would remain unchanged if we allowed the agents to share the total costs of establishing a link arbitrarily.[24] This is because for any link, the Myerson value rewards the two agents establishing the link symmetrically. Hence the agents can find a split of the link-formation cost such that establishing the link is profitable for both of them if and only if it is profitable for both of them to form the link with an equal split of the cost. Given this we stick with the simpler model with exogenously given costs.

A network $L$ is *efficient* when there is no other network $L'$—and no risk sharing agreement on $L'$—that can make everyone at least as well off as they were on $L$ and someone strictly better off. Let $|L_w|$ be the number of within-group links, and let $|L_a|$ be the number of across-group links. As expected utility is transferable in certainty-equivalent units, efficient networks must maximize the net total surplus $NTS(L)$:

$$NTS(L) := TS(L) - 2|L_w|\kappa_w - 2|L_a|\kappa_a, \tag{8}$$

Clearly, two necessary conditions for a network to be efficient are that the removal of a set of links does not increase $NTS(L)$ and the addition of a set of links does not increase $NTS(L)$. If there exists a set of links the removal of which increases $NTS(L)$, we will say there is *overinvestment* inefficiency. If there exists a set of links the addition of which increases $NTS(L)$, we will say there is *underinvestment* inefficiency.[25] A network is *robust to underinvestment* if there is no underinvestment inefficiency and no agent can strictly benefit from deleting a link that would result in underinvestment inefficiency. A network is *robust to overinvestment* if there is no overinvestment inefficiency and no pair of agent $i, j$ can both strictly benefit from creating the link $l_{ij}$.

We will say that a link $l_{ij}$ is *essential* if after its removal $i$ and $j$ are no longer path-connected while it is *superfluous* if after its removal $i$ and $j$ are still path-connected.

**Remark 3.** *Preventing overinvestment requires that all links be essential. Superfluous links create no social surplus and are costly. In all efficient networks, therefore, every component must be a tree.*

[24]More precisely, we could allow agents to propose a division of the costs of establishing each link as well as indicating who they would like to link to, and a link would then form only if both agents indicate each other and they propose the same split of the cost. A network would then be stable if it is a Nash equilibrium of this expanded network formation game and if there is no new link $l_{ij} \notin L$ , and some split of the cost of forming this link, that would make both $i$ and $j$ strictly better off if formed.

[25]Note that these definitions are not mutually exclusive (there can be both underinvestment and overinvestment inefficiency) or collectively exhaustive (inefficient networks can have neither underinvestment nor overinvestment inefficiency if an increase in the net total surplus is only possible by the simultaneous addition and removal of edges).

Real world networks among villagers are a long way from being trees (see Section 7). If our model perfectly captured network formation Remark 3 would imply that there is substantial overinvestment. However, our model is stylized, and this result needs to be applied with caution. For example, while there may be overinvestment, our assumption that all links are costly to form is unlikely to hold. Family ties or the time villagers spend working together might permit relationships to be formed without any additional investment. We discuss in Appendix II how, what we view as the main insights of our results, extend to a setting in which some links are free to form.

In most of the analysis below, we focus on investigating the relationship between stable networks and efficient networks. Additionally, we investigate the amount of inequality prevailing in equilibria in our model. For this, we will use the Atkinson class of inequality measures (Atkinson, 1970). Specifically we consider a welfare function $W : \mathbb{R}^{|\mathbf{N}|} \to \mathbb{R}$ that maps a profile of expected utilities into the real line such that

$$W(u) = \sum_{i \in \mathbf{N}} f(u_i), \tag{9}$$

where $f(\cdot)$ is assumed to be an increasing, strictly concave and differentiable function. The concavity of $f(\cdot)$ captures the social planner's preference for more equal income distributions. Supposing all agents instead received the same expected utility $u'$, we can pose the question what aggregate expected utility is required to keep the level of the welfare function constant.[26] In other words we find the scalar $u' : |\mathbf{N}| f(u') = \sum_{i \in \mathbf{N}} f(u_i)$. Letting $\overline{u} = (1/|\mathbf{N}|) \sum_{i \in \mathbf{N}} u_i$ be the mean expected utility, Atkinson's inequality measure (or index) is given by

$$I(f) = 1 - \frac{u'}{\overline{u}} \in [0, 1]. \tag{10}$$

We let $\mathcal{I}$ be the set (class) of Atkinson inequality measures and note that any $I(f) \in \mathcal{I}$ equals zero if and only if all agents receive the same expected utility.[27] Two different inequality measures from the Atkinson class can rank the inequality of two distributions differently. However, certain pairs of distributions are ranked the same way by all members of the class, such as when one distribution is a mean-preserving spread of the other one.

## 4. Within-Group Network Formation

In this section we assume that $|\mathbf{M}| = 1$, that is, that agents are ex-ante symmetric, and any differences in their outcomes stem from their stable positions on the social network. This will lay the foundations for the more general case considered in the next section.

We begin our investigation by proving a general characterization of the set of stable networks. Recall that a *path* between $i$ and $j$ is a walk in which no agent is visited more

[26] This exercise is analogous to the certainty equivalent exercise that can be undertaken for an agent facing stochastic consumption.

[27] As $f(\cdot)$ approaches the linear function the social planner cares less about inequality and $I(f) \to 0$. Nevertheless, strict concavity prevents $I(f)$ equaling 0 unless all agents receive the same expected utility.

than once. If there are $K$ paths between $i$ and $j$ on the network $L$, we let $\mathbf{P}(i,j,L) = \{P_1(i,j,L),\ldots,P_K(i,j,L)\}$ be the set of these paths. For every $k \in \{1,\ldots,K\}$, let $|P_k(i,j,L)|$ be the cardinality of the set of agents on the path $P_k(i,j,L)$.[28] We can now use these definitions to define a quantity that captures how far away two agents are on a network in terms of the probability that for a random arrival order they will be connected without a direct link when the second of the two agents arrives. We will refer to this distance as the agents' Myerson distance:

$$md(i,j,L) := \frac{1}{2} - \sum_{k=1}^{|\mathbf{P}(i,j,L)|} (-1)^{k+1} \left( \sum_{1 \le i_1 < \cdots < i_k \le |\mathbf{P}(i,j,L)|} \left( \frac{1}{|P_{i_1} \cup \cdots \cup P_{i_k}|} \right) \right). \tag{11}$$

This expression calculates the probability that for a random arrival order the link $l_{ij}$ will be essential immediately after $i$ arrives,[29] using the classic *inclusion–exclusion principle* from combinatorics. This probability is important because it affects $i$'s incentives to link to $j$.

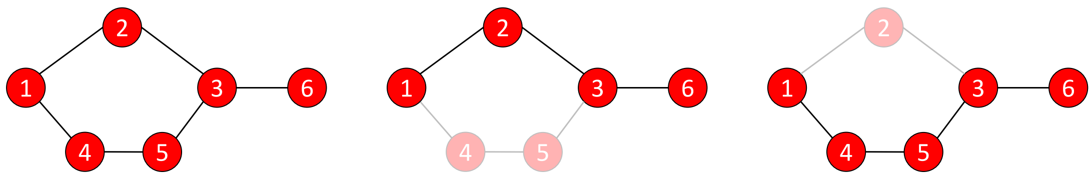
As an illustration, consider the network shown in Panel (A) of Figure 2. The Myerson value allocates each agent their average marginal contribution to total surplus, where the average is taken over all possible arrival orders. For example, for the network shown in Figure 2 consider the arrival order $1,2,5,6,3,4$. When agent 1 is added there are no other agents and so no links are formed. Thus 1's marginal contribution to total surplus is 0. Then agent 2 is added and the link $l_{12}$ is formed. This link is essential on this network permitting risk sharing between agents 1 and 2 that wasn't previously possible. As a result, by equation 4, the total surplus generated by risk sharing increases from 0 to $V$. Thus 2's marginal contribution to total surplus, for this arrival order, is $V$. When 5 and 6 are added no new links are formed and no additional risk sharing is possible—their marginal contributions are 0. However, the arrival of 3 results in the formation of the links $l_{23}, l_{35}$ and $l_{36}$. All of these links are essential and risk sharing among agents $1,2,5,6$ and 3 becomes possible. This increases total surplus to $4V$ by equation 4, so 3's marginal contribution to total surplus is $3V$. Finally, adding 4 the links $l_{14}$ and $l_{45}$ are formed, and this permits risk sharing to also include 4 increasing total surplus to $5V$. So 4's marginal contribution to total surplus is $V$.

Whenever a link is formed that is essential for a given arrival order, it contributes $V$ to total surplus, while whenever a link is superfluous for a given arrival order, it makes a marginal contribution of 0 to total surplus.[30] Consider now the incentives agent 1 has to form a superfluous link to agent 6. To calculate this we need to know the probability with which such a link would be essential for a random arrival order. There are three ways in which the link $l_{16}$ might not be essential upon $i$'s arrival. First, with probability $1/2$ agent

[28] For example, for a path $P_k(i,j,L) = \{i, i', i'', j\}$, $|P_k(i,j,L)| = 4$ and for a path $P_{k'}(i,j,L) = \{i, i', i''', i'''', j\}$, $|P_{k'}(i,j,L)| = 5$. Finally, we will let $|P_k(i,j,L) \cup P_{k'}(i,j,L)| = 5$ denote number of different agents on path $P_k(i,j,L)$ or path $P_{k'}(i,j,L)$.

[29] If for a given arrival order, agents $\mathbf{S} \subseteq \mathbf{N}$ arrive before $i$, then $l_{ij}$ is essential immediately after $i$ arrives if it is essential on the network $L(\mathbf{S} \cup \{i\})$.

[30] Note that in the arrival order considered in the preceding paragraph, 4's marginal contribution to total surplus would still have been $V$ without the link $l_{14}$ ($l_{45}$) as long as the link $l_{45}$ ($l_{14}$) was still formed.




(A) Network (B) Path 1 (C) Path 2

FIGURE 2. Paths connecting nodes 1 and 6.

6 arrives after agent 1 and the link $l_{16}$ will be formed on 6's arrival instead of 1's. Second, Path 1 shown in Panel (B) of Figure 2 might be present. This will be the case if and only if agents 2, 3 and 6 arrive before agent 1. The probability that agent 1 is last to arrive of these 4 agents is $1/4$. Finally, Path 2 shown in Panel (C) of Figure 2 might be present. This occurs if and only if agents 3, 4 ,5 and 6 arrive before 1. The probability that 1 is last to arrive of these 5 agents is $1/5$.

If these three possibilities were mutually exclusive, then the probability the link $l_{16}$ would be formed and essential upon 1's arrival would be: $1-1/2-1/4-1/5$. The probability that agent 6 arrives after agent 1 is mutually exclusive from the probability that either Path 1 or Path 2 is present, because both these paths need agent 6 to arrive before agent 1. However, it is possible for both Path 1 and Path 2 to be formed upon 1's arrival. Indeed, this occurs if and only if agent 1 is the last agent to arrive, which happens with probability $1/6$. So the probability that at least one of the two paths to agent 6 is present upon 1's arrival is $1/4+1/5-1/6$. We need to subtract the probability $1/6$ to avoid double counting the event that both paths are present. Thus, the probability that the link $l_{16}$ will be essential upon 1's arrival, is $1-1/2-1/4-1/5+1/6 = md(1,6,L)$.

**Lemma 4.** *If all agents are from the same group network $L$ is pairwise stable if and only if*

$$\begin{array}{llll} (i) & md(i,j,L\setminus\{l_{ij}\}) & \geq \kappa_w/V & \text{for all } l_{ij}\in L, \text{ and} \\ (ii) & md(i,j,L) & \leq \kappa_w/V & \text{for all } l_{ij}\notin L. \end{array}$$

The proof is relegated to Section A of the Supplementary Appendix. Recall from equation 3 that the social benefits of a link is proportional to the variance reduction it generates. For a single group, if a link $l_{ij}$ is essential in the network $L\cup\{l_{ij}\}$, then this variance reduction is $\Delta\operatorname{Var}(L\cup\{l_{ij}\},L) = (1-\rho_w)\sigma^2$.

The crucial feature of this expression is that it does not depend on size of the network components the link $l_{ij}$ connects on $L$. Although in general the size of these components does affect the consumption variance, two effects exactly offset each other.[31] On the one hand,

[31]Let $L(\mathbf{S}_1)$ and $L(\mathbf{S}_2)$ be the network components of agent $i$ and agent $j$ on network $L\setminus\{l_{ij}\}$, and let $|\mathbf{S}_1| = s_1$ and $|\mathbf{S}_2| = s_2$. Then the sum of consumption variances on $L(\mathbf{S}_1)$ and $L(\mathbf{S}_2)$ (with Pareto efficient risk sharing) are $\frac{s_1+s_1(s_1-1)\rho_w}{s_1}\sigma^2$ and $\frac{s_2+s_2(s_2-1)\rho_w}{s_2}\sigma^2$, respectively. Once $S_1$ and $S_2$ are connected through $l_{ij}$, the sum of consumption variances on $L(\mathbf{S}_1\cup\mathbf{S}_2)$ becomes $\frac{s_1+s_2+(s_1+s_2)(s_1+s_2-1)\rho_w}{s_1+s_2}\sigma^2$. This implies that the consumption variance reduction induced by the link $l_{ij}$ is $\Delta\operatorname{Var}(L\cup\{l_{ij}\},L) = (1-\rho_w)\sigma^2$.

in larger components there are more people to benefit from the essential link. On the other hand, people are already able to smooth there consumption more effectively.

As the social value of a non-essential, or superfluous link, is always zero the total surplus generated by a network $L$ takes a very simple form. Let $\Upsilon(L)$ be the number of network components on $L$. Then

$$TS(L) = CE\Big(\Delta\,\mathrm{Var}(L,\emptyset)\Big) = \Big(|\mathbf{N}| - \Upsilon(L)\Big)\frac{\lambda}{2}\Big(1-\rho_w\Big)\sigma^2 = \Big(|\mathbf{N}| - \Upsilon(L)\Big)V. \tag{12}$$

Since the surplus created by any essential link is $V$, the total gross surplus is equal to this constant times the number of network component reductions obtained relative to the empty network.

To consider individual incentives to form links we can use the definition of the Myerson value and consider the average marginal contribution an agent makes to total surplus over all possible arrival orders. Specifically, we want to consider the increase in $i$'s Myerson value due to a link $l_{ij}$. The link $l_{ij}$ will reduce the number of components in the graph by one when $i$ arrives, relative to the counterfactual component reduction without $l_{ij}$, if and only if $j$ has already arrived and there is no other path between $i$ and $j$. In other words, the link increases $i$'s marginal contribution to total surplus if and only if it is essential when $i$ is added. Moreover, for the permutations in which $l_{ij}$ is essential it contributes $V$ to $i$'s marginal contribution to total surplus. Averaging over arrival order, the value to $i$ of the link $l_{ij} \in L$ is $md(i,j,L\setminus\{l_{ij}\})V$, while the value to establishing a new link $l_{ij} \notin L$ is $md(i,j,L)V$.

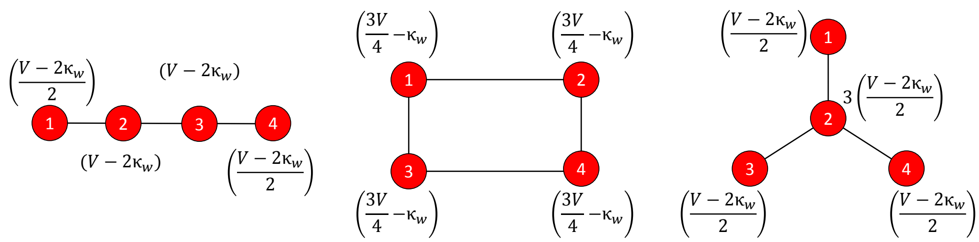
If a link $l_{ij}$ is essential on $L$ then for any arrival order, there will always be a component reduction of 1 when the later of $i$ or $j$ is added. Therefore, $md(i,j,L) = 1/2$, and $l_{ij}$ will be formed as long as $V > 2\kappa_w$. As $V$ is the social value of forming the link and $2\kappa_w$ is the total cost of forming it, when all agents are from the same group there is never underinvestment in a stable network or overinvestment in an essential link.

**Proposition 5.** *If all agents are from the same group then there is never underinvestment in a stable network. Furthermore, there is never overinvestment in an essential link.*

The proof is relegated to Section A of the Supplementary Appendix. When all agents are from the same group Proposition 5 establishes that there is never overinvestment in an essential link, but overinvestment in superfluous links is possible. If the costs of link formation are low enough then agents will receive sufficient benefits from establishing superfluous links to be incentivized to do so. Even if a link $l_{ij}$ is superfluous on $L$, for some arrival orders it will be essential on the induced subnetwork at the moment when $i$ is added and make a positive marginal contribution to total surplus.[32] An example of such overinvestment is shown in Section C of the Supplementary Appendix.

An immediate implication of Proposition 5 is that if all agents are from the same group and $2\kappa_w > V$ then the only stable network is the empty one and this network is efficient,

[32]Consider, for examples, arrival orders in which $i$ arrives first and $j$ arrives second.




(A) Line (B) Circle (C) Star

FIGURE 3. Three possible network structures for connecting 4 agents and the resulting net payoffs.

while if $2\kappa_w < V$ then all stable networks have only one network component (all agents are path-connected). For the remainder of the paper we focus on the parameter range for which the empty network is inefficient for a single group and assume $2\kappa_w < V$. We refer to this as our *regularity condition* and omit it from the statement of subsequent results.

Under this regularity condition the set of efficient networks are the set of tree networks in which all agents are path-connected. In other words, all agents must be in the same component and all links must be essential. We will now focus on which, if any, of these efficient networks are stable. As by Proposition 5 there is never any underinvestment in any stable network the only reason an efficient network will not be stable is if two agents have a profitable deviation by forming an additional (superfluous) link. We therefore focus on investigating what network structures minimize incentives for overinvestment. As we will see, this question is also related to the issue of inequality that different network structures imply.

Figure 3 illustrates three networks: A line (Figure 3a); a circle (Figure 3b) and a star (Figure 3c). While the line and star networks are efficient, the circle network is not as it includes a superfluous link. Among the two efficient networks, the star is more stable than the line. Applying Lemma 4, whenever the line is stable so is the star but there are parameter values for which the star is stable and the line is not. While the star is more stable than the line, it also results in more inequality. The expected utility distribution obtained on the line network can be generated from that obtained on the star network by the best off agent (agent 2) transferring $(V - 2\kappa_w)/2 > 0$ units of expected utility to one of the worst off agents (agent 3). This is enough to ensure that the expected utility distribution on the star is more unequal than the expected utility distribution on the line for any inequality measure in the Atkinson class. We generalize these insights in Proposition 6.

**Proposition 6.** *Suppose all agents are from the same group.*

(i) *If there exists an efficient stable network then star networks are stable, and for a range of parameter specifications only star networks are stable. If a line network is stable then all efficient networks are stable.*

(ii) *For all inequality measures in the Atkinson class, among the set of efficient network, star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.*

The proof is in Section A of the Supplementary Appendix but we provide some intuition after we discuss the result. Proposition 6 states that, in a certain sense, among the set of efficient networks the star is the most stable but maximizes inequality, while the line minimizes inequality but is least stable. This indicates a novel tension between stability/efficiency and inequality. For example, in contrast, Pycia (2012) studies when stable coalitional structures exist and finds that stable coalitions are more likely to exist when the bargaining functions of agents are more equal.

To gain intuition for Proposition 6, recall that Proposition 5 implies that an efficient network will be stable if and only if no pair of players have a profitable deviation in which they form a superfluous link. By Lemma 4 the incentives for two agents to form such a link are strictly increasing in their Myerson distance. Thus, a network is stable if and only if the pair of agents furthest apart from each other, in terms of their Myerson distance, cannot benefit from forming a link. As efficient networks are tree networks, the Myerson distance between any two agents depends only the length of the unique path between them.[33] The longest path between any pair of agents is, by definition, the diameter of the network $d(L)$. So, an efficient network is stable if and only if its diameter is sufficiently small. More precisely, an efficient network $L$ is stable if and only if its diameter is weakly less than $\overline{d}(\kappa_w, V)$, where $\overline{d}(\kappa_w, V)$ is increasing in $\kappa_w$, decreasing in $V$ and integer valued.[34]

Let $\mathcal{L}^e(\mathbf{N})$ be the set of efficient networks. Star networks have the smallest diameter among networks within this set, while line networks have the largest diameter among networks within this set. This establishes part (i) of Proposition 6.

To gain intuition for part (ii) a first step is noting that on any efficient network agents' net payoffs are proportional to their degrees (i.e., the number of neighbours they have):[35] $u_i(L) = |\mathbf{N}(i; L)|(V/2 - \kappa_w)$. The key insight is then showing that for any network in the set of efficient networks $\mathcal{L}^e(\mathbf{N})$, the star network can be obtained by rewiring the network (deleting a link $l_{ij} \in L$ and adding a link $l_{ik} \notin L$) in such a way that at each step we increase the degree of the agent who already has the highest degree, reduce the degree of some other agent and obtain a new network in $\mathcal{L}^e(\mathbf{N})$. This process transfers expected utility to the agent with the highest expected payoff from some other agent, thereby increasing inequality for any inequality measure in the Atkinson class. Likewise, we can obtain the line network

[33] Suppose $d$ is the number of agents on the unique path connecting $i$ and $j$. The probability that this path exists when agent $i$ arrives is $1/d$. In addition, if agent $j$ has not yet arrived, which occurs with probability $1/2$, $i$ would not benefit from the link $l_{ij}$, so $i$'s expected payoff from forming a superfluous link to $j$ is $(1 - 1/2 - 1/d)V$. We also note that as $d$ gets large, this converges to $V/2$ which is the value $i$ receives from forming an essential link.

[34] We show in the proof of Proposition 6 that $\overline{d}(\kappa_w, V) = \lfloor 2V/(V - 2\kappa_w) \rfloor$.

[35] This is also known as an agent's degree centrality.

from any network in the set $\mathcal{L}^e(\mathbf{N})$ by rewiring the network to decrease the degree of the agent with the highest degree at every step. This transfers expected utility from the agent with the highest expected payoff to some other agent, thereby decreasing inequality for any inequality measure in the Atkinson class.

To summarize, this section identifies two novel downsides to informal risk sharing agreements. First, they promote a misallocation of villagers time towards excessive social capital accumulation. Villagers have incentives to form links with a view to becoming more central within the risk sharing network in order to appropriate a larger share of the surplus generated by risk sharing. Secondly, even when investments into social capital are efficient the networks that can be supported in equilibrium generate social inequality, and this translates into (potentially severe) financial inequality.

## 5. Connections Across Groups

We now generalize our model by permitting multiple groups. These different groups might correspond to people from different villages, different occupations, or different social status groups, such as castes. We will first show that (under our regularity condition) there is still never any underinvestment within a group. However, this does not apply to links that bridge groups. As, by assumption, incomes are more correlated within a group than across a group, there can be significant benefits from establishing such links and not all these benefits accrue to the agents forming the link. Intuitively, an agent establishing a bridging link to another group provides other members of his group with access to a less correlated income stream, which benefits them. As agents providing such bridging links are unable to appropriate all the benefits these links generate, and these links are relatively costly to establish, there can be underinvestment.

To analyze the incentives to form links within a group, we first need to consider the variance reduction obtained by a within-group link. Such a link may now connect two otherwise separate components consisting of arbitrary distributions of agents from different groups. Suppose the agents in $\mathbf{S}_0 \cup \cdots \cup \mathbf{S}_k$ and the agents in $\widehat{\mathbf{S}}_0 \cup \cdots \cup \widehat{\mathbf{S}}_k$ form two distinct network components, where for every $i \in \{0, ..., k\}$, the agents in $\mathbf{S}_i$ and those in $\widehat{\mathbf{S}}_i$ are all from group $i$. Consider now a potential link $l_{ij}$ connecting the two otherwise disconnected components. Letting $s_0$ be the number of agents in group 0, the variance reduction obtained is:[36]

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \left[ (1-\rho_w) + \frac{\sum_{i=0}^k \left( \hat{s}_i \sum_{j=0}^k s_j - s_i \sum_{j=0}^k \hat{s}_j \right)^2}{\left(\sum_{i=0}^k s_i\right) \left(\sum_{i=0}^k \hat{s}_i\right) \left(\sum_{i=0}^k s_i + \hat{s}_i\right)} (\rho_w - \rho_a) \right] \sigma^2. \tag{13}$$

[36]By definition

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \operatorname{Var}(L(\mathbf{S}_0, ..., \mathbf{S}_k)) + \operatorname{Var}(L(\widehat{\mathbf{S}_0}, ..., \widehat{\mathbf{S}_k})) - \operatorname{Var}(L(\mathbf{S}_0 \cup \widehat{\mathbf{S}_0}, ..., \mathbf{S} \cup \widehat{\mathbf{S}_k})).$$

Recalling that

$$\operatorname{Var}(L(\mathbf{S}_0, \mathbf{S}_1, ..., \mathbf{S}_k)) = \left( \sum_{i=0}^k (s_i + s_i(s_i-1)\rho_w) + 2\rho_a \sum_{i=0}^{k-1} (s_i \sum_{j=i+1}^k s_j) \right) \sigma^2 \Bigg/ \sum_{i=0}^k s_i \,,$$

some algebra yields the result.

The key feature of this variance reduction is that it is always weakly greater than $(1-\rho_w)\sigma^2$, which is the variance reduction we found in the previous section when all agents were from the same group. Thus, the presence of across-group links only increases the incentives for within-group links to be formed. A within-group link can now give (indirect) access to less correlated incomes from other groups and so is weakly more valuable. This implies that there will still be no underinvestment under our regularity condition that $2\kappa_w < V$.[37] The above reasoning is formalized by Proposition 7.

**Proposition 7.** *There is no underinvestment between any two agents from the same group in any stable network.*

The proof of Proposition 7 is in Section A of the Supplementary Appendix. While underinvestment is not possible within group, it is possible across groups. An example of this is shown in Section C of the Supplementary Appendix. Although when all agents are from the same group the value of an essential link does not depend on the sizes of the components it connects, the value of an essential link connecting two different groups of agents increases in the sizes of the components. To demonstrate this formally, consider an isolated group that has no across-group connections and consider the incentives for a first such connection to be formed. Thus the first component consist of agents from a single group, say group 0. We let the second component consist of agents from one or more of the other groups (1 to $k$). The variance reduction obtained by connecting these two components is

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \left[ (1-\rho_w) + \frac{\hat{s}_0 \left( \left( \sum_{i=1}^k s_i \right)^2 + \sum_{i=1}^k s_i^2 \right)}{\left( \sum_{i=1}^k s_i \right) \left( \hat{s}_0 + \sum_{i=1}^k s_i \right)} (\rho_w - \rho_a) \right] \sigma^2, \tag{14}$$

which is increasing in $\hat{s}_0$:

$$\frac{\partial \Delta \operatorname{Var}(L \cup l_{ij}, L)}{\partial \hat{s}_0} = \frac{\left( \sum_{i=1}^k s_i \right)^2 + \sum_{i=1}^k s_i^2}{\left( \hat{s}_0 + \sum_{i=1}^k s_i \right)^2} (\rho_w - \rho_a)\sigma^2 > 0. \tag{15}$$

The inequality follows since $\rho_w > \rho_a$. Thus if agents $i$ and $j$ who connect two otherwise unconnected groups they receive a strictly smaller combined private benefit than the social value of the link. To see why, suppose that on the network $L$ the link $l_{ij}$ is essential, and without $l_{ij}$ there would be two components, the first connecting agents from group $G(i)$ and the second connecting agents from group $G(j) \neq G(i)$. Consider the Myerson value calculation. For arrival orders in which $i$ or $j$ is last to arrive, the value of the additional variance reduction due to $l_{ij}$ obtained upon the arrival of the later of $i$ or $j$, is the same as its marginal social value, i.e., the value of variance reduction obtained by $l_{ij}$ on $L$. For any other arrival order the value of variance reduction due to $l_{ij}$ when the later of $i$ or $j$ arrives

[37] Recall that this regularity condition just requires that it is efficient for two agents in the same group, both without any other connections, to form a link.

is strictly less. Averaging over these arrival orders, the link $l_{ij}$ contributes less to $i$ and $j$'s combined Myerson values than its social value, leading to the possibility of underinvestment.

Besides underinvestment, overinvestment is also possible across groups. Forming superfluous links will increase an agent's share of surplus without improving overall risk sharing and can therefore create incentives to overinvest. Nevertheless, when $\kappa_a$ is relatively high, underinvestment rather than overinvestment in across-group links will be the main efficiency concern. In many settings, within-group links are relatively cheap to establish in comparison to across-group links. For example, when the different groups correspond to different castes, as in our data, it can be quite costly to be seen interacting with members of the other caste (e.g., Srinivas (1962), Banerjee et al. (2013b)). Motivated by this, and because across-group links are considerably sparser in our data (to be described in the next section) than within-group links, we focus our attention on this parameter region. More concretely, below we investigate what within-group network structures create the best incentives to form across-group links and what network structures minimize the incentives for overinvestment within group. Remarkably, we find that these two forces push within-group network structures in the same direction, and in both cases towards inequality in the society.

We begin by considering within-group overinvestment, which corresponds to the formation of superfluous links within a group. We found in the previous section that when all agents are from the same group the star is the efficient network that minimized the incentives for overinvestment. However, once we include links to other groups, the analysis is more complicated. The variance reduction a within-group link generates is still 0 if the link is superfluous, but when the link is essential it depends on the distribution of agents across the different groups the link grants access to. Moreover, the variance reduction may be decreasing or increasing in the numbers of people in those groups.[38] This makes the Myerson value calculation substantially more complicated. When all agents were from the same group all that mattered was whether the link was essential when added. Now, for each arrival order in which the link is essential, we also need to keep track of the distribution of agents across the different groups that are being connected. Nevertheless, our earlier result generalizes to this setting, although the argument establishing the result is more subtle.

To state the result, it is helpful to define a new network structure. A *center-connected star* network is a network in which all within-group network structures are stars and all across-group links are held by the center agents in these stars. We denote the set of center-connected star networks by $\mathcal{L}^{CCS}$.

**Proposition 8.** *If any efficient network $L$ is robust to overinvestment within group, then any center-connected star network $L' \in \mathcal{L}^{CCS}$ is also robust to overinvestment within group.*

[38]In the case of an essential across-group link that connects agents from just one group to agents from other groups, the comparative statics are unambiguous. In this case, the variance reduction is increasing in the sizes of the groups connected (see inequality (15)).

*Moreover, if $L \notin \mathcal{L}^{CCS}$, then for a range of parameter specifications any center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to overinvestment within group but $L$ is not.*

The proof of Proposition 8 is in Section A of the Supplementary Appendix. In Proposition 6 we found that when all agents are from the same group, incentives for overinvestment (within group) are minimized by forming a (within-group) star. However, the incentives to form superfluous within-group links are weakly greater when someone within the group holds an across-group link (see equation 13). We can therefore think of the incentives for over-investment we found in Proposition 6 as a lower bound on the minimal incentives we can hope to obtain once there are across-group links. A key step in the proof of Proposition 8 shows that this lower bound is obtained by all center-connected star networks.

Consider a center-connected star network $L'$. As the agent at the center of a within-group star, agent $k$, has a link to all agents within the same group, we can focus on the incentives of two non-center agents from the same group, $i$ and $j$, to form a superfluous link. Consider any subset of agents $\mathbf{S} \subseteq \mathbf{N}$ such that $i, j \in \mathbf{S}$. On the induced subnetwork $L'(\mathbf{S})$ either $l_{ij}$ is superfluous or else $k \notin \mathbf{S}$. This implies that no across-group links are present whenever the additional link $l_{ij}$ makes a positive marginal contribution. Hence considering different arrival orders, the average marginal contribution of such a link when it is added is the same on the star network with no across-group links as for a center-connected star network: The lower bound on within-group overinvestment incentives is obtained.

We now consider the within-group network structures that maximize the incentives for an across-group link to be formed. We have already established that the marginal contribution of a first bridging link to the total surplus is increasing in the sizes of the groups it connects. By the Myerson calculation, the agents with the strongest incentives to form such links are then those who will be linked to the greatest number of other agents within their group when they arrive. The result below formalizes this intuition.

Let $\mathcal{A}(\mathbf{S}_k)$ be the set of possible arrival orders for the agents in $\mathbf{S}_k$. For any arrival order $A \in \mathcal{A}(\mathbf{S})$, let $\mathbf{T}_i(A)$ be the set of agents to whom $i$ is path-connected on $L(\mathbf{S}')$, where $\mathbf{S}'$ is the set of agents (including $i$) that arrive weakly before $i$. Let $T_i^{(m)}$ be a random variable, taking values equal to the cardinality of $\mathbf{T}_i(A)$, where $A$ is selected uniformly at random from those arrival orders in which $i$ is the $m$-th agent to arrive.

We will say that agent $i \in \mathbf{S}_k$ is more *Myerson central* (from now on, simply more central, for brevity) within his group than agent $j \in \mathbf{S}_k$ if $T_i^{(m)}$ first-order stochastically dominates $T_j^{(m)}$ for all $m \in \{1, 2, ..., |\mathbf{S}_k|\}$.[39] In other words, considering all the arrival orders in which $i$ is the $m$-th agent to arrive, and all the arrival orders in which $j$ is the $m$-th agent to arrive, the size of $i$'s component at $i$'s arrival is larger than that of $j$'s at $j$'s arrival in the sense of

[39]We also use this notion of centrality to compare the within-group centrality of the same agent on two different network structures. To avoid repetition we do not state the slightly different definition that would apply this situation.

first-order stochastic dominance.[40] This measure of centrality provides a partial ordering of agents.

**Lemma 9.** *Suppose agents in $\mathbf{S}_0$ form a network component, and all other agents in $\mathbf{N}$ form another network component. Let $i, i' \in \mathbf{S}_0$ and let $j \notin \mathbf{S}_0$. If $i$ is more central within group than $i'$, then $i$ receives a higher payoff from forming $l_{ij}$ than $i'$ receives from forming $l_{i'j}$:*

$$MV(i; L \cup l_{ij}) - MV(i; L) > MV(i'; L \cup l_{i'j}) - MV(i'; L).$$

The proof is relegated to Section A of the Supplementary Appendix. The key step in the proof pairs the arrival orders of a more central agents with a less central agent, so that in each case the more central agent is connected to weakly more people in the same group upon his arrival, and to the same set of people from other groups. Such a pairing of arrival orders is possible from the definition of centrality, and in particular the first-order stochastic dominance it requires.

Lemma 9 shows that more central agents have better incentives to form intergroup links. We can then consider the problem of maximizing the incentives to form intergroup links by choosing the within-group network structures (networks containing only within-group links). We will say that the within-group network structures that achieve these maximum possible incentives are most robust to underinvestment inefficiency across groups.

**Proposition 10.** *If any efficient network $L$ is robust to underinvestment across group, then some center-connected star network $L' \in \mathcal{L}^{CCS}$ is also robust to underinvestment across group. Moreover, if $L \notin \mathcal{L}^{CCS}$, then for a range of parameter specifications the center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to underinvestment across group but $L$ is not.*

The proof of Proposition 10 is in Section A of the Supplementary Appendix. Intuition can be gained from Lemma 9. This Lemma shows that agents have better incentives to provide a bridging link across group when they are more central within their own group. Thus to maximize the incentives of an agent to provide an across-group link, we need to maximize the centrality of this agent within group. This is achieved by any network that directly connects this agent to all others in the same group. However, only one of these within-group network structures can be part of an efficient network, and this is the star network, with the agent providing the across-group link at the center.

Figure 4 shows a center-connected star network when there are two groups. As long as it is efficient for these groups to be connected, center-connected star networks and only the center-connected star networks minimize the incentives for within-group overinvestment (by Proposition 9) and minimize the inventives for across-group underinvestment (by Proposition 10).

[40] An alternative and equivalent definition is that $i$ is more central than $j$ if there exists a bijection $B : \mathcal{A}(\mathbf{S}_k) \to \mathcal{A}(\mathbf{S}_k)$ such that $|\mathbf{T}_i(A)| \geq |\mathbf{T}_j(B(A))|$ and $A(i) = A'(j)$, where $A(i)$ is $i$'s position in the arrival order $A$ and $A' = B(P)$.

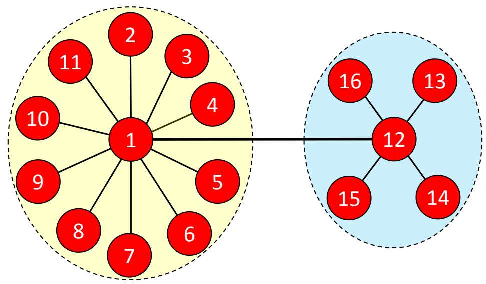



FIGURE 4. Center-connected within-group stars, in a context with two groups.

The above results further reinforce the tension between efficiency and equality. However, one subtlety relative to the one group case is that while the center-connected star network maximizes inequality among all efficient networks with respect to degree distribution, additional assumptions are required on the parameters of the model to ensure that such networks maximize inequality.

## 6. Testable Predictions

We now turn our attention to taking our model to the data. The model is intentionally stylized and intended to illustrate key economic forces in the simplest possible way. To take it to data we need to consider the general insights. The broad predictions of our model thus far are that (1) there are endogenously arising asymmetries among agents, (2) there is no underinvestment within groups, (3) agents cannot be too far away from each other (in terms of the Myerson distance), (4) agents that have across-group links should be more Myerson central within group. However, these predictions provide neither a clean nor powerful test of our theory. Predictions (2) and (3) depend on an unobservable linking cost parameter, while there are many alternative stories, including ones not directly connected to risk sharing, that generate similar predictions. For example, if individuals have heterogenous time budgets and made random links within and across groups, predictions (1) and (4) are mechanically generated.

We therefore turn to more subtle predictions that rely on the comparative statics under our model, as we exogenously change parameters of the economic environment. These are more demanding predictions from the theory, with richer empirical content.

The prediction we consider describes how the network positions of those agents with across-group bridging links varies with the environment. Lemma 9 shows that more Myerson central agents have better incentives to provide an across-group link. However, the importance of centrality will depend on the overall strength of the incentives to form across-group links. When income variance is higher there are stronger incentives to form an across-group link, and so network position will be less important; villagers in more varied locations will have sufficient incentives to form across-group links. More formally, from the variance reduction given in equation 13 it is straightforward to show that for an essential across-group bridging link $l_{ij}$:

$$\frac{\partial \Delta \operatorname{Var}(L, L \cup \{l_{ij}\})}{\partial \sigma^2} > 0.$$

This means that the incentives to form an across-group link are increasing in $\sigma^2$ leading to the following prediction:

P1. In villages with higher $\sigma^2$, the association between within-group centrality and the formation of across-group links is lower.

P1 provides a prediction that can be tested empirically, if there is exogenous variation in $\sigma^2$. The exogenous variation we will use is the randomized introduction of micro finance. A central aim of micro finance is to smooth incomes. We thus assume that in those villages where micro finance is introduced income variability decreases. The theoretical model then predicts that the association between within-group centrality and the presence of an across group link to be stroger in those villages randomly selected for the introduction of microfinance.

Of course, our simple theoretical model treats people within the village homogenously, while in practice there are likely to be heterogeneities and the impact of micro finance is likely to be felt differently by different villagers. The empirical test is valuable because it helps to determine whether the simple model is able to account for real world correlations, despite its parsimony.

## 7. Empirical Analysis

We make use of a unique social network dataset from 185 villages in Tamil Nadu, India. This dataset is particularly well-suited for our analysis as it (i) involves numerous independent villages (essential for inference, though most network-based studies have just one or a handful of villages), (ii) includes complete network data across both financial and social connections for almost all households in every village, and (iii) captures both within village contacts and outside-village contacts, which are rarely contained in datasets of this kind. However, the major advantage of these data for testing the empirical predictions of our model is that they were collected in the context of a large field experiment in village banking, in which a randomly chosen half of villages gained access to local banking services 1-2 years prior to data collection. This gives us indisputably exogenous variation in access to formal financial services with which to test one of the model's key predictions: that the correlation between Myerson Centrality and number of costly links is more positive when the value of outside risk-sharing links is lower, all else equal. The setting is unique in the sense that reliance on informal networks varies randomly with the introduction of banking services, which allows us to study how truly exogenous variation in the value of network links across villages influences network composition.

7.1. **Setting and Data.** The data we employ were collected from 2014 to 2016 in conjunction with a large-scale impact evaluation of access to formal financial services in rural Tamil

Nadu, India (Binzel et al., 2017). The implementing partner was a large financial institution (henceforth, LFI) that offers group-based and individual loans to both men and women through local village branches with the explicit goal of reaching individuals in financially marginalized (previously unbanked) rural communities. Beginning in 2008, LFI expanded bank infrastructure across villages from the districts of Thanjavur, Thiruvarur and Pudukkotai (Tamil Nadu). Prior to this rolling-out, 102 potential branch service areas (henceforth, SAs) were identified by LFI as potential expansion areas. The average SA spans 10 villages within a radius of 4-5 km from the branch and covers a population of roughly 10,000 people. Once all feasible branch locations in the district had been designated, SAs were matched into pairs using a minimum distance matching algorithm, and 51 bank branches were randomly assigned to one SA in each pair.

Bank operations began soon after treatment assignment. By the onset of network data collection efforts, bank penetration had reached a average level of 41% in treatment SAs. An early evaluation of this expansion of financial activity conducted in 2013 shows that households living in treatment villages were 32% more likely to have borrowed in the previous year compared with households in control villages.

Beginning in 2014, a full social network mapping survey was administered in a randomly chosen subset of 204 villages from the 102 service areas (2 villages per service area).[41] In these villages, all households were asked to name all social and financial contacts both within and outside the village, enabling us to map the full network of social and financial connections within each sampled village.[42] Households were surveyed 18 to 24 months after the opening of the branch.[43] The network survey was administered to both the head and spouse of each household, when available.[44] In the survey, the head of the household and spouse were asked to identify all individuals within the village with whom they: (i) spend leisure time; (ii) could borrow in case of emergency; and (iii) could borrow to finance a business investment.[45] In addition, respondents were asked to list all individuals living *outside* of the village from whom they could borrow in case of emergency. In addition to naming each link both inside

[41] At baseline, i.e. before branch openings in treatment service areas, two villages per service area were selected as follows. First, the sample was limited to villages with 40-250 households, excluding the designated branch location. For each pair, one village was randomly selected and then matched with the village in the corresponding treatment or control service area that had as close to the same distance from its respective branch as the first picked village had from its branch. For control area villages, the planned branch location was used as benchmark.

[42] Links named by respondents were immediately matched to names within a database of village members collected at baseline. Information on outside contacts cannot be mapped since household living in villages that are not included in our sample cannot be identified by name and location.

[43] In 85 villages, an additional round of network data was also collected at Baseline (prior to the opening of the bank branch) in addition to Endline. Because less than half of villages have panel network data, baseline data are excluded from the current analysis.

[44] For 23% of the households, only the head of the household has been interviewed. For 13% only the spouse has been interviewed.

[45] In all questions, households could list up to 15 individuals, which results in very little censoring of networks. The maximum number of links was reached in only very few cases (less than 0.01% of cases).

and outside the village, respondents were asked their relationship to the link (friend, family, employer, moneylender), the actual amount borrowed from each link, the amount they could borrow from each link in case of emergency, the amount they could borrow from each link to finance a business investment, and the number of contact days with each link (out of past 7). For outside links, respondents were also asked the distance to each link (for our purposes, whether the link lives within walking distance). Information on outside links was only collected in 189 villages, and village-level controls are missing from 4 villages. As a consequence, this analysis is limited to the 185 villages with complete data.[46] We also exclude the 38 households that moved into the study area between baseline and endline.

Although financial and leisure ties are elicited separately for both the head and spouse, in order to analyze household-level networks we aggregate observations within the household in the following manner. First, we only consider "OR" networks - that is, those containing either a social or a financial tie.[47] Second, we aggregate the two layers of edges between households using the following rule: If two people from household A report two persons in household B, there is a *unique* directed edge from A to B. Thus, it will be equivalent to the case where only one person from household A reports a person from household B.

Second, to aggregate characteristics of the interaction between two households, we consider an aggregation rule that avoids double-counting. The continuous value characterizing a link between household A and household B is the maximum of all the values that the head and the spouse of household A have reported for *anyone* belonging to household B.[48]

These aggregation rules are only applicable for inside village contacts in which we know whether two declared contacts belong to the same household. For outside contacts, we consider all the outside contacts listed by the household, excluding only contacts that are classified by respondents as money-lenders.

In Section B.2 of the Supplementary Appendix, Table 2-3 provide summary statistics of the sampled villages in order to verify that the sample is balanced across treatment and control arms. The average number of households per village is 112, the average node degree is 4.56, and the density is 0.04. Tables 4-6 show the treatment effects of village-level banking services on within-village and outside-village links, which are discussed extensively in Binzel et al. (2017). As predicted, the introduction of banking services generates an exogenous

[46]Inclusion in the sub-sample is balanced across treatment and control.

[47]In particular, our analysis focuses on two types of networks: the *financial graph* $L^f$ and the *social graph* $L^s$. The financial graphs represent risk-sharing connections, and the social graph represents friendships and ties used to socialize (see survey questionnaire in Section B.1 of the Supplementary Appendix), which are not mutually exclusive. Our empirical test utilizes both types of links and considers $L^{all}$ the network of either risk-sharing or friendship connections. We favor "OR" networks because of the high degree of overlap between social and financial networks, and because of the concern that network links are self-censored due to imperfect recall or insufficient recall effort. The results are almost identical when MC is computed on the financial or social networks alone.

[48]For instance, if the head of household A report that she can borrow Rs. 150 from the spouse of household B and the spouse of household A reports that he can borrow Rs. 100 from the head of the household B, we will consider that household A can borrow Rs. 150 from household B.

reduction in households' within-village network links and their reliance on informal transfers, as measured by the difference in real and potential borrowing levels between treatment and control villages (Table 4).

In Tables 5 and 6 we observe that banking does not reduce the number of outside links, but does lead to less informal borrowing from outside sources, and to changes in the composition of outside links. In particular, in banked villages, outside links are younger, closer in distance, and more social than outside links in control villages. All of these suggest a shift away form more financially valuable outside links.

Overall, these results demonstrate that the randomized introduction of formal banking services reduced the value of informal links both within and outside the village, as would be expected. This allows us to rigorously test the more nuanced predictions on the relationship between bridging links and Myerson Centrality. Our main test of the theoretical model, presented in Table 1, utlizes the following specification:

$$\#OutContact_{ji} = \alpha_0 + \alpha_1 T_i + \alpha_2 MC_{ji} + \alpha_3 MC_{ji} \times T_i + \alpha_4 X_{ij} + \gamma_{s_i} + \epsilon_{it} \tag{16}$$

Where $\#OutContact_{ij}$ is a binary indicator of whether household $j$ in village $i$ has any links outside the village, $T_i$ is the treatment indicator equals to 1 if village $i$ was in a service area that was randomly given access to the LFI's services, $MC_{ji}$ is the Myerson Centrality of individual $j$ computed on the ALL network, $X_{ij}$ is a set of control variables either at the household or the village level, $\gamma_{s_i}$ is a pair fixed effect that accounts for the experimental stratification, and $\epsilon_{it}$ is an error term. Since the treatment is assigned at the service area level (encompassing several villages), this error term is clustered at the service area level.

In Table 1 we see clear evidence that an exogenous reduction in the value of outside links brought about by the introduction of formal banking services leads to a significantly more positive relationship between Myerson Centrality and link formation. The negative and significant coefficient estimate on the treatment indicator implies that banking services encouraged an overall reduction in outside links for all individuals in the village. Meanwhile, the positive and significant coefficient estimate on the interaction between MC and treatment implies that the impact of access to formal banking on outside links was less extreme among more central individuals. That is, although outside links are less common in the new regime in which those links are less financially valuable, Myerson central individuals are significantly more likely to retain outside links compared to less central individuals when their value declines.[49] Section B.3 of the Supplementary Appendix also shows that villagers' incomes are also positively correlated with their Myerson centralities, as predicted by the theory.

[49]The negative coefficient on Myerson centrality is not predicted by the theory, but it is also not inconsistent with it: when the benefits of across group links are large then all agents, including those who are not central in their own group's network, have incentives to establish outside links, and our model does not give sharp predictions on the structure of the network. Furthermore, there might be individuals who have more recently moved to the village and are both more likely to have "free" outside links (as considered in Appendix II) and to be less central within the village community. Unfortunately, we do not have information on how long a household has resided in the village to test this hypothesis.

Table 1. Treatment effect on whether household has outside contacts

| | Has any outside contact (1) | Has any outside contact (excluding contacts within walking distance) (2) | Has any outside contact (3) | Has any outside contact (excluding contacts within walking distance) (4) | Has any outside contact (5) | Has any outside contact (excluding contacts within walking distance) (6) |
|---|---|---|---|---|---|---|
| Treatment | -0.0310* | -0.0288 | -0.0274* | -0.0255 | -0.0512* | -0.0614** |
| | (0.0165) | (0.0191) | (0.0157) | (0.0176) | (0.0285) | (0.0270) |
| MC | -0.000201*** | -0.000156*** | -0.000108*** | -0.0000880** | -0.000415 | -0.000920* |
| | (0.0000469) | (0.0000528) | (0.0000306) | (0.0000339) | (0.000419) | (0.000501) |
| MC × Treatment | 0.000113** | 0.0000830 | 0.0000788*** | 0.0000558* | 0.000433* | 0.000463** |
| | (0.0000449) | (0.0000531) | (0.0000294) | (0.0000332) | (0.000221) | (0.000197) |
| Nr. Observations | 18,648 | 18,648 | 18,648 | 18,648 | 18,648 | 18,648 |
| Nr. villages | 185 | 185 | 185 | 185 | 185 | 185 |
| R2 | 0.164 | 0.143 | 0.163 | 0.142 | 0.162 | 0.142 |
| MC Calculation | Method 1, Undirected | Method 1, Undirected | Method 2, Undirected | Method 2, Undirected | Method 3, Undirected | Method 3, Undirected |
| Mean Control | 0.60 | 0.52 | 0.60 | 0.52 | 0.60 | 0.52 |

Note: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses
Control variables include, at the household level, a dummy of whether the only respondent of the household was the head of the household, a dummy of whether the only respondent of the household was the head of the household's spouse, the average age of the household's respondents, a dummy of whether the household is involved in agriculture. Control variables at the village level are: the number of households in the village, the distance to the bank branch, and the proportion of people belonging to the same caste in the village. Control variables take a value of zero when missing values, and regressions include an indicator of missing data corresponding to each control. Control *HH* refers to a regression including only the first two dummies of the household controls.

## 8. Conclusion

Our paper provides a relatively tractable model of network formation and surplus division in a context of risk sharing that allows for heterogeneity in correlations between the incomes of pairs of agents. Such correlations have a sizeable impact on the potential of informal risk sharing to smooth incomes. We investigate the incentives for relationships that enable risk sharing to be formed both within a group (caste or village) and across groups, giving access to less correlated income streams. We find that overinvestment into social relations is likely within a group, but there is potential underinvestment into more costly social connections that bridge different groups. We also find a novel trade-off between equality and efficiency. Thus we identify new downsides to informal risk sharing arrangements that can have important policy implications.

Using a unique dataset of 185 Indian villages, we find some empirical support for our model. The premise of our empirical investigation is that the (randomized) introduction of microfinance makes risk sharing links going outside the treated villages less valuable. Congruent with our model's predictions we then find that people have to be more central within treated villages to maintain risk sharing links outside the village, in comparison to control villages.

Although we focus our analysis on risk sharing, our conclusions regarding network formation could apply in other social contexts too, as long as the economic benefits created by the social network are distributed similarly to the way they are in our model—a question that requires further empirical investigation. Within the context of risk sharing, a natural next step would be to provide a dynamic extension of the analysis that allows for autocorrelation between income realizations.

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## Appendix I. Generalized Theoretical Results

In this section we examine under what conditions our main conclusions extend to more general utility functions, endowment distributions and surplus division rules. The environment with CARA utilities and jointly normally distributed endowments facilitates a convenient transferrable (expected) utilities environment that is particularly tractable to analyze when social surplus is divided in accordance with the Myerson value. While analytical tractability requires a series of strong assumptions, below we show that some of the main qualitative insights of the model extend to much more general specifications.

For general specifications of the model utilities are nontransferable and social surplus (as a single number) is undefined, hence we need a more general approach. Let $v_i$ be the utility function for agent $i$, mapping second period consumption into utility. We assume that $v_i = v_j$ for all $i$ and $j$ in the same group, and that $v_i$ is strictly increasing and strictly concave for all $i \in \mathbf{N}$. These properties imply that for any number of agents more than one, and for any point of the Pareto frontier of feasible consumption plans that can be reached via risk-sharing arrangements, there is a direction along the Pareto frontier in which a given agent's expected utility is strictly increasing. Let $\mathcal{P}_k$ be the endowment distribution of agents in group $k \in \mathbf{M}$.

Let $\mathcal{L}$ be the set of all possible networks for agents in $\mathbf{N}$. We assume that agents correctly foresee what risk-sharing arrangement they would agree upon for any possible $L \in \mathcal{L}$. These risk-sharing arrangements, which depend on the social network, might be dictated by social conventions, or they can be outcomes of negotiation processes for transfer arrangements once the network is formed. Let $\tau(L)$ be the transfer arrangement and $u_i^\tau(L)$ be the expected second period consumption utility of agent $i$ implied by $\tau(L)$.[50] We refer to $\tau(\cdot)$ as the surplus division rule.

We assume that for every $L \in \mathcal{L}$, $\tau(L)$ specifies a pairwise-efficient risk-sharing arrangement $\tau_{ij}(L)$ for every pair of agents $i, j$ linked in $L$. As shown earlier, this is equivalent to $\tau(L)$ being Pareto efficient at a component level.

Agent $i$ maximizes the difference between expected utility from the second period risk sharing (given by $u_i^\tau$) and her costs of establishing links.

Let $C_i(L)$ be the set of agents on the same component as $i$ given $L$, and recall that $G$ is a function mapping agents in $\mathbf{N}$ to groups in $\mathbf{M}$.

Next we impose a series of assumptions on $\tau(\cdot)$. We do not claim that the above assumptions hold universally when informal risk-sharing takes place, but they are relatively weak requirements that are natural in many settings. Our main objective is to demonstrate that our qualitative results hold for a much broader class of models than the CARA-normal setting with surplus division governed by the Myerson value.

The first assumption requires that establishing a link always strictly increases the connecting agents' expected consumption utilities.

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[50] More precisely, utility function $v_i$, the distribution of endowment realizations and transfer arrangement $\tau(L)$ jointly determine $u_i^\tau(L)$.

**Assumption 11.** $u_i^\tau(L \cup \{l_{ij}\}) > u_i^\tau(L)$ *for every* $L \in \mathcal{L}$, $i, j \in \mathbf{N}$ *and* $l_{ij} \notin L$.

The next assumption requires that establishing an essential link does not impose a negative externality on other agents. This implies that while both $i$ and $j$ privately benefit from essential link $l_{ij}$, in terms of second period expected utility, they do not benefit over and beyond the enhancement of risk-sharing opportunities that the link facilitates.

**Assumption 12.** $u_k^\tau(L \cup \{l_{ij}\}) \geq u_k^\tau(L)$ *for every* $L \in \mathcal{L}$, $i, j, k \in \mathbf{N}$ *and* $C_i(L) \neq C_j(L)$.

Next we extend the idea that the private benefit that two agents receive from establishing a link should be increasing in the distance between them in the absence of the link. In the previous analysis these private benefits depended specifically on the Myerson distance between the two agents, while here we allow for a general class of distance measures. Before defining the class of distance measures we allow for, some additional notation is required. For two sets $S$ and $S'$ we define $\mathcal{M}(S, S')$ as the set of matching functions $\mu : S \to S' \cup \{\emptyset\}$, such that for $s \in S$ if $\mu(s) \neq \emptyset$ then $\mu(s) \neq \mu(t)$ for all $t \in S \setminus \{s\}$. Thus every $\mu \in \mathcal{M}(S, S')$ maps each element of $S$ into a different element of $S'$, or else the empty set.

Let $\overline{\mathbf{N}}^2 = \{(i,j) | i, j \in \mathbf{N}, i \neq j\}$.

**Definition (Distance measure):** A distance measure is a mapping $d : \overline{\mathbf{N}}^2 \times \mathcal{L} \to \mathbb{R}_{++}$ satisfying the following properties:

**Assumption 13.**

(i) *If $i$ and $j$ are in different components on $L$, then $d(i,j,L) = \overline{d}$, with $\overline{d}$ strictly greater than the maximum possible distance between any two path-connected agents.*

(ii) *The distance measure depends only on paths (thus ignoring walks with cycles).*

(iii) *Let $S_{ij}$ be the set of paths between $i$ and $j$ and $S_{kl}$ be the set of paths between $k$ and $l$. We assume $d(i,j;L) > d(k,l;L)$ if there exists a matching function $\mu \in \mathcal{M}(S, S')$ such that each path between $i$ and $j$ is matched to a shorter path between $k$ and $l$, and all such paths between $k$ and $l$ are independent (do not pass through any of the same nodes as each other).*

Assumption 13 places only weak restrictions on the distance measure. In particular, part (iii) in general only provides a very weak partial ordering of the distances between agents. However, there is a special case in which the ordering is complete. On a tree network, there is a unique path between any two agents, so this determines the ordering of distances between pairs of agents. In what follows, let $d(\cdot)$ be any distance measure satisfying the above requirements.

While we will use the concept of distance between agents in the general case of multiple groups, first we focus on extending our earlier results for the case of homogeneous agents. Next we make assumptions on how distance in the absence of a link influences the private benefits of two agents within the same group establishing that link.

The next assumption requires that if all agents are from the same group then the private benefit two agents receive when establishing a link only depends (positively) on their distance in the absence of the link, and on the sizes of the components they are on. Recall that in our benchmark model in the CARA-normal setting these private benefits only depended on the Myerson-distance between the agents. The requirement below allows the private benefit to depend on different distance measures, and also on the sizes of the agents' components (which for general utilities influences the difference between the Pareto frontiers of feasible consumption plans with and without the link).

**Assumption 14** (Only Distance and Size Matter)**.** *If $G(i) = G(j)$ for all $i, j \in \mathbf{N}$ and $l_{ij} \notin L$, then*

$$u_i^{\tau}(L \cup \{l_{ij}\}) - u_i^{\tau}(L) = g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|),$$

*Moreover, $g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|)$ is increasing in $d(i,j,L)$.*

As Assumption 14 does not apply when there are multiple groups, in the multiple group case the composition of each component, in terms of the groups the constituent agents come from, and their network positions, can matter.

The last assumption we need for recreating the results of the benchmark model for homogeneous agents is that the cost of link formation within a group is sufficiently small relative to the private benefits from establishing an essential link. In the CARA-normal framework with the surplus allocated according to the Myerson value and all agents being homogeneous, a pair forming an essential link received the full social surplus created by the link. This implies that the social and private benefits coincide in the benchmark model for essential links, and therefore there is no within group underinvestment for any cost of link formation. For general utility functions and surplus allocation rules such equivalence does not hold, therefore no within group underinvestment cannot be expected to hold for all possible costs of link formation. However, for any specification of the general model that satisfies the assumptions above (in particular that the private benefit of establishing any link is always strictly positive), there is no within group underinvestment *if* the cost of establishing a link between agents from the same group is small enough. While this is a nontrivial assumption, it is realistic in many settings. Indeed, in the data we consider, within group underinvestment does not appear to be a problem.

**Assumption 15** (Within Group Cost of Link-formation Small)**.** *For all networks $L$,*

$$c_w/2 < \min_{L,i,j \ st. \ C_i(L) \neq C_j(L)} u_i^{\tau}(L \cup \{l_{ij}\}) - u_i^{\tau}(L).$$

Assumption 15 immediately implies that if all agents are from the same group then in all stable networks there is a single component. The next proposition shows that the same holds for all efficient networks. For the rest of the section, Assumptions 11-15 are maintained.

A network is Pareto efficient if there is a feasible transfer agreement that could be reached on that network such that there is no other network, feasible transfer agreement pair in which all agents are weakly better off and some agents are strictly better off.

**Proposition 16.** *If all agents are from the same group then a network is Pareto efficient if and only if it is a tree connecting all agents.*

**Proof:**

First, we consider the "only if" direction. In any Pareto efficient network, every component has to be a tree. This is because if any component was not a tree then a link could be deleted and the same risk-sharing arrangement can be achieved as before, but the costs of establishing the link saved. Now suppose there are two components of a Pareto efficient network $L$ that are not connected. Let agents $i$ and $j$ be on different components. By Assumption 15, total expected utilities (that is, taking into account the costs of network formation, too) of both $i$ and $j$ are strictly higher for network $L \cup \{l_{ij}\}$ than for network $L$, while by Assumption 12 all other agents' total expected utilities are weakly higher for $L \cup \{l_{ij}\}$ than for $L$. This contradicts that $L$ is Pareto efficient.

We now consider the "if" direction. Consider a tree network and suppose we implement a risk sharing agreement in which $c_i(\omega) = c_j(\omega)$, for all $i$ and $j$ and all states $\omega$. As all agents' consumptions are equalized in all states, there is then no way in which link formation costs can be redistributed and the risk sharing arrangement changed, without making someone worse off. Suppose, towards a contradiction, that we can redistribute the link formation costs, by forming a different tree network, and find a new feasible consumptions that together constitute a Pareto improvement. Holding consumptions fixed, the change in network will make some agents worse off if any agents are made better off. Thus, to achieve a Pareto improvement, consumptions will have to be changed. Let $c'(\omega)$ be the new consumption vector. As the utility function $v(\cdot)$ is concave, Jensen's inequality implies that

$$\frac{1}{n}\sum_i v(c_i'(\omega)) < v\left(\frac{1}{n}\sum_i c_i'(\omega)\right) = \frac{1}{n}\sum_i v(c_i(\omega)),$$

for all $\omega$. Thus the average expected utility from consumption will decrease. As total link formation costs have remained constant, this implies that at least one agent must be worse off. This is a contradiction.■

**Corollary 17.** *When all agents are from the same group, there is no underinvestment.*

Given Proposition 16, Corollary 17 follows immediately from Assumption 15 and we omit a proof.

Note that for any non-essential link $|C_i(L)| = |C_i(L) \cup \{l_{ij}\})|$. Thus the marginal benefits from $i$ and $j$ forming a superfluous links depend only on the distance between $i$ and $j$ on $L$ and the number of agents in their component. The latter is $n$ for any efficient network, by Proposition 16. Thus the marginal benefit from $i$ and $j$ receive from forming a superfluous link depend only on the distance between $i$ and $j$, and are increasing in this distance. Thus an efficient network will be stable if and only the maximum distance between any two agents is sufficiently low. The next Corollary formally states this result.

**Corollary 18.** *If all agents are from the same group then an efficient network is stable if and only if its diameter is sufficiently small.*

**Proof:** Consider an efficient network $L$. As $L$ is efficient there exists a unique path between $i$ and $j$ for all $i$ and all $j \neq i$. Consider two such agents $i$ and $j \neq i$. Assumption 13 implies that $d(i,j,L)$ is strictly increasing in the path length between $i$ and $j$, and that $d(i,j,L) = d(j,i,L)$. Further, as $|C_i(L)| = |C_i(L \cup \{l_{ij}\})| = n$, by Assumption 14

$$u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L) = g(d(i,j,L), n, n) = g(d(j,i,L), n, n) = u_j^\tau(L \cup \{l_{ij}\}) - u_j^\tau(L).$$

Moreover, by Assumption 14, $g(d(i,j,L), n, n)$ is strictly increasing in $d(i,j,L)$. Thus for all $i$ and $j \neq i$, there exists a threshold $\hat{d}$ such that $i$ and $j$ benefit from forming a superfluous link if and only $d(i,j,L) > \hat{d}$.

As there is never any underinvestment by Corollary 17, no agent can benefit from deleting a link on $L$. Thus the network $L$ is stable if and only if no two agents can benefit from forming a superfluous link. Hence $L$ is stable if and only if $\max_{i,j} d(i,j,L) \leq \overline{d}$. As $d(i,j,L)$ is strictly increasing in the (unique) path length between $i$ and $j$, this is equivalent to the diameter of $L$ being sufficiently small.■

A network is *least* stable within a class of networks, when its stability implies the stability of any other network in that class. A network is *most* stable within a class of networks, when its instability implies the instability of any other network in that class.

**Proposition 19.** *If all agents are from the same group then*

(i) *the most stable efficient network is the star,*
(ii) *the least stable efficient network is the line.*

*Proof.* By Corollary 18 an efficient network is stable if and only if its diameter is sufficiently low. It follows that if a network with diameter $d$ is stable, all efficient networks with weakly lower diameter will also be stable. As the line network maximizes diameter among efficient networks, its stability implies the stability of all other efficient networks and it is least stable.

Similarly, if a network with diameter $d$ is unstable, Corollary 18 implies that all network with a weakly higher diameter are unstable. As the star network minimizes the diameter within the class of efficient networks, its instability implies the instability of all other efficient networks, and it is most stable within the class of efficient networks. □

Inequality measures within the Atkinson class will often rank utility vectors differently. In the simpler setting, with CARA utilities, normally distributed incomes and the Myerson value allocation rule we were able to identify the star as the least equitable networks for any inequality measure in the Atkinson class. This was achieved by showing that any efficient network could be transformed into a star by rewiring it in a way such that, at each step of the rewiring, the utility of the center agent increased, the utility of one other agent decreased and the utility of the remaining agents remained constant. Specifically, the act of removing a link $l_{ij}$ and adding a link $l_{jk}$, increased the utility of agent $k$, decreased the utility of agent $i$ and held constant the utility of all other agents.

In the more general setting, this rewiring need not hold constant the utility of the other agents. This creates problems. Consider the four agent line network and suppose utilities, after link formation costs, are $(10, 25, 25, 10)$. Now suppose we remove link $l_{34}$ and add link $l_{24}$ to create a star network. In the more general model, utilities after this rewiring might be $(11, 35, 11, 11)$. These two vectors will be ranked differently by different inequality measures within the Atkinson class. However, if we make an additional assumption that this kind of rewiring only affects those agents who gain or lose a link, then we can relate inequality to network structure in the more general setting.

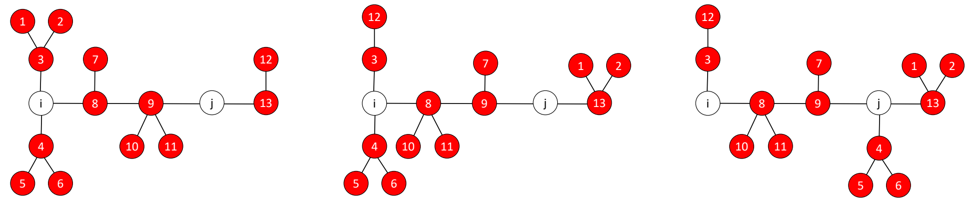
**Proposition 20.** *Suppose there is one group, and for all pairs of efficient networks $L$ and $L'$ such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then for all inequality measures in the Atkinson class, among the set of efficient network, star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.*

*Proof.* We begin with a Lemma:

**Lemma 21.** *Suppose there is one group, and for all pairs of efficient networks $L$ and $L'$ such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then agents with a higher degree in $L$ have a higher utility.*

*Proof.* Consider an efficient network $L$ and suppose agent $i$ has higher degree than $j$. We will show that we can rewire a network in a way that weakly reduces $i$'s utility and increases $j$'s utility, but swaps the positions of $i$ and $j$ in the network such that on this new network $i$ should have the same utility $j$ had on the initial network. This will imply that $i$ must have had a higher utility on the initial network.

Consider the following rewiring, an example of which is illustrated in Figure 5. As $L$ is efficient it is a tree by Proposition 16 and there is a unique path between $i$ and $j$. If $i$ is




(A) Initial Network $L$

(B) $L'$

(C) Rewired network $L''$

FIGURE 5. An example of the rewiring used to find a contradiction in the proof of Lemma 21 is shown. Panel (i) shows the initial network, Panel (ii) the interim network and Panel (iii) the final network after the rewiring is complete.

directly connected to $j$ we do not need to do any rewiring along this path. Otherwise, let there be $l \geq 1$ agents on this path, other than $i$ and $j$, and create the following two labelings of these agents: $i, i1, \ldots, il, j$ and $i, jl, \ldots, j1, j$. Thus $i1 = jl$, $i2 = j(l-1)$, and so on. Now, if agent $i1$ has a link to an agent $k$ on $L$, and $k$ is not on the path between $i$ and $j$, we remove the link $l_{i1,k}$ and add the link $l_{j1,k}$. Repeat until all of $i1$'s links to agents not on the shortest path between $i$ and $j$ have been rewired. We now repeat for $ik$, with $k = 2, \ldots l$. Note that at each step of this rewiring we reach a connected tree network.

Consider now the neighbors of $j$ not on the path between $i$ and $j$. Match each of these neighbors to a different neighbor of $i$'s who is also not on this path. As $i$ has a higher degree than $j$, such a matching exists. For each such pair we start with $j$'s neighbor. Letting this neighbor of $j$ be $k$, one by one, we rewire each of $k$'s links on $L$, except $l_{ij}$, to the neighbor of $i$ agent $k$ was matched to. Let this agent be $l$. We then rewire each of $l$'s links on $L$, except $l_{il}$, to agent $k$. Repeat for all of $j$'s neighbors on $L$ not on the path between $i$ and $j$. Note again that at each step of this rewiring we reach a tree network. After all this rewiring, let the network that has been reached be denoted $L'$.

As in all the rewiring so far $i$ and $j$ have kept the same links, and as at each step an efficient network has been reached, by the premise of the Proposition, $\tau_i(L) = \tau_i(L')$ and $\tau_j(L) = \tau_j(L')$, so $u_i^\tau(L) = u_i^\tau(L')$ and $u_j^\tau(L) = u_j^\tau(L')$.

Finally, we consider the neighbors of $i$ who were not on the shortest path to $j$, and were not matched to one of $j$'s neighbors. As $i$'s degree is higher than $j$'s there exists at least one such agent. For all agents in this set, we remove their link to $i$ and add a link to $j$. Let the network reached after this be denoted $L''$.

By Assumption 11, this increases $j$'s utility and decreases $i$'s utility, so $u_i^\tau(L) = u_i^\tau(L') > u_i^\tau(L'')$ and $u_j^\tau(L) = u_j^\tau(L') < u_j^\tau(L'')$. However, by construction, after this rewiring is complete $i$'s position in $L''$ is identical to $j$'s position in $L$ (up to a relabeling of agents), while

$j$'s position in $L''$ is identical to $i$'s position in $L$. Thus by Assumption 14 $u_i^\tau(L) = u_i^\tau(L'')$ and $u_j^\tau(L) = u_j^\tau(L'')$. We then have that

$$u_i^\tau(L) = u_i^\tau(L') > u_i^\tau(L'') = u_j^\tau(L).$$

□

We can now prove the Proposition. As shown in the proof of Proposition 6, the star network can be reached from any efficient network $L$ by rewiring links to the highest degree agent in $L$. By Lemma 21 the agent with the highest utility on $L$ is the agent with the highest degree, and by Assumption 15, the net expected utility of this agent increases at each such step of the rewiring, while the net expected utility of all other agents weakly decreases. The argument from the proof of Proposition 6 can then be applied again, and utilities become more unequal for any inequality measure in the Atkinson class.

The argument for the line network is equivalent. From any efficient network $L$, there is a rewiring to the line network that decreases the utility of the highest degree agent at each step, which by Lemma 21 is also the highest utility agent, and increases the utility of all other agents. Thus, utilities become more equal for any inequality measure in the Atkinson class. □

We will now consider the multiple group case. With one group it was efficient for a network to form in which all agents are path-connected to each other. We now make an assumption to ensure this remains the case with multiple groups.

**Assumption 22** (Efficient Risk Sharing Across Group)**.** *For any network $L$ with at least two components there exists a risk sharing agreement $\tau$, and a pair of agents $i$ and $j \notin C_i$, such that all agents are weakly better off on $L \cup \{l_{ij}\}$ and some agents are strictly better off.*

Relative to the single group case, agents from different groups provide each other with access to less correlated income streams. This increases the total surplus generated by risk sharing conditional on a given network being formed. Moreover, the presence of across group links provides positive externalities to others insofar as it increases the marginal value of within group links. This raises the question of how the additional surplus generated by across group risk sharing should be split among the agents. We take a parsimonious approach to this issue by making two assumptions. The first assumption builds on the single group analysis. It requires that agents receive at least the same marginal benefits they would receive were all agents from the same group. The additional surplus generated must be split in a way such that each agent receives a weakly positive share.

**Assumption 23** (Lower Bound)**.** *Consider a network $L$, such that $l_{ij}$ is essential on $L \cup \{l_{ij}\}$, and two allocations of the agents to groups $G, G'$. If all agents are from the same group under $G$, such that $G(i) = G(j)$ for all $i \neq j$, then*

$$u_i^\tau(L \cup \{l_{ij}\}, G') - u_i^\tau(L, G') \geq u_i^\tau(L \cup \{l_{ij}\}, G) - u_i^\tau(L, G).$$

This assumption requires that the additional benefits an agent $i$ gets from risk sharing, in terms of the second period agreement reached relative to the payoff $i$ would have got were everyone from the same group, strictly increase if a link $l_{jk}$ is removed form the network and replaced by a link $l_{ij}$ without changing the set of agents in each component.

**Assumption 24** (Link Increasing Additional Benefits)**.** *Consider two networks $L$ and $L'$ connecting the same sets of agents, and two allocations of the agents to groups $G, G'$. If $L'$ can be reached from $L$ by rewiring a link to $i$ such that, $L' = \{L \setminus l_{jk}\} \cup l_{ij}$, $i \neq j \neq k$, $l_{ij} \notin L$, $l_{jk} \in L$, $G'$ contains agents from different groups and under $G$ all agents are from the same group, then*

$$u_i^\tau(L', G') - u_i^\tau(L, G') > u_i^\tau(L', G) - u_i^\tau(L, G).$$

Assumption 24 is only a coarse partial ordering on utilities. While it implies that an agent's share of the additional surplus generated by across group risk sharing increases as links are rewired to that agent, it makes no comparison between networks that cannot be reached by rewiring links to a single agent. In particular, following a rewiring to $i$, it does not pin down how the payoffs of other agents changes.

**Proposition 25.** *Suppose all groups have the same utility functions, such that $v_i = v_j$ for all $i, j$. With $k$ different groups, there exist a $\bar{\kappa}_W > 0$ such that for all $\kappa_W < \bar{\kappa}_W$ a network is Pareto efficient if and only if it is a tree with $k-1$ across group links.*

*Proof.* We begin by showing the "only if" direction. All Pareto efficient networks are trees. First, by Assumption 22, risk sharing among all agents is efficient so $L$ must connect all agents. Second, a Pareto improvement can be achieved on any connected non-tree network by implementing the same risk sharing arrangement and deleting a superfluous link, thereby saving these costs.

We now show that efficient networks must also have exactly $k-1$ across group links. We will show, by construction, that for any tree network with strictly more than $k-1$ across group links, there exists a Pareto improvement.

If there are more than $k-1$ across group links in a tree network, we claim that there must exist an across-group link $l_{ij}$ which, upon its removal, will result in a network $L' = L \setminus \{l_{ij}\}$ such that there exists two agents $(k, l)$, with $G(k) = G(l)$ and $C_k(L') \neq C_l(L')$.

Towards a contradiction, let there be $k' > k-1$ across group links and suppose this is not true. As $L$ is a tree network, removing all across group links must then result in there being $k'+1$ components. If there are no agents from the same group in different components, this implies that there must be at least $k'+1 > k$ different groups, which would be a contradiction. Thus there exist two components each containing an agent from the same group. Denote these agents $k, l$. As $L$ is a tree there exists a unique path between $k$ and $l$ on $L$, and as $k$ and $l$ are in different components following the removal of across group links there exists at least one across group link on this path. Letting this link be $l_{ij}$ proves the claim.

As $k, l$ are in different components on $L'$, but from the same group, the network $L'' = L' \cup l_{kl}$ will be a connected tree network with one less across-group link, and one more within-group link than $L$.

On the network $L''$ we implement the same risk-sharing arrangement as before, with one exception. First we identify the vector of consumptions for agents $i$ and $j$ that make them just as well off as on the original network, and continue to satisfy the Borch rule:

$$\frac{\partial v_i(c_i(\omega))/\partial c_i(\omega)}{\partial v_i(c_i(\omega'))/\partial c_i(\omega')} = \frac{\partial v_j(c_j(\omega))/\partial c_j(\omega)}{\partial v_j(c_j(\omega'))/\partial c_j(\omega')} = \frac{\partial v_{i'}(c_{i'}(\omega))/\partial c_{i'}(\omega)}{\partial v_{i'}(c_{i'}(\omega'))/\partial c_{i'}(\omega')},$$

for all states $\omega, \omega'$ and all $i' \neq k, l$.

As $i$ and $j$ save the cost of an across group link, and utility is strictly increasing and concave in consumption, this implies that $c_i(\omega)$ and $c_j(\omega)$ must strictly decreases in all states $\omega$. This additional consumption is passed onto agents $k$ and $l$. As there is a strictly positive amount of remaining consumption in all states of the world, and utilities are strictly increasing in consumption, there exist feasible consumption vectors for agents $k$ and $l$ that strictly increase $\mathrm{E}(v(c_k))$ and $\mathrm{E}(v(c_l))$. Thus, for all $\kappa_w$ sufficiently small, we have $\mathrm{E}(v(c_k)) > \kappa_w$ and $\mathrm{E}(v(c_l)) > \kappa_w$. We have therefore constructed a Pareto improvement.

We now show the "if" direction. Consider a tree network with $k-1$ across group links. Suppose we implement a risk sharing agreement in which $c_i(\omega) = c_j(\omega)$, for all $i$ and $j$. As all agents' consumptions are equalized in all states, there is then no way in which link formation costs can be redistributed and the risk sharing arrangement changed, without making someone worse. Suppose towards a contradictions that we can redistribute the link formation costs, by forming a different tree network with $k-1$ across group links, to generate a Pareto improvement. Holding consumption fixed, on the new network if some agents are better off, then some will be worse off. Thus, to achieve a Pareto improvement, consumptions will have to be changed. Let $c'(\omega)$ be the new consumption vector. As the utility function $v(\cdot)$ is concave, Jensen's inequality implies that

$$\frac{1}{n}\sum_i v(c_i'(\omega)) < v(\frac{1}{n}\sum_i c_i'(\omega)) = \frac{1}{n}\sum_i v(c_i(\omega)),$$

for all $\omega$. Thus the average expected utility from consumption will decrease, and total link formation costs have remained constant, so at least one agent must be worse off. This is a contradiction.

□

In our simple CARA utility, normally distributed incomes, Myerson value allocation rule model, underinvestment across group is possible but there is no underinvestment within group. The same example establishes the possibility of underinvestment across group in our more general setting. There is also never any underinvestment within group in our more general setting as we now show.

**Proposition 26.** *There is never any underinvestment within group.*

*Proof.* Consider any stable network $L'$ and allocation to groups $G'$. Suppose, towards a contradiction, there is underinvestment within a group in $L'$. There must then be an essential link $l_{ij}$ the planner could form to achieve a Pareto improvement. Stability of $L'$ implies that either $u_i^\tau(L' \cup \{l_{ij}\}, G') - u_i^\tau(L', G') < c_w$ or else $u_j^\tau(L' \cup \{l_{ij}\}, G') - u_j^\tau(L', G') < c_w$. Without loss of generality suppose $u_i^\tau(L' \cup \{l_{ij}\}, G') - u_i^\tau(L', G') < c_w$. Consider now the alternative grouping $G$ in which all agents are from the same group. In this case, by Assumption 15 and as $l_{ij}$ is essential, $u_i^\tau(L' \cup \{l_{ij}\}, G) - u_i^\tau(L', G) \geq c_w$. Thus, combining inequalities, $u_i^\tau(L' \cup \{l_{ij}\}, G) - u_i^\tau(L', G) > u_i^\tau(L' \cup \{l_{ij}\}, G') - u_i^\tau(L', G')$. This contradicts Assumption 23. □

Consider the partial ordering in which an agent $i$ is more central in a network $L'$ than in network $L$ if and only if $L'$ can be reached from $L$ by rewiring links only to $i$. The following result generalizes the result in the benchmark model that more centrally located agents within a group have higher incentive to create across group links.

**Proposition 27.** *Suppose that*

(i) *when there is one group, for all efficient networks $L \cup \{l_{ij}\}$, $g(\overline{d}, |C_i(L)|, |C_i(L \cup \{l_{ij}\})|) = g(\overline{d}, |C_j(L)|, |C_j(L \cup \{l_{ij}\})|)$; and*
(ii) *there are two groups.*

*Then, for any efficient network $L$ with across group link $l_{ij}$, if it is profitable for an agent $i$ to form $l_{ij}$, and the alternative efficient network $L'$ can be reached from $L$ by rewiring within group links to $i$, then it is also profitable for $i$ to form the link $l_{ij} \in L'$.*

*Proof.* Let $G'$ be the grouping of agents. Agent $i$ is weakly better incentivized to invest in the across group link $l_{ij}$ on the network $L'$ than the network $L$ if and only if

$$u_i^\tau(L, G') - u_i^\tau(L \setminus \{l_{ij}\}, G') \leq u_i^\tau(L', G') - u_i^\tau(L' \setminus \{l_{ij}\}, G'). \tag{17}$$

As $L$ and $L'$ are efficient, and $l_{ij}$ is an across group link on both $L$ and $L'$, all agents who are path-connected to $i$ on $L \setminus \{l_{ij}\}$ are from the same group as $i$, as are all agents path connected to $i$ on $L' \setminus \{l_{ij}\}$. Thus, on the networks $L' \setminus \{l_{ij}\}$ and $L \setminus \{l_{ij}\}$, by Assumption 23 agent $i$ must then get exactly the same payoffs as he would do in the one group case: $u_i^\tau(L \setminus \{l_{ij}\}, G') = u_i^\tau(L \setminus \{l_{ij}\}, G)$ and $u_i^\tau(L' \setminus \{l_{ij}\}, G') = u_i^\tau(L' \setminus \{l_{ij}\}, G)$, where $G$ is the grouping in which all agents are from the same group. We can therefore rewrite equation 17 as

$$
\begin{aligned}
u_i^\tau(L,G') - u_i^\tau(L,G) + u_i^\tau(L,G) - u_i^\tau(L \setminus \{l_{ij}\},G) \quad &\leq \quad u_i^\tau(L',G') - u_i^\tau(L',G) \\
&\quad +u_i^\tau(L',G) - u_i^\tau(L' \setminus \{l_{ij}\},G).
\end{aligned} \tag{18}
$$

Repeatedly applying Assumption 24, $u_i^\tau(L,G') - u_i^\tau(L,G) < u_i^\tau(L',G') - u_i^\tau(L',G)$. Thus a sufficient condition for equation 18 to hold is that:

$$u_i^\tau(L,G) - u_i^\tau(L \setminus \{l_{ij}\},G) \leq u_i^\tau(L',G) - u_i^\tau(L' \setminus \{l_{ij}\},G).$$

As we are in the one group case and $l_{ij}$ is essential on both $L$ and $L'$, $u_i^\tau(L,G) - u_i^\tau(L \setminus \{l_{ij}\},G) = u_i^\tau(L',G) - u_i^\tau(L' \setminus \{l_{ij}\},G) = g(\overline{d})$. This completes the proof.

□

## Appendix II. Permitting some free links

In our model we assume that each link costs a fixed amount to form, but in practice, certain relationships will already exist permitting risk sharing without any investment. We now permit this possibility by assuming that there are a set of within-group links that can be formed for free. These links might represent family relationships or close friendships formed in childhood. Within this context we re-examine the structure of efficient networks, and those networks that are most stable to underinvestment and overinvestment.

More formally, we let $\widehat{L}$ denote the set of within group links that can be formed for free. As, by the Myerson Value calculation, a link strictly increases the expected utility an agent receives in a risk sharing arrangement, we let all such links be always formed. The network $\widehat{L}$ will consist of a set of components, each of which contains agents from the same group. For each such component $C$, we identify an agent $i^*(C) \in \text{argmin}_i \max_j md_{ij}(C)$. This is agent who has the lowest maximum Myerson Distance to any other agent in the component $C$. We will refer to agent $i^*(C)$ as the Myerson distance central agent in component $C$ and let $C_i$ denote the component to which $i$ belongs. Considering all components, we then have a set of Myerson distance central agents $I^* = (i^*(C))_C$. Finally, we identify a Myerson distance central agent associated with the largest distance, $i^{**} \in \text{argmax}_{i^* \in I^*} \max_{j \in C_{i^*}} md_{i^*j}$.

When there is one group, we dub a network generated by forming all free links, and the links $l_{i^*i^{**}}$ for all $i^* \neq i^{**}$ a *central connections network*.

Suppose there are $k$ different groups and $k' \geq k$ initial components. The set of efficient network then comprises of the set of networks in which all free links are formed, $k-1$ across group links are formed and $k'-k$ within group links are formed, such that a there is a single component. This is the lowest cost way to form a single component, and by assumption it is efficient for all agents to risk share with each other.[51] We now consider those

[51]As before, the same set of risk sharing arrangements can be implemented on any given component, and as expected utility is transferable, given that formation costs have been minimized, any point on the Pareto frontier can be obtained.

efficient networks that are most robust to underinvestment. When there is one group, central connections networks are always efficient. They are are also most stable within the class of efficient networks.

**Proposition 28.** *Suppose there is one group. If any efficient network is stable, then all central connections networks are also stable.*

*Proof.* Consider two components $C$ and $C'$. For two agents $i, j$ in component $C$, recall that $md(i, j, C)$ equals $1/2$ less the probability that a path exists between $i$ and $j$ on $C$ upon the arrival of $i$. Suppose now we take two components $C$ and $C'$. Let agents $i, k$ be in component $C$ and agents $j, k'$ be in component $C'$, and form the bridging link $l_{kk'}$. The probability a path exists between $i$ and $j$ upon $i$'s arrival is now is equal to the probability that a path exists between $i$ and $k$ on $C$ multiplied by the probability that a path exists between $k'$ and $j$ on $C'$. This is because these events are independent, and when both path exist agents $k$ and $k'$ must have arrived before $i$ and so the link $l_{kk'}$ must be present. It follows that

$$\operatorname*{argmax}_{i,j} md_{ij}(C \cup C' \cup \{l_{kk'}\}) = \{i, j : i \in \operatorname*{argmax}_{l} md_{lk}(C), j \in \operatorname*{argmax}_{l} md_{lk'}(C')\}.$$

Thus the network generated by forming all free links, and the links $l_{i^* i^{**}}$ for all $i^* \neq i^{**}$ minimizes the maximum Myerson distance on an efficient network and, by Lemma 4, is stable if any other efficient network is stable. □

Proposition 28 shows that when some within-group are formed for free, the most stable efficient network continues to create additional links that increase the centrality of the most central agents.

When there are multiple groups, central connections networks within group with the agent $i^{**}$ providing the across group link(s) continue to work well. With multiple groups, agents' incentives to form superfluous within-group links depend on two things. First, as before, whether the link will be essential for a random arrival order, and second, unlike before, how many agents from other groups the link provides access to upon $i$'s arrival when it is essential. Incentives to form a superfluous within-group links are increasing in the number of agents from other groups the link provides access to, and decreasing in the number of agents within-group the link provides access to. These considerations make superfluous links to the agent providing the across group link(s) particularly valuable. However, by construction the network generated by forming a central connections network within-group, with the agent $i^{**}$ providing the across group link(s), minimizes the maximum probability that a superfluous link to the agent providing the across group link(s) will be essential for a random arrival order. It thus minimizes the maximum incentives for an agent to form a superfluous link within-group to the agent providing the across group link(s).

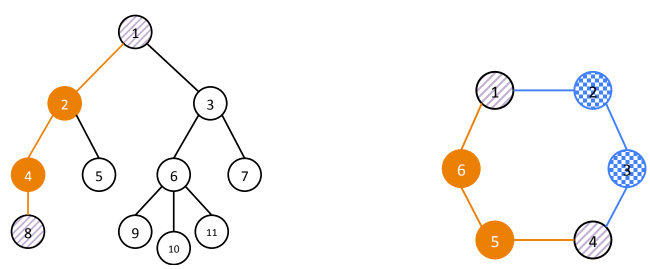
Considering the incentives within a group to efficiently form an across-group essential link, a central connections networks within-group is also likely to do well. By Lemma 9 more Myerson central agents have better incentives to form across group links. While central connections networks maximize a slightly different notion of the centrality of the most central agent, in this case agent $i^{**}$, these measures of centrality are likely to be highly correlated. We therefore expect central connections networks within-group to provide relatively good incentives for across group links to be formed

## Appendix III. Variable Construction

III.1. **Approximating the Myerson distance and centrality.** We would like to compute the Myerson distance of every pair in every village and the Myerson centrality for all nodes. Unfortunately, this is computationally infeasible for the sample sizes of our data (see Algaba et al. (2007)), presenting a new challenge. Thus, we develop an approximation, described below.

Let $\mathbf{md}(L)$ be the matrix of Myerson distances and define $\mathbf{q}(L) := 1/2 - \mathbf{md}(L)$. So $\mathbf{q}(L)$ is a matrix with the $ij$th entry capturing the probability that, upon his arrival agent $i$ will not be connected to agent $j$. It is difficult to directly characterize $\mathbf{md}(L)$ (or equivalently, $\mathbf{q}(L)$) as each village typically consists of around 230 households and the number of candidate paths between each $i$ and $j$ is exponential in the size of the network. Correctly accounting for paths that share nodes is computationally very intensive (see Lemma 4), and it has to be done for all pairs of agents without a direct connection.[52] Instead, we develop a computationally feasible approximation of $\mathbf{md}(L)$, which is exact for trees.

To approximate $\mathbf{q}$, we use the following idea. The algorithm works by starting with a node, moving to its neighbors, then move to its neighbors' neighbors, and so on, never returning to a previously used node along a given walk. This helps us to avoid counting walks that revisit nodes and are therefore not paths. All the while, we keep track of how many ways we have moved from the original node to any given node. We denote our approximation of $\mathbf{q}$ by $\widehat{\mathbf{q}}$.




(A) Tree (B) Circle

Figure 6. The nodes $i$,$j$ for which we are computing $md(i, j, L)$ have purple stripes. The tree contains a single path (solid orange nodes), whereas the circle contains two paths (solid orange nodes and chequered blue nodes).

[52]Further, due to presumed measurement error (see Banerjee et al. (2013)), there are likely to be missing paths. In fact, the data have occasional disconnected components, and so measures that are precisely based on exact paths or even maximal path lengths are likely to be problematic (Chandrasekhar and Lewis (2014)).

The inclusion–exclusion principle weights paths that are longer less and a path that shares many nodes with another less. With this in mind, we choose the following two approximation strategies. Let the shortest path between two nodes be of length $l$. We first count the paths of length $l$ and length $l + 1$. We then count paths of length $l + 2$.[53] If there are fewer than $k$ such paths, we use them all. Otherwise, we consider only the $k$ shortest and in practice we set $k = 4$.[54] Discarding longer paths in this way biases downwards our approximation of $\mathbf{q}$. As we cannot keep track of exactly which nodes feature in each path, we also have to make an assumption about the overlap of nodes in order to apply the inclusion–exclusion principle to these paths. Each path must share the same first and last node. We perform the inclusion–exclusion principle assuming that only these nodes are shared (see Section 4). Assuming no other nodes are shared introduces a second bias, but this time upwards in our approximation.

To explain these concepts, we provide some illustrations. Figure 6 presents two examples: a tree and a circle. The tree has a single path between nodes 1 and 8, whereas the circle has two paths between nodes 1 and 4. Figure 7 shows how links are removed for the case of a tree. Once a node has been reached, links back into that node are deleted before the nodes neighbors are "infected." This ensures only paths, and not other walks, are included in the calculation.

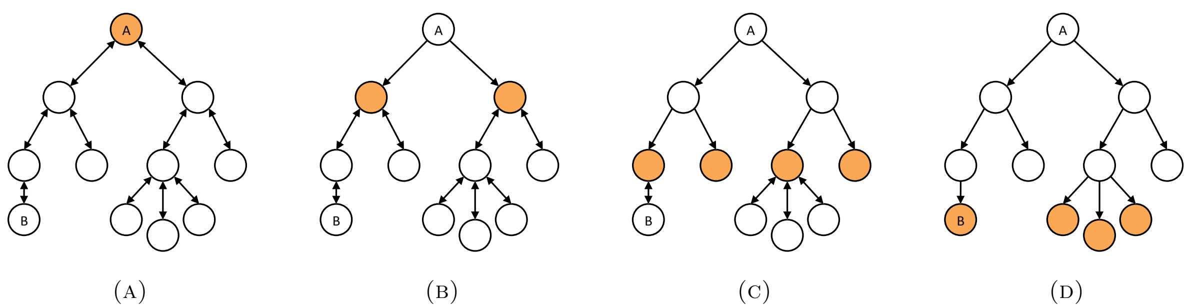



FIGURE 7. As the algorithm progresses, directed links into nodes that are reached are deleted. This ensures that only paths, and not other walks, are included. In this case, as in all tree networks, there is a unique path from A to B.

In the case of the circle shown in Figure 6, our algorithm is also exact for paths between 1 and 4. There are two paths (which in this case are both shortest paths too), and we find both in the initial run of our algorithm. Following the inclusion–exclusion principle, we add 1/4 to 1/4 and subtract 1/6. In this case our assumption that the two paths share only two nodes is accurate. We are also exact for paths between 1 and 3, but in this case there is a

[53]Counting more paths greatly (exponentially) increases the running time of our algorithm.

[54]We need a fixed (small) truncation. Otherwise both the memory requirements and the run-time of the algorithm grow exponentially. Results are not sensitive to the truncation point.

path of length $l+2$. To find this path, we look for paths of length $l$ from 1 to nodes other than 3. In this case there is one such path to node 5. We then look for paths from 5 to 3 that pass through one other node. There is one such path and so the calculation we perform is: $1/3+1/5-1/6$. While we are accurate for all pairs of nodes in the circle shown, in larger circles we will miss the longer paths.

The following algorithm finds the length of the shortest path between two nodes, how many paths of that length there are and how many paths there are that are one longer. From this information, we also find paths of length $l+2$.[55]

**Algorithm 29** (Incoming Link Deletion)**.** *Let $e^i$ be the ith basis vector. This will represent the root (starting) node. Initialize $\widehat{\mathbf{q}} = zeros(n,n)$, a matrix of zeros. Initialize $z^{t,i} = zeros(n,1)$ and $x^{t,i} = zeros(n,1)$ to be n-vectors of zeros, indexed by $i = 1,...,n$ and $t = 1,...,T$. Repeat steps 1–4 for each of $(e^1,...,e^n)$.*

(1) *Period 1: There is no identification or updating steps.*
  (a) *Percolation: $x^{1,i} = \mathbf{A}e^i$.*
  *(Identifies who is connected to the root node)*
(2) *Period 2, given $(x^{1,i}, \mathbf{A})$:*
  (a) *Identification: $z^{2,i} = e^i$.*
  (b) *Update graph:*[56] $\mathbf{A}_2 = zeros(n,n)$, $\mathbf{A}_2(\neg z^{2,i},:) = \mathbf{A}(\neg z^{2,i},:)$.
  *(Deletes links into the root node)*
  (c) *Percolation: $x^{2,i} = \mathbf{A}_2 x^{1,i}$.*
  *(Records number of paths from root node to other nodes passing through one other)*
(3) *Period t, given $(x^{t-1,i}, \mathbf{A}_{t-1})$:*
  (a) *Identification: $z^{t,i} = \mathbf{1}\left\{\sum_{s=3}^{t} x^{s-2,i} > 0\right\}$.*
  *(Identifies nodes already visited)*
  (b) *Update graph: $\mathbf{A}_t = zeros(n,n)$, $\mathbf{A}_t(\neg z^{t,i},:) = \mathbf{A}_{t-1}(\neg z^{t,i},:)$.*
  *(Deletes links into all nodes that have already been visited)*
  (c) *Percolation: $x^{t,i} = \mathbf{A}_t x^{t-1,i}$.*

By construction $x_j^{t,i}$, the $j$th entry of $x^{t,i}$, records paths from $i$ to $j$ that pass through $t$ nodes. If $t'$ is the lowest $t$ with a positive entry in this matrix, then the shortest path from $i$ to $j$ passes through $t'$ nodes. In this case, $x_j^{t',i}$ tells us how many such paths there are and $x_j^{t'+1,i}$ tells us how many paths there are that pass through one more node. However, by construction $x_j^{t'+k,i} = 0$ for all $k > 1$ and longer paths are not recorded. This is because the incoming links to node $j$ will have been deleted by this step of the algorithm. Deletion of incoming links helps prevents walks that are not paths from being recorded. Using this

[55] For paths from $i$ to $j$, this is done by looking at paths of length $l$ to agents other than $j$, and then looking at paths from these agents to $j$.
[56] Let $\mathbf{A}(:,v)$ denote $(A(1,j),...,A(n,j))$.

information for all seed nodes, the number of paths of length $t'+2$ between $i$ and $j$ are also found as described above. The inclusion-exclusion principle is then applied to this combined set of paths, assuming each path shares only the first and last nodes, to calculate $\widehat{\mathbf{q}}(L)$.

To approximate Myerson centrality we use $\widehat{MC}_i = \sum_j \widehat{q}_{ij}$. While this approximation generates a cardinal measure of what is an ordinal concept, it does correctly order people when the Myerson distance approximation is exact as shown in Proposition 30

**Proposition 30.** *If $i$ is more Myerson central than $j$, then $\sum_k q_{ik} > \sum_k q_{jk}$.*

The proof is relegated to Section A of the Supplementary Appendix. We now show that the Myerson distance approximation is exact for trees.

**Proposition 31.** *Let $L$ be a tree. Then $\widehat{\mathbf{q}}(L) = \mathbf{q}(L)$.*

Again, the proof is in Section A of the Supplementary Appendix. In combination, Propositions 30 and 31 lead directly to the following corollary.

**Corollary 32.** *Let $L$ be a tree. If $i$ is more Myerson central than $j$, then $\widehat{MC}_i > \widehat{MC}_j$.*

A limitation of the Incoming Link Deletion algorithm is that longer paths are excluded. To address this, we construct an alternative algorithm. This Outgoing Link Deletion algorithm is identical to the one described, except that it deletes outgoing links instead of incoming links. The Outgoing Link Deletion algorithm finds longer paths, and does an especially good job of picking up longer paths that share few nodes with other paths. However, it also includes additional short walks that are not paths and is not exact for tree networks. As longer paths are found, we directly use the output of the algorithm without constructing any additional longer paths. Nevertheless, for the set of paths we find, it is computationally infeasible to compute the Myerson distances using the inclusion-exclusion principle. Censoring these paths would defeat the point of the Outgoing Link Deletion algorithm. Instead, we use an approximation of the inclusion–exclusion principle which makes the computation much simpler. This approximation treats every path as completely independent, assuming that no nodes are shared (even though we know at two must be). For example, if we find 3 paths from $i$ to $j$ that pass through $l$ nodes, $l'$ nodes and $l''$ nodes respectively, our approximation of $q_{ij}$ will be $1/l + 1/l' + 1/l''$. Finally, we also consider a hybrid of the Incoming Link Deletion algorithm and the Outgoing Link Deletion algorithm. We refer to this as the Link Deletion algorithm.

## Supplementary Appendix: For Online Publication Only

### Appendix A. Omitted Proofs

*Proof of Proposition 1.* To prove the first statement, consider villagers' certainty-equivalent consumption. Let $\hat{K}$ be some constant, and consider the certain transfer $K'$ (made in all states of the world) that $i$ requires to compensate him for keeping a stochastic consumption stream $c_i + \hat{K}$ instead of another stochastic consumption stream $c_i' + \hat{K}$:

$$\begin{aligned} \mathbf{E}[v(c_i + \hat{K} + K')] &= \mathbf{E}[v(c_i' + \hat{K})] \\ -\frac{1}{\lambda}e^{-\lambda \hat{K}}e^{-\lambda K'}\mathbf{E}[e^{-\lambda c_i}] &= -\frac{1}{\lambda}e^{-\lambda \hat{K}}\mathbf{E}[e^{-\lambda c_i'}] \\ e^{\lambda K'} &= \frac{\mathbf{E}[e^{-\lambda c_i}]}{\mathbf{E}[e^{-\lambda c_i'}]} \\ K' &= \frac{1}{\lambda}\left(\ln\left(\mathbf{E}[e^{-\lambda c_i}]\right) - \ln\left(\mathbf{E}[e^{-\lambda c_i'}]\right)\right) \qquad (19) \end{aligned}$$

This shows that the amount $K'$ needed to compensate $i$ for taking the stochastic consumption stream $c_i + \hat{K}$ instead of $c_i' + \hat{K}$ is independent of $\hat{K}$. As a villager's certainty-equivalent consumption for a lottery is independent of his consumption level, certainty-equivalent units can be transferred among the villagers without affecting their risk preferences, and expected utility is transferable.

Next, we characterize the set of Pareto efficient risk sharing agreements. Borch (1962) and Wilson (1968) showed that a necessary and sufficient condition for a risk-sharing arrangement between $i$ and $j$ to be Pareto efficient is that in almost all states of the world $\omega \in \Omega := \mathbf{R}^{|S|}$,

$$\left(\frac{\partial v_i(c_i(\omega))}{\partial c_i(\omega)}\right) \Big/ \left(\frac{\partial v_j(c_j(\omega))}{\partial c_j(\omega)}\right) = \alpha_{ij} \qquad (20)$$

where $\alpha_{ij}$ is a constant. Substituting in the CARA utility functions, this implies that

$$\begin{aligned} \frac{e^{-\lambda c_i(\omega)}}{e^{-\lambda c_j(\omega)}} &= \alpha_{ij} \\ c_i(\omega) - c_j(\omega) &= -\frac{\ln(\alpha_{ij})}{\lambda} \\ \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] &= -\frac{\ln(\alpha_{ij})}{\lambda} \\ c_i(\omega) - c_j(\omega) &= \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] \qquad (21) \end{aligned}$$

Letting $i$ and $j$ be neighbors such that $j \in \mathbf{N}(i)$, equation 21 means that when $i$ and $j$ reach any Pareto-efficient risk-sharing arrangement their consumptions will differ by the same constant in all states of the world. Moreover, by induction the same must be true for all pairs of path-connected villagers.

Consider now the problem of splitting the incomes of a set of villagers $\mathbf{S}$ in each state of the world to minimize the sum of their consumption variances:

$$\min_{\mathbf{c}} \sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) \qquad \text{subject to} \quad \sum_{i\in\mathbf{S}} y_i(\omega) = \sum_{i\in\mathbf{S}} c_i(\omega) \quad \text{for all } \omega. \tag{22}$$

If we denote a CDF of income probability distribution on $\Omega = \mathbf{R}^{|S|}$ by $F(\cdot)$,

$$\sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) = \int_{\Omega} \sum_{i\in\mathbf{S}} (c_i(\omega) - \mathbf{E}[c_i])^2 dF(\omega). \tag{23}$$

Since $\mathrm{Var}(c_i(\omega) + a_i) = \mathrm{Var}(c_i(\omega))$, the sum of variances is invariant to state-independent changes in a consumption profile, and the variance-minimizing consumption profile exists for any profile of expected consumptions $\{\mathbf{E}[c_i]\}_{i\in\mathbf{S}}$: $\sum_{i\in\mathbf{S}} \mathbf{E}[c_i] = \sum_{i\in\mathbf{S}} \mathbf{E}[y_i]$. Fix any such profile of expected consumptions, $\{\mathbf{E}[c_i]\}_{i\in\mathbf{S}}$. Similarly to Wilson (1968), we apply Theorem 1 from Zahl (1963) to our minimization problem. We denote a Lagrange multiplier attached to constraint $\sum_{i\in\mathbf{S}} y_i(\omega) = \sum_{i\in\mathbf{S}} c_i(\omega)$ by $\gamma(\omega)$. Then, the corresponding Lagrangian of the problem is

$$\int_{\Omega} \left[ \sum_{i\in\mathbf{S}} (c_i(\omega) - \mathbf{E}[c_i])^2 - \gamma(\omega) \sum_{i\in\mathbf{S}} c_i(\omega) \right] dF(\omega).$$

By pointwise minimization with respect to $c_i(\omega)$ we obtain that for each $i \in \mathbf{S}$ and almost every $\omega \in \Omega$, $2(c_i^*(\omega) - \mathbf{E}[c_i]) = \gamma(\omega)$. Thus, $c_i^*(\omega) - c_j^*(\omega) = \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]$ for all $i, j \in \mathbf{S}$. Note that this equality as well implies that $\mathbf{E}[c_i^*(\omega)] = \mathbf{E}[c_i]$, and $\{c_i^*(\omega)\}$ indeed solves the minimization problem. Thus, the condition $c_i^*(\omega) - c_j^*(\omega) = \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]$ for almost all $\omega$ is exactly the same as the necessary and sufficient condition for an ex-ante Pareto efficiency. Hence, a risk-sharing agreement is Pareto efficient if and only if the sum of the consumption variances for all path-connected villagers is minimized.

Using the necessary and sufficient condition for efficient risk sharing, we obtain

$$\sum_{k\in\mathbf{S}} y_k(\omega) = \sum_{k\in\mathbf{S}} c_k(\omega) = |\mathbf{S}| c_i(\omega) - \sum_{k\in\mathbf{S}} \left(\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_k(\omega)]\right), \tag{24}$$

which implies that

$$c_i(\omega) = \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega) + \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} \left(\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]\right) = \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega) + \tau_i, \tag{25}$$

where $\tau_i = \mathbf{E}[c_i(\omega)] - \mathbf{E}[\frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega)]$. □

*Proof of Lemma 4.* Agent $i$'s net benefit from forming link $l_{ij}$ is $(MV_i(L) - MV_i(L \setminus \{l_{ij}\}) - \kappa_w)$. We need to show that

$$MV_i(L) - MV_i(L \setminus \{l_{ij}\}) = MV_j(L) - MV_j(L \setminus \{l_{ij}\}) = md(i, j, L)V. \tag{26}$$

Some additional notation will be helpful. Suppose agents arrive in a random order, with a uniform distribution on all possible arrival orders. The random variable $\widehat{\mathbf{S}}_i \subseteq \mathbf{N}$ identifies the set of agents, including $i$, who arrive weakly before $i$. For each arrival order, we then have an associate network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ that describes the network formed upon $i$'s arrival (the subnetwork of $L$ induced by agents $\widehat{\mathbf{S}}_i$). Let $q(i,j,L)$ be the probability that $i$ and $j$ are path-connected on network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$.

The certainty-equivalent value of the reduction in variance due to a link $l_{ij}$ in a graph $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ is $V$ if the link is essential and 0 otherwise. The change in $i$'s Myerson value, $MV_i(L) - MV_i(L \setminus \{l_{ij}\})$, is then $(q(i,j,L) - q(i,j,L \setminus \{l_{ij}\}))\,V$. However, $q(i,j,L) = 1/2$. To see this, note that $l_{ij} \in L$ and therefore in every order of arrival in which $i$ arrives after $j$ (which happens with probability 1/2), $i$ and $j$ are path-connected on the network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$, while $i$ and $j$ are never path-connected on $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ when $j$ arrives after $i$.

Probability $q(i,j,L \setminus \{l_{ij}\})$ can be computed by the inclusion-exclusion principle, using the fact that the probability of a path connecting $i$ and $j$ existing on network $\mathcal{L}_{L\setminus\{l_{ij}\}}(\widehat{\mathbf{S}}_i)$ is equal to the probability that for some path connecting $i$ and $j$ on $L \setminus \{l_{ij}\}$ all agents on the path are present in $\widehat{\mathbf{S}}_i$. Thus

$$q(i,j,L \setminus l_{ij}) = \sum_{k=1}^{|\mathbf{P}(i,j,L\setminus l_{ij})|} (-1)^{k+1} \left( \sum_{1 \le i_1 < \cdots < i_k \le |\mathbf{P}(i,j,L)|} \left( \frac{1}{|P_{i1} \cup \cdots \cup P_{ik}|} \right) \right). \tag{27}$$

We therefore have that

$$MV_i(L) - MV_i(L \setminus l_{ij}) = (1/2 - q(i,j,L \setminus l_{ij}))V = md(i,j,L)V, \tag{28}$$

where the last equality follows from the definition of Myerson distance.

□

*Proof of Proposition 5.* For there to be underinvestment in a pairwise-stable network $L$, there must exist a link $l_{ij} \notin L$ for which $TS(L \cup l_{ij}) - TS(L) > 2\kappa_w$. This can only happen if $l_{ij}$ is essential on $L \cup l_{ij}$ as otherwise $TS(L \cup l_{ij}) - TS(L) = 0$. Thus $TS(L \cup l_{ij}) - TS(L) = V$ and so $V > 2\kappa_w$. As $l_{ij} \notin L$ and $L$ is pairwise stable, Lemma 4 implies that $md(i,j,L) \le \kappa_w/V$. However, as $l_{ij}$ is essential on $L \cup l_{ij}$, $md(i,j,L) = 1/2$. Substituting this into the condition from Lemma 4 we get $V \le 2\kappa_w$, leading to a contradiction.

For the second part of the proposition, let $L$ be a pairwise-stable network and let $l_{ij} \in L$ be an essential link on $L$ in which there is overinvestment. Thus $TS(L) - TS(L \setminus \{l_{ij}\}) = V < 2\kappa_w$. Since $l_{ij}$ is essential, $md(i,j,L \setminus \{l_{ij}\}) = 1/2$. But Lemma 4 implies that $md(i,j,L \setminus \{l_{ij}\}) \ge \kappa_w/V$. We therefore have that $V \ge 2\kappa_w$, leading to a contradiction. □

*Proof of Proposition 6.*

**Part (i):** By remark 3 and under our regularity condition, all efficient networks are tree networks. By definition, in all tree networks any pair of agents $i$ and $j$ have a unique path between them. Thus, for a tree network $L$ with diameter $d(L)$, there exist agents $i$ and $j$ with a unique path between them of length $d(L)$ and all other pairs of agents have a weakly shorter path between them. Thus by equation 11:

$$md(i,j,L) = \frac{1}{2} - \frac{1}{d(L)} \geq md(k,k',L) \qquad \text{for all } k,k' \in \mathbf{N}. \tag{29}$$

By Proposition 5 there is no underinvestment in any stable network. Lemma 4 therefore implies that the efficient network $L$ is stable if and only if $md(k,k',L) \leq \kappa_w/V$ for all $k,k'$ such that $l_{kk'} \notin L$. As $md(i,j,L) \geq md(k,k',L)$ and $md(i,j,L) = 1/2 - 1/d(L)$ (see equation 29), this condition simplifies and the efficient network $L$ is stable if and only if

$$\frac{V - 2\kappa_w}{V} \leq \left(\frac{2}{d(L)}\right). \tag{30}$$

As $d(L)$ gets large, the right-hand side converges from above to 0 and so in the limit, the condition for stability becomes $V \leq 2\kappa_w$, which is violated by our regularity condition. Thus, there exists a finite $\overline{d}(L)$ such that the efficient network $L$ is stable if and only if $d(L) \leq \overline{d}(L)$.

Rearranging equation 30, $L$ is stable if and only if

$$d(L) \leq 2\left(\frac{V}{V - 2\kappa_w}\right). \tag{31}$$

So the key threshold is $\overline{d}(\kappa_w) = \lfloor 2V/(V - 2\kappa_w) \rfloor$.

Fixing the number of agents $|\mathbf{N}|$ in an efficient (tree) network $L$, the star network is the unique (tree) network (up to a relabeling of players) that minimizes the diameter $d(L)$ while the line network is the unique (tree) network (up to a relabeling of players) that maximizes the diameter $d(L)$. The result now follows immediately.

**Part (ii):** On any efficient networks all links are essential and generate a net surplus of $V - 2\kappa_w > 0$, where the inequality follows from our regularity condition. As $i$ and $j$ must benefit equally at the margin from the link $l_{ij}$ (see condition (ii) in the definition of agreements that are robust to split-the-difference renegotiation), agent $i$'s expected payoff on an efficient network $L$ is

$$u_i(L) = |\mathbf{N}(i;L)|(V/2 - \kappa_w) > 0. \tag{32}$$

Thus $i$'s net payoff is proportional to his degree.

For any tree network $L$ other than the star network let agent $k$ be one of the agents with the highest degree. Consider a link $l_{ij} \in L$ such that $i, j \neq k$. As $L$ is a tree there is a unique path from $i$ to $k$ and a unique path length from $j$ to $k$. As we are on a tree network, either the path from $j$ to $k$ passes through $i$, or else the path from $i$ to $k$ passes through $j$. Hence either $i$ or $j$ is closer to $k$ and without loss of generality we let $i$ have a longer path to $k$ than $j$. We now delete the link $l_{ij}$ and replace it with the link $l_{ik}$. This operation generates a new tree network. Moreover, repeating this operations until there are no links $l_{ij}$ such that $i, j \neq k$, defines an algorithm.

This algorithm terminates at star networks as the operation cannot be applied to this network; There are no links of $l_{ij}$ such that $i, j \neq k$. Moreover the operation can be applied to any other tree network because on all other tree networks there exists an $l_{ij}$ such that $i, j \neq k$. Finally, in each step of the algorithm the degree of $k$ increases and so the algorithm must terminate in a finite number of steps. Moreover, the algorithm must terminates at the star network with $k$ at the center.

By construction, at each step of the above algorithm we decrease the degree of some agent $j \neq k$ and increase the degree of $k$. Suppose we start with a network $L$ and consider a step of this rewiring where the link $l_{ij}$ is deleted and replaced by the link $l_{ik}$. Only the expected payoff of agents $j$ and $k$ on $L$ and $L \cup l_{ik} \setminus l_{ij}$ change; The degrees of all other agents remain constant and thus by equation 32 so do their payoffs. Letting $\alpha = (V/2 - \kappa_w)$, we have $u_j(L) = \alpha d_j(L)$, $u_k(L) = \alpha d_k(L)$, $u_j(L \cup l_{ik} \setminus l_{ij}) = \alpha(d_j(L) - 1)$ and $u_k(L \cup l_{ik} \setminus l_{ij}) = \alpha(d_k(L) + 1)$.

It follows that welfare $W(u) = \sum_i f(u_i)$ (see equation 9) decreases through the rewiring in this step if and only if

$$f(\alpha(d_j - 1)) + f(\alpha(d_k + 1)) - f(\alpha d_j) - f(\alpha d_k) < 0, \tag{33}$$

which is equivalent to:

$$f(\alpha(d_k + 1)) - f(\alpha d_k) < f(\alpha d_j) - f(\alpha(d_j - 1)) \tag{34}$$

As $f(\cdot)$ is increasing, strictly concave and differentiable $f'(\alpha d_j)\alpha < f(\alpha d_j) - f(\alpha(d_j - 1))$ and $f'(\alpha d_k)\alpha > f(\alpha(d_k + 1)) - f(\alpha d_k)$. Moreover, by concavity $f'(\alpha d_j) \geq f'(\alpha d_k)$ (as $d_k \geq d_j$). Combining these inequalities establishes the claim that $f(\alpha(d_k + 1)) - f(\alpha d_k) < f(\alpha d_j) - f(\alpha(d_j - 1))$.

Thus at each step of the rewiring welfare $W(u)$ decreases. For each network $L'$ reached during the algorithm we can consider the average expected utility $u'(L')$ which if distributed equally would generate the same level of welfare as obtained on $L$. As aggregate welfare is decreasing at each step of the rewiring $u'(L)$ must be decreasing too. However, the total surplus generated by risk sharing remains constant and so average expected utility $\overline{u}$ remains constant. Recall that Atkinson's inequality measure / index is given by $I(L) = (1 - (u'(L)/\overline{u})$. Thus at each step of the rewiring the inequality measure $I(L)$ increases. As this rewiring can be used to move from any tree network to the star network, stars network and only star networks maximize inequality among the set of tree networks, which correspond to the set of efficient networks under our regularity condition. As this argument holds for any strictly increasing and differentiable, concave function $f$ it holds for all inequality measures in the Atkinson class.

Consider now an alternative rewiring of a tree network $L$. Let $k$ be one of the agents with highest degree on $L$ and let $j$ be one of the agents with degree 1 on $L$. As tree networks contain no cycles, there always exists agents with degree 1 (leaf agents). Pick one of $k$'s neighbors $i \in \mathbf{N}(k; L)$, remove the link $l_{ik}$ from $L$ and add the link $l_{ij}$ to $L$. This operation generates a new tree network. Repeating this operation until the highest degree agent has degree 2 defines an algorithm. As the unique tree network with a highest degree of 2 is the line network, the algorithm terminates at line networks and only line networks. At each stage of the rewiring we either reduce the degree of the highest degree agent $k$ or reduce the number of agents who have the highest degree. Thus the algorithm must terminate in a finite number of steps at a line network. Moreover, reversing the argument above, inequality is reduced at each step of the rewiring for any inequality measure in the Atkinson class.

□

*Proof of Proposition 7.* By definition, underinvestment within group for a network $L$ requires that there exists an $l_{ij} \notin L$ such that $G(i) = G(j)$ and for which $TS(L \cup l_{ij}) - TS(L) > 2\kappa_w$. As $TS(L \cup l_{ij}) - TS(L) = 0$ for all non-essential links, $l_{ij}$ must be essential on $L \cup \{l_{ij}\}$. Thus $l_{ij}$ is also essential on $\widehat{L} \cup \{l_{ij}\}$ for any $\widehat{L} \subseteq L$. Equation 13 then implies that $TS(\widehat{L} \cup l_{ij}) - TS(\widehat{L}) \geq V$ for any $\widehat{L} \subseteq L$.

Consider any arrival order in which $i$ arrives after $j$ and let $\mathbf{S}_i$ be the agents that arrive (strictly) before $i$. Agent $i$'s marginal contribution to total surplus without $l_{ij}$ when $i$ arrives is then $TS(L(\mathbf{S}_i \cup \{i\})) - TS(L(\mathbf{S}_i))$ while with $l_{ij}$ it is $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i))$. So $i$'s additional marginal contribution to total surplus when $l_{ij}$ has been formed is $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i \cup \{i\}))$. As $L(\mathbf{S}_i \cup \{i\}) \subseteq L$, by the above argument $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i \cup \{i\})) \geq V$. As $i$ arrives after $j$ in half the arrival orders, $i$'s average additional incremental contribution to total surplus when $l_{ij}$ has been formed is at least $V/2$. Thus $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \geq V/2$. An equivalent argument establishes that $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \geq V/2$. Under our regularity condition $V/2 > \kappa_w$ and so $i$ and $j$ have a profitable deviation to form $l_{ij}$ and the network $L$ is not stable. As $L$ was an arbitrary network within underinvestment within group, there is no stable network with underinvestment within group. □

*Proof of Proposition 8.* The proof of the first part of the statement has four steps.

*Step 1:* Consider any efficient network $L$ that is robust to overinvestment inefficiency within group. This implies that for all path-connected agents $i, j$ such that $G(i) = G(j)$ and $l_{ij} \notin L$, either $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \leq \kappa_w$ or $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \leq \kappa_w$. However, by condition (i) in the definition of agreements that are robust to split-the-difference renegotiation, $MV_i(L \cup \{l_{ij}\}) - MV_i(L) = MV_j(L \cup \{l_{ij}\}) - MV_j(L)$ and so both $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \leq \kappa_w$ and $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \leq \kappa_w$.

*Step 2:* Let a network $\widehat{L} := \{l_{ij} : G(i) = G(j), l_{ij} \in L\}$ be a network formed from $L$ by deleting all across-group links. Consider any subset of agents $\mathbf{S} \subseteq \mathbf{N}$ such that $i, j \in \mathbf{S}$. As the network $L$ is efficient, it is a tree network that minimizes the number of across-group links conditional on a given set of agents being in a component. This implies that the unique path between $i$ and $j$ cannot contain an across-group link. So, $i$ is path-connected to $j$ on the induced subnetwork $L(\mathbf{S})$ if and only if $i$ is path-connected to $j$ on the induced subnetwork $\widehat{L}(\mathbf{S})$. Thus, by equation 13, the additional variance reduction that $i$ and $j$ can now achieve by forming a superfluous across-group link on $\widehat{L}(\mathbf{S})$ is weakly lower than on $L(\mathbf{S})$. So, by the Myerson value definition (equation 6), $MV_i(\widehat{L}\cup\{l_{ij}\}) - MV_i(\widehat{L}) \leq MV_i(L\cup\{l_{ij}\}) - MV_i(L)$ and $MV_j(\widehat{L}\cup\{l_{ij}\}) - MV_j(\widehat{L}) \leq MV_i(L\cup\{l_{ij}\}) - MV_i(L)$. This implies that $\widehat{L}$ is robust to overinvestment within group.

*Step 3:* Let a network $\widehat{L}'$ be a network formed from $\widehat{L}$ by rewiring (alternately deleting then adding a link) each within-group network into a star (for an algorithm that does this, see the part (ii) of the proof of Proposition 6). Consider any two agents $i', j'$ such that $G(i') = G(j')$, $l_{i'j'} \notin \widehat{L}'$. By part (i) of Proposition 6, $MV_{i'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{i'}(\widehat{L}') \leq MV_i(\widehat{L} \cup \{l_{ij}\}) - MV_i(\widehat{L})$ and $MV_{j'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{j'}(\widehat{L}') \leq MV_j(\widehat{L} \cup \{l_{ij}\}) - MV_j(\widehat{L})$. Thus $\widehat{L}'$ is robust to overinvestment within group.

*Step 4:* Finally, consider any network $L' \in \mathcal{L}^{CSS}$. This network can be formed by adding a set of across-group links to a network $\widehat{L}'$ such that $\widehat{L}' \subseteq L'$ and if $l_{kk'} \in L' \setminus \widehat{L}'$ then $G(k) \neq G(k')$. Consider any subset of agents $\mathbf{S}' \subseteq \mathbf{N}$ such that $i', j' \in \mathbf{S}'$. Recall that $G(i') = G(j')$ and note that by the construction of $L'$, $l_{i'j'} \notin L'$. On the induced subnetwork $L'(\mathbf{S}')$, either $i'$ is path-connected to $j'$, in which case $l_{i'j'}$ would be superfluous if added, or else $i'$ and $j'$ are isolated nodes. This is because the within-group network structure for group $G(i')$ is a star. Thus, whenever $l_{i'j'}$ would not be superfluous, the change in $i'$ and $j'$'s Myerson value if it were added is independent of the across-group links that are present: $MV_{i'}(L'\cup\{l_{i'j'}\}) - MV_{i'}(L') = MV_{i'}(\widehat{L}'\cup\{l_{i'j'}\}) - MV_{i'}(\widehat{L}')$ and $MV_{j'}(L'\cup\{l_{i'j'}\}) - MV_{j'}(L') \leq MV_{j'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{j'}(\widehat{L}')$. Thus $L'$ is robust to overinvestment within group.

We turn now to the second part of the result. If $L \notin \mathcal{L}^{CCS}$, then there will be agents $i, j$ such that $G(i) = G(j)$ and $l_{ij} \notin L$ such that either the within-group network structure for $G(i)$ is not a star, or else it is a star but there are across-group links being held by an agent who is not the center agent. In the first case, the inequality in step 3 will be strict by Proposition 6. In the second case, we can without loss of generality let agent $i$ be the non-center agent holding the across-group link. Then, by equation 13, the inequality in step 2 will be strict. Thus for some parameter values $L$ will not be robust to overinvestment within group, but $L'$ will be. □

*Proof of Lemma 9.* Denote the set of all possible arrival orders for the set of agents $\mathbf{N}$, by $\mathcal{A}(\mathbf{N})$. Order this set of $|\mathbf{N}|!$ arrival orders in any way, denoting the $k$th arrival order by $\widehat{A}_k \in \mathcal{A}(\mathbf{N})$. We will then construct an alternative ordering, in which we denote the $k$th arrival order by $\widetilde{A}_k \in \mathcal{A}(\mathbf{N})$, such that for arrival order $\widetilde{A}_k$,

(i) $i$ arrives at the same time as agent $i'$ does for the arrival order $\widehat{A}_k$;

(ii) when $i$ arrives he connects to exactly the same set of agents from $\mathbf{N} \setminus \mathbf{S}_0$ that $i'$ connects to upon his arrival for the arrival order $\widehat{A}_k$;

(iii) when $i$ arrives he connects to weakly more agents from $\mathbf{S}_0$ that $i'$ connects to upon his arrival for the arrival order $\widehat{A}_k$.

Equation 15 shows that the risk reduction, and hence the marginal contribution made by an agent $k \in \mathbf{S}_0$ from providing the across-group link $l_{kj}$, is an increasing function of the component size of $k$'s groups. It then follows that

$$MV(i; L \cup l_{ij}) - MV(i; L) > MV(i'; L \cup l_{i'j}) - MV(i'; L). \tag{35}$$

To construct the alternative ordering of the set $\mathcal{A}(N)$ as claimed we will directly adjust individual arrival orders, but in a way that preserves the set $\mathcal{A}(N)$. First, for each arrival order, we switch the arrival positions of $i'$ and $i$. This alone is enough to ensure that conditions (i) and (ii) are satisfied. There are $|\mathbf{S}_0|!$ possible arrival orders for the set of agents $\mathbf{S}_0$. Ignoring for now the other agents, we label these arrival orders lexicographically. First we order them, in ascending order, by when $i$ arrives. Next, we order them in ascending order by the number of agents $i$ is connected to upon his arrival. Breaking remaining ties in any way, we have labels $1_i, 2_i, \ldots, |\mathbf{S}_0|!_i$. We then let every element of $\mathcal{A}(N)$ inherit these labels, so that two arrival orders receive the same label if and only if the agents $\mathbf{S}_0$ arrive in the same order. We now construct a second set of labels by doing the same exercise for $i'$, and denote these labels by $1_{i'}, 2_{i'}, \ldots, |\mathbf{S}_0|!_{i'}$. We are now ready to make our final adjustment to the arrival orders. For each original arrival order $\widehat{A}_k$ we find the associated (second) label. Suppose this is $x_{i'}$. We then take the current $k$th arrival order (given the previous adjustment), and reorder (only) the agents in $\mathbf{S}_0$, so that the newly constructed arrival order now has (first) label $x_i$. Because of the lexicographic construction of the labels, the arrival position of agent $i$ will not change as a result of this reordering of the arrival positions of agents in $\mathbf{S}_0$, so conditions (i) and (ii) are still satisfied. In addition, condition (iii) will now be satisfied from the definition of $i$ being more central than $i'$. The only remaining thing to verify is that the set of arrival orders we are considering has not changed (i.e. that we have, as claimed, constructed an alternative ordering of the set $\mathcal{A}(N)$) and this also holds by construction. □

*Proof of Proposition 10.* Let $L$ be an efficient network that is robust to underinvestment across group. This implies that for any across-group link $l_{ij} \in L$ between groups $g = G(i)$ and $\hat{g} = G(j) \neq g$, $MV_i(L) - MV_i(L \setminus \{l_{ij}\}) = MV_j(L) - MV_j(L \setminus \{l_{ij}\}) \geq \kappa_a$, where the inequality follows from condition (i) in the robustness to split-the-difference renegotiations definition.

We now rewrite $L$. As the network $L$ is efficient, it is a tree network that minimizes the number of across-group links conditional on a given set of agents being in a component. This implies that the unique path between any two agents from the same group cannot contain an across-group link. We can therefore rewrite the within-group network structures of $L$ to obtain a star by sequentially deleting and then adding within-group links (an algorithm that does this is presented in the proof of part (ii) of Proposition 6). Do this rewiring so that agent $i$ is the agent at the center of the within-group network for group $G(i)$ and let $j$ be the agent at the center of the within-group network for group $G(j)$. Finally, we rewire across-group links so that the same groups remain directly connected, but all across-group links are held by the center agents. Let the network obtained be $L'$. By construction, $L' \in \mathcal{L}^{CCS}$.

Under our definition of Myerson centrality, it is staightforward to verify that both $i$ and $j$ are weakly more Myerson central within their respective groups on network $L'$ than on network $L$. An argument almost identical to that in the proof of Lemma 9 then implies that $i'$ and $j'$ have better incentives to keep the link $l_{i'j'}$ than $i$ and $j$ have to keep the link $l_{ij}$ (because the argument is more or less identical we skip it). Hence,

$$MV_{i'}(L) - MV_{i'}(L \setminus \{l_{i'j'}\}) \geq MV_i(L) - MV_i(L \setminus \{l_{ij}\}) \tag{36}$$

$$MV_{j'}(L) - MV_{j'}(L \setminus \{l_{i'j'}\}) \geq MV_j(L) - MV_j(L \setminus \{l_{ij}\}) \tag{37}$$

Network $L'$ is therefore robust to underinvestment. Moreover, whenever the within-group networks of $i$ and $j$ on network $L$ are not both stars with $i$ and $j$ at the centers, the inequality is strict because both $i$ and $j$ are strictly more Myerson central within-group on $L'$ than on $L$. There then exists a range of parameter specifications for which any center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to underinvestment across group but $L$ is not. □

*Proof of Proposition 31.* We will say that agent $k$ is a distance-$t$ neighbor of $i$ if the shortest path from $i$ to $k$ take exactly $t$ steps (and contain $t+1$ agents, including $i$ and $k$).

Consider the implementation of the Incoming Link Deletion algorithm to find $\widehat{q}_{ij}$. We begin by calculating $x^{1,i} = \mathbf{A}e^i$, where $e^i$ is the $i$th basis vector. This identifies all agents connected to $i$. We then set all entries in the $i$th row from the adjacency matrix $\mathbf{A}$ to 0 and call this new matrix $\mathbf{A}_2$. This deletes the inward links to$i$ in the network $L$. Starting from $i$'s neighbors, we then find their neighbors on $\mathbf{A}_2$. In other words we calculate $x^{2,i} = \mathbf{A}_2 x^{1,i}$. This identifies the distance-2 neighbors of $i$. We then delete the rows of $\mathbf{A}_2$ that are indexed by one of $i$'s neighbors, and so on.

In the $t$th round the algorithm identifies the distance-$t$ neighbors of $i$. Thus, for $t < l$, $x_j^{l,i} = 0$; for $t = l$, $x_j^{l,i} = 1$; and for all $t > l$, $x_j^{t,i} = 0$. Deleting incoming links ensures for all $t > l+1$, $x_j^{t,i} = 0$. As $L$ is a tree there, there is no path of length $l+1$ to $j$ and so $x_j^{l+1,i} = 0$. The algorithm therefore finds the unique path from any $i$ to any $j$ and records its length; If the unique path from $i$ to $j$ has length $l$, $\widehat{q}_{ij} = 1/l$. From equation 11 it is also easily checked that $q_{ij} = 1/l$. Thus $\widehat{\mathbf{q}}(L) = \mathbf{q}(L)$. □

*Proof of Proposition 30.* By definition, if $i$ is more Myerson central than $j$, then there exists a pairing or arrival orders, such that for each arrival order in which $j$ is path-connected to $k$ agents, $i$ is path-connected to weakly more than $k$ agents. Thus, if $i$ is more Myerson central than $j$, then the expected number of agents that $i$ is connected to upon her arrival is greater than the expected number of agents that $j$ is connected to upon her arrival. The expected number of agents that $i$ is connected to upon her arrival is $\sum_k q_{ik}$. We therefore have that $\sum_k \widehat{q}_{ik} > \sum_k \widehat{q}_{jk}$ as claimed. □

## Appendix B. Supplementary Empirical Analysis

B.1. **Survey questions and definition of networks.** As part of the social and informal risk-sharing network mapping, we asked about three types of ties: inside their village leisure contact, inside their village borrowing contact and outside their village contact.

Inside leisure contacts were elicited by asking respondents to "think of the people, within their gramum with whom *they* spend the most leisure time (non-household members above the age of 18). These are people with whom *they* may have spent time for your relaxation, during breaks at work, discussing in person or on the phone, at festivals, drinking tea, or whenever *they* have made time for yourself (free time)".

Additional borrowing contacts within the village were elicited by asking respondents to list "additional people (outside of their household) who *they* could borrow from in case of emergency, other than those people *they* have already listed".

Finally, contacts living outside the village were elicited by asking respondents to "list people outside the gramum from whom *they* could borrow in the case of an emergency".

For each contact, we asked the respondent to specify:

(1) The number of days in the last 7 when you have met or spent time with this person face-to-face or spoken to this person on the phone.
(2) The total amount of money actually borrowed from this person in the last 12 months.
(3) The maximum amount of money that this person would have been willing to lend you over the last 12 months.
(4) The amount of money the respondent could borrow from this person if she was going to start a business or expand an existing business, over the past 12 months.

For outside contacts we also ask where the contact lives.
Based on this set of question, we define two types of inside ties:

- Leisure contact are any household listed as an *inside leisure* contact.
- Borrowing contact are any household listed as an *inside leisure* contact or as an *inside borrowing* contact such as one of the answer to questions 2, 3 or 4 is strictly

positive, i.e. it is a contact the respondent can either borrow money from, has already borrowed from or could borrow from in case she wants to start a business.

Finally, from these two types of links we draw the final graphs used in the analysis:

- *Bidirected financial network:* there is a link between household A and B iif A declares B and B declares A as a borrowing contact.
- *Bidirected social network:* there is a link between household A and B iif A declares B and B declares A as a leisure contact.
- *Bidirected all network:* there is a link between household A and B iif A declares B as either a leisure or a borrowing contact and B declares A as either a leisure or a borrowing contact.

B.2. **Additional Tables.**

TABLE 2. Balance statistics: Village characteristics at Endline - Villages with more than 40 households

| | **Observations** $N_C + N_T$ [1] | **Control Mean [SD]** [2] | **Treatment Mean diff. (SE)** [3] |
|---|---|---|---|
| **Village characteristics** | | | |
| Number of households (Census) | N= 185 | 119.52 | -8.771 |
| | 93C + 92T | [ 55.54] | ( 6.688) |
| Number of heads and spouses (Census) | N= 185 | 214.77 | -16.793 |
| | 93C + 92T | [ 99.04] | ( 11.788) |
| Number of surveyed households (SNM) | N= 185 | 110.71 | -9.974 |
| | 93C + 92T | [ 53.47] | ( 6.479) |
| Number of surveyed heads and spouses (SNM) | N= 185 | 175.51 | -18.774* |
| | 93C + 92T | [ 87.80] | ( 9.829) |
| Pct. of surveyed households | N= 185 | 0.92 | -0.006 |
| | 93C + 92T | [ 0.09] | ( 0.010) |
| Pct. of surveyed heads and spouses | N= 185 | 0.81 | -0.011 |
| | 93C + 92T | [ 0.10] | ( 0.009) |
| Population estimate (Indian Census, 2001) | N= 185 | 488.83 | -34.688 |
| | 93C + 92T | [ 213.33] | ( 25.550) |
| Distance to the bank branch, *kms* | N= 185 | 2.31 | -0.051 |
| | 93C + 92T | [ 1.32] | ( 0.114) |

**Note :** The sample is restricted to villages with outside contact information. . ***, **, and * indicate significance at the 1%, 5%, and 10% levels respectively. Column (1) reports the total number of observation and its decomposition by groups. Column (2) reports the average outcome (standard deviation) for the control group. Column (3) reports the regression coefficent associated to the treatment dummy when controlling for pair fixed effects and with error terms cluster at the Service Area level.

Table 3. Descriptive Statistics - Undirected networks

| | N<br>[1] | Mean (SD)<br>[2] | Min-Max<br>[3] |
|---|---|---|---|
| **Average Myerson Centrality - Incoming link deletion algorithm (Method 1)** | | | |
| Leisure network | 185 | 169.49 ( 142.91) | 0.77 - 569.05 |
| Financial network | 185 | 188.02 ( 167.11) | 3.67 - 768.19 |
| All network | 185 | 227.90 ( 184.56) | 6.37 - 805.90 |
| **Std Dev Myerson Centrality -Incoming link deletion algorithm (Method 1)** | | | |
| Leisure network | 185 | 35.71 ( 27.11) | 0.84 - 212.86 |
| Financial network | 185 | 42.82 ( 27.98) | 2.92 - 149.75 |
| All network | 185 | 44.35 ( 26.67) | 3.98 - 140.46 |
| **Average Myerson Centrality - Outgoing link detection algorithm (Method 2)** | | | |
| Leisure network | 185 | 236.11 ( 207.57) | 0.64 - 801.46 |
| Financial network | 185 | 258.25 ( 243.28) | 2.27 - 1189.14 |
| All network | 185 | 315.79 ( 269.74) | 4.77 - 1237.11 |
| **Std Dev Myerson Centrality - Outgoing link detection algorithm (Method 2)** | | | |
| Leisure network | 185 | 65.58 ( 63.47) | 1.01 - 546.02 |
| Financial network | 185 | 71.86 ( 59.44) | 1.72 - 329.28 |
| All network | 185 | 77.95 ( 59.94) | 3.09 - 321.49 |
| **Average Myerson Centrality - Link detection algorithm (Method 3)** | | | |
| Leisure network, bidirected | 185 | 56.49 ( 39.36) | 13.67 - 215.75 |
| Financial network | 185 | 53.78 ( 31.93) | 13.58 - 181.04 |
| All network | 185 | 50.22 ( 29.33) | 13.54 - 179.86 |
| **Std Dev Myerson Centrality - Link detection algorithm (Method 3)** | | | |
| Leisure network | 185 | 9.07 ( 11.10) | 0.50 - 58.53 |
| Financial network | 185 | 12.97 ( 12.45) | 0.51 - 57.42 |
| All network | 185 | 9.71 ( 12.51) | 0.55 - 55.90 |
| **Average Node Degree** | | | |
| Leisure network | 185 | 4.14 ( 1.65) | 0.75 - 7.11 |
| Financial network | 185 | 4.31 ( 1.64) | 1.38 - 7.14 |
| All network | 185 | 4.84 ( 1.68) | 1.58 - 7.61 |
| **Std Dev Node Degree** | | | |
| Leisure network | 185 | 2.31 ( 0.60) | 1.02 - 3.88 |
| Financial network | 185 | 2.55 ( 0.67) | 1.18 - 4.33 |
| All network | 185 | 2.65 ( 0.68) | 0.71 - 4.41 |

**Note :** The sample is restricted to villages with outside contact information and control variables.

Table 4. First stage effect at the Household level - Inside contact characteristics

| | **Observations** $N_C + N_T$ [1] | **Control Mean [SD]** [2] | **Treatment Mean diff. (SE)** [3] |
|---|---|---|---|
| **Household declaring inside contact** | | | |
| Household having least one inside contact | N= 18642 | 0.86 | -0.006 |
| | 9816C + 8826T | [ 0.35] | ( 0.011) |
| Number of inside contacts | N= 18642 | 2.60 | -0.129** |
| | 9816C + 8826T | [ 1.93] | ( 0.058) |
| **Borrowing capacity** | | | |
| Total emergency borrowing capacity - *Rs 1,000* | N= 18642 | 20.51 | -2.078* |
| | 9816C + 8826T | [ 41.72] | ( 1.185) |
| Total business borrowing capacity - *Rs 1,000* | N= 18642 | 24.08 | -2.569* |
| | 9816C + 8826T | [ 50.53] | ( 1.387) |
| Maximum total borrowing capacity - *Rs 1,000* | N= 18642 | 25.90 | -2.457* |
| | 9816C + 8826T | [ 52.15] | ( 1.421) |
| Total actual borrowed amount - *Rs 1,000* | N= 18642 | 6.96 | -0.799** |
| | 9816C + 8826T | [ 16.78] | ( 0.379) |

**Note :** The sample is restricted to villages with outside contact information and at least 40 households. Borrowing capacity amounts have been top-coded using the $99^{th}$ percentile.. ***, **, and * indicate significance at the 1%, 5%, and 10% levels respectively. Column (1) reports the total number of observation and its decomposition by groups. Column (2) reports the average outcome (standard deviation) for the control group. Column (3) reports the regression coefficent associated to the treatment dummy when controlling for pair fixed effects and with error terms cluster at the Service Area level.

TABLE 5. First stage effect at the Household level - Outside contact characteristics

| | **Observations** $N_C + N_T$ [1] | **Control Mean [SD]** [2] | **Treatment Mean diff. (SE)** [3] |
|---|---|---|---|
| **Household declaring outside contact** | | | |
| Household having least one outside contact | N= 18642 | 0.52 | -0.008 |
| | 9816C + 8826T | [ 0.50] | ( 0.012) |
| Number of outside contacts | N= 18642 | 0.90 | -0.023 |
| | 9816C + 8826T | [ 1.15] | ( 0.029) |
| **Borrowing capacity** | | | |
| Total emergency borrowing capacity - *Rs 1,000* | N= 18642 | 31.44 | -2.108 |
| | 9816C + 8826T | [ 73.34] | ( 1.449) |
| Total business borrowing capacity - *Rs 1,000* | N= 18642 | 35.14 | -2.753* |
| | 9816C + 8826T | [ 83.47] | ( 1.629) |
| Total actual borrowed amount - *Rs 1,000* | N= 18642 | 12.30 | -1.154** |
| | 9816C + 8826T | [ 32.38] | ( 0.537) |
| Maximum total borrowing capacity - *Rs 1,000* | N= 18642 | 37.22 | -2.096 |
| | 9816C + 8826T | [ 86.92] | ( 1.709) |

**Note :** The sample is restricted to villages with outside contact information and at least 40 households. Borrowing capacity amounts have been top-coded using the $99^{th}$ percentile.. ***, **, and * indicate significance at the 1%, 5%, and 10% levels respectively. Column (1) reports the total number of observation and its decomposition by groups. Column (2) reports the average outcome (standard deviation) for the control group. Column (3) reports the regression coefficent associated to the treatment dummy when controlling for pair fixed effects and with error terms cluster at the Service Area level.

Table 6. First stage effect at the contact level - Outside contact characteristics

| | **Observations** $N_C + N_T$ [1] | **Control Mean [SD]** [2] | **Treatment Mean diff. (SE)** [3] |
|---|---|---|---|
| **Demographics** | | | |
| Respondent's Age | N= 18103 | 41.59 | -0.245 |
| | 9555C + 8548T | [ 12.15] | ( 0.239) |
| Male Respondent | N= 18103 | 0.44 | -0.003 |
| | 9555C + 8548T | [ 0.50] | ( 0.008) |
| Contact's Age | N= 18101 | 43.27 | -0.582*** |
| | 9554C + 8547T | [ 12.37] | ( 0.207) |
| Male Contact | N= 18103 | 0.67 | -0.006 |
| | 9555C + 8548T | [ 0.47] | ( 0.007) |
| **Type of contact** | | | |
| Type of contact: Family and other relatives | N= 18103 | 0.70 | -0.016 |
| | 9555C + 8548T | [ 0.46] | ( 0.011) |
| Type of contact: Employer | N= 18103 | 0.04 | 0.004 |
| | 9555C + 8548T | [ 0.19] | ( 0.005) |
| Nr. of days over the last 7 spent with the contact | N= 18103 | 2.86 | 0.126*** |
| | 9555C + 8548T | [ 2.50] | ( 0.039) |
| **Location** | | | |
| In the respondent's panchayat but not their gramum | N= 18103 | 0.02 | 0.005* |
| | 9555C + 8548T | [ 0.12] | ( 0.003) |
| In the respondent's district but not their panchayat | N= 18103 | 0.66 | 0.034** |
| | 9555C + 8548T | [ 0.47] | ( 0.014) |
| In Tamil Nadu but not the respondent's district | N= 18103 | 0.30 | -0.038*** |
| | 9555C + 8548T | [ 0.46] | ( 0.013) |
| In India but not in Tamil Nadu | N= 18103 | 0.01 | 0.000 |
| | 9555C + 8548T | [ 0.09] | ( 0.001) |
| Outside of India | N= 18103 | 0.01 | -0.002 |
| | 9555C + 8548T | [ 0.12] | ( 0.002) |

**Note :** The sample is restricted to villages with outside contact information.. ***, **, and * indicate significance at the 1%, 5%, and 10% levels respectively. Column (1) reports the total number of observation and its decomposition by groups. Column (2) reports the average outcome (standard deviation) for the control group. Column (3) reports the regression coefficent associated to the treatment dummy when controlling for pair fixed effects and with error terms cluster at the Service Area level.

B.3. **Household Income and Myerson Centrality.** The household income has been collected as part of the auxiliary survey in a subset of villages. The exact question was:

*How much rupees, in total, did household members earn in the last 30 days from all income-generating activities including household business, farming, income from other sources of labour, transfers and government schemes? Include in-kind earnings, but first convert to cash and then add to the total.*

TABLE 7

| | **N** [1] | **Mean (SD)** [2] | **Min-Max** [3] |
|---|---|---|---|
| **Household Income over the last 30 days** | | | |
| Monthly Income, *Rs* | 8735 | 7603.17 (11293.11) | 0.00 - 4.0e+05 |

**Note :** The sample is restricted to villages with outside contact information and control variables.

TABLE 8. Pearson Correlation Coefficient - Undirected networks, excluding Money Lenders

| | **Income over the last 30 days** [1] |
|---|---|
| **Myerson Centrality - Incoming link deletion algorithm (Method 1)** | |
| Leisure network | 0.037 *** |
| Financial network | 0.034 *** |
| Financial & leisure network | 0.027 *** |
| **Myerson Centrality - Outgoing link detection algorithm (Method 2)** | |
| Leisure network | 0.032 *** |
| Financial network | 0.030 *** |
| Financial & leisure network | 0.023 *** |
| **Myerson Centrality - Link detection algorithm (Method 3)** | |
| Leisure network | 0.048 *** |
| Financial network | 0.043 *** |
| Financial & leisure network | 0.050 *** |
| **Node Degree** | |
| Leisure network | 0.060 *** |
| Financial network | 0.089 *** |
| Financial & leisure network | 0.081 *** |

**Note :** The sample is restricted to villages with outside contact information and control variables.

## Appendix C. Over and Under Investment Examples

In this Appendix we provide an example of over-investment within group in the unique stable network and a related example of underinvestment across group in the unique stable network.

We begin by assuming there is one group with $s$ members connected by a network $L$. Equation 11 implies that Myerson distance of two agents $i, j$ such that $l_{ij} \notin L$ is greater than $1/2$, while the Myerson distanace between $i$ and $j$ if they form the link $l_{ij}$ would be $1/2$. Thus $i$ and $j$'s gross payoff strictly increases if the link $l_{ij}$. So, for $\kappa_w$ sufficiently close to 0, in all stable networks for any pair of agents $i, j$ the link $l_{ij}$ must be formed; The unique stable network is the complete network and there is overinvestment.

Suppose now there two groups, $g, g'$ both with $s$ members and keep the same parameter values from the previous example. By equation 13 the incentives for form within group links are weakly increased by any across group links. Thus in all stable networks the network structures within-group must be complete networks; All possible within-group links must be formed. Suppose these are the only links formed so that no across-group links are formed. Denote this network $L$. From equation 15 the change in total variance achieved by connecting an agent $i$ from group $g$ to an agent $j$ from group $g'$ is strictly increasing in the size of both groups $s$. Considering the Myerson value calculation (equation 6), this means that the marginal contribution of the link $l_{ij}$ to total surplus (the certainty equivalent value of the variance reduction) is strictly greater on $L \cup \{l_{ij}\}$ than it is on any strict subgraph, including all those formed when the later of $i$ and $j$ arrives in the Myerson calculation. This implies that $(MV(i; L \cup l_{ij}) - MV(i; L)) + (MV(j; L \cup l_{ij}) - MV(j; L)) < TS(L \cup l_{ij}) - TS(L)$ for all $l_{ij} : i \in \mathbf{S}_g, j \in \mathbf{S}_{g'}$. So, setting $\kappa_a$ such that

$$MV(i; L \cup l_{ij}) - MV(i; L) + MV(j; L \cup l_{ij}) - MV(j; L) < 2\kappa_a < TS(L \cup l_{ij}) - TS(L),$$

the network $L$ is the unique stable network and there is underinvestment (in across-group links) in all stable networks.