

The Strategy and Technology of Conflict*

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Abstract

Using a simple conflict bargaining game, we study how the strategic interaction is shaped by underlying preferences over a contested territory and by the technology of conflict. Our model predicts a non-monotonic relationship between the cost of conflict and the probability of war. We find conditions under which a conflict becomes more likely when a militarily weaker player catches up with a stronger one (the “Thucydides trap”). With risk-averse players, the game has strategic complements if the cost of conflict is small and there is a large first-mover advantage, and strategic substitutes otherwise. This characterization generates predictions about the strategic advantages of *ex ante* tactics. As a whole, our paper provides a theoretical foundation for important ideas of Thomas Schelling, as well as for more recent formal models inspired by his work.

1 Introduction

Russia’s annexation of Crimea, and China’s island-building in the South China Sea, are recent examples of “strategic moves” (Schelling [26]): the other side is presented with the stark choice of either conceding and losing

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the territory for sure, or resisting and risking a major confrontation. Such strategic moves have a rich history. After World War II, the Soviet Union gained the first-mover advantage in Eastern Europe by occupying it in violation of the Yalta agreement.¹ If the West had not conceded, for example in Czechoslovakia or Hungary, a military confrontation would have been quite likely, because the Soviets could not have retreated from these countries without a massive loss of reputation. Conversely, US soldiers stationed in Western Europe represented “the pride, the honor, and the reputation of the United States government and its armed forces” (Schelling [26], p. 47). They had no graceful way to retreat, so the Soviet Union had every reason to believe that the United States would resist a Russian attack (Schelling [26], p. 47). An East-West confrontation was avoided because the Soviets conceded Western Europe just as the West had conceded Eastern Europe.

We study a simple two-player bargaining game that captures the idea of a strategic move as a “voluntary but irreversible sacrifice of freedom of choice” (Schelling [25]). Each player may challenge the status quo division of a contested territory. A costly conflict occurs if both players challenge (since this implies mutually incompatible commitments), or if only one player challenges but the other refuses to concede. In the latter case, the challenger was the one who took the initiative, for example by placing his soldiers on the contested territory, which may give him a first-mover advantage. As the optimal challenge is the largest demand the opponent would concede to, the game can be represented by a two-by-two matrix game, with actions Hawk (the optimal challenge) and Dove (no challenge). A player who chooses Hawk incurs a cost, which is his private information.

We use this framework to make three main contributions. First, by “microfounding” conflict on primitives, such as the cost of conflict and the magnitude of first-mover advantage, we are able to study how the probability of conflict depends on these primitives. Second, we are able to characterize environments where conflict is caused by escalation (strategic complements) rather than by a failure of deterrence (strategic substitutes). This characterization is important for both positive and normative analysis, as these features determine the strategic nature of conflict and the effects of policy interventions. Third, we connect the classic but informal work of Schelling

¹At the Yalta conference in February 1945, it was agreed that the Soviet Union would recover the territory it had lost after 1941. Elsewhere there were supposed to be free elections and democratic governments.

([25], [26]) and Jervis ([16]) with modern bargaining models. We hope this can advance the theory of conflict in the same way that game theory has advanced Industrial Organization theory.

If conflicts are not too costly, then an increase in the cost of conflict can encourage a player to challenge the status quo, since the opponent will be willing to make larger concessions to avoid a conflict. However, when conflicts become extremely costly, challenges become too risky and will be avoided. Therefore, our model predicts that the relationship between the cost of conflict and the probability of conflict is non-monotonic. This may shed some light on the “stability-instability” paradox discussed by Hart [18], and on why many provocations and challenges have occurred despite a potentially high cost of conflict. For example, Khrushchev assisted the Cuban revolution in 1960, in defiance of the “Truman doctrine”. Apparently he was convinced that the U.S. would not risk a major war.² Similarly, Pakistan has employed terrorist groups to attack India under the safety of a nuclear umbrella, and North Korea has attacked South Korean assets after conducting nuclear tests.

Changes in relative military strength also impact the probability of conflict. For example, China’s military and economic power soon may rival the United States. Analysts such as Allison [2] have warned that this may trigger conflict: the “Thucydides trap”.³ Even China is concerned about this possibility.⁴ In Section 5, we assume the players have identical preferences but unequal military capabilities. We first consider how the status quo must be divided to offset differences in military strength and create a “balanced” situation. Next, we study what happens if the weaker side (the rising power), who controls a smaller share of the contested territory under the status quo, becomes militarily stronger. If utility functions are strictly concave, and if the rising power’s increased military strength is not reflected by an adjust-

²During the Berlin crisis, Khrushchev told an American visitor that Berlin was not worth a war to the US. Khrushchev was then asked whether it was worth a war to the Soviet Union. “No”, he replied, “but you are the ones that have to cross a frontier” (Schelling [26], p. 46) – implying that it would be the West’s decision to risk a war by entering East Germany which the Soviets already occupied.

³Thucydides described how a rising power can create a fear spiral: “It was the rise of Athens and the fear that this inspired in Sparta that made war inevitable” (Thucydides [27]).

⁴President Xi Jinping recently commented: “We all need to work together to avoid the Thucydides trap - destructive tensions between an emerging power and established powers . . . Our aim is to foster a new model of major country relations ([29]).”

ment in the status quo, then the probability of conflict will increase. Strict concavity is crucial here: it means that the rising power (who controls less territory) would gain a lot from increasing his share, and therefore becomes very likely to challenge the status quo when his military strength increases.

In applied game theory, a key distinction is made between strategic complements and strategic substitutes. This distinction can also be applied to conflicts, for example, to distinguish World Wars I and II.⁵ When actions are strategic complements, conflicts are caused by a lack of trust, as in Schelling's [25] analysis of the reciprocal fear of surprise attack. With strategic substitutes, conflicts are instead caused by a failure of deterrence. The analysis and policy must be adjusted, depending on which scenario is at play.⁶ Jervis ([17] p. 96) asks, "[W]hat are the conditions under which one model rather than the other is appropriate?"⁷

Our conflict bargaining game has strategic substitutes if the cost of conflict is high. This is consistent with the notion that the Cold War was a sequence of chicken races (c.f. footnote 2). If the cost of conflict is low, the problem is more subtle. It turns out that the game has strategic complements if utility functions are strictly concave and there is a significant first-mover advantage. The result is not obvious, because there are two opposing effects: when the first-mover advantage increases, the cost of choosing Dove when the opponent chooses Hawk increases, but so does the benefit from choosing Hawk when the opponent chooses Dove. The first effect tends to generate strategic complements, while the second effect does the opposite. The first effect dominates when the marginal utility of land is decreasing, so the cost of losing territory exceeds the benefit of acquiring the same amount. Therefore, in the expected-utility calculations, losses suffered by Doves who encounter Hawks outweigh gains enjoyed by Hawks who encounter Doves, so the game

⁵Nye ([23], p. 111) argues, "World War I was an unwanted spiral of hostility... World War II was not an unwanted spiral of hostility-it was a failure to deter Hitler's planned aggression,"

⁶For example, in previous work (Baliga and Sjöström [5]), we found that with strategic complements, hawkish third parties ("extremists") can trigger conflicts by sending messages that create "fear spirals". With strategic substitutes, hawkish extremists cannot do this – instead dovish third parties ("pacifists") can prevent conflicts by persuading one side to back down. But the analysis left open the question of what factors determine whether actions are strategic substitutes or complements.

⁷Jervis [17] continues: "When will force work and when will it create a spiral of hostility? When will concessions lead to reciprocations and when will they lead the other side to expect further retreats?"

has strategic complements. This analysis hints at how factors such as the shape of the utility function, the cost of conflict, and first-mover advantages jointly determine whether actions are strategic substitutes or complements. These factors in turn depend on the weapons being used to challenge and defend, and on the properties of the contested resource or territory.

Mapping the technology of war onto the parameters of our model can help provide some intuition about which scenario was at play in a specific situation. For example, McNeil [19] discusses how first-mover advantages changed over time. During the era when forts could withstand a siege for many years, the offensive advantage was not large. Then, mobile and powerful siege cannon, developed in France in the late 1400s, gave the advantage to the attacker. But this was later neutralized by the *trace italienne*, a fort design that combined wide ditches reinforced with soft earth and a star-shaped structure that facilitated counter-attack (Duffy [12]).

By the end of the nineteenth century, Napoleon’s use of trained mass armies and Prussia’s rapid, well-planned attacks with breech-loading, long range guns led to short wars and a significant first-mover advantage. There was “offense dominance” (Bueno de Mesquita [7]). By World War I, trench warfare instead made defense dominant, and wars were long and costly. But Germany and Great Britain may have miscalculated, believing offense was still dominant (Jervis [16]). Their escalating conflict shows how policies can backfire if based on an incorrect identification of the underlying strategic nature of the game. Nuclear warfare with its destructiveness and second-strike capability seems closer to trench warfare than to the Napoleonic mass army.⁸

In Section 7 we discuss the incentives to make strategic moves *ex ante* (before the bargaining), as suggested by Schelling’s [25] work on commitment tactics. Some results have an intuitive interpretation that is reminiscent of Fudenberg and Tirole [15]. For example, if the game has strategic substitutes, then it is advantageous to reduce one’s cost of conflict in order to “look tough” and make the opponent back down. This could be achieved, for example, by delegating decision-making to an agent who is commonly known

⁸For the analogy between nuclear war and World War I and the lack of foresight of policymakers before the war, see Clark [10]: “In the 1950s and 1960s, decision-makers and the general public alike grasped in a visceral way the meaning of nuclear war - images of mushroom clouds over Hiroshima and Nagasaki entered the nightmares of ordinary citizens....[T]he protagonists of 1914 were sleepwalkers, watchful but unseeing, haunted by dreams, yet blind of the reality of the horror they were about to bring to the world.”

to have a low cost of conflict. However, with strategic complements, the reverse strategy of raising one’s cost of conflict in order to “look soft” is not necessarily optimal. The reason is that a player who raises his own cost of conflict becomes more willing to concede territory, and this tends to make the opponent more hawkish (as he has more to gain from a challenge). This direct external effect on another player’s payoff function, which was not present in Fudenberg and Tirole [15], means that while a low cost of conflict is surely a strategic advantage under strategic substitutes, it may also be an advantage under strategic complements.

A more subtle *ex ante* move is for a player to raise the value he puts on his own territory, for example, by building settlements on it. Intuition may suggest that this is just another way to “look tough”, but in our model this intuition is misleading. Since a conflict may lead to the loss of *all* the (divisible) territory, the settlements make the player *more* willing to give up *some part* of it, to avoid a conflict. This encourages the opponent to challenge the status quo. Therefore, building settlements is not necessarily optimal even if the game has strategic substitutes.

2 Related Literature

The game-theoretic literature on commitment in bargaining traces its origins to Nash’s [22] demand game and Schelling’s [25] seminal discussion. Fearon [13] studied a model of *one-sided* commitment. This did not address the coordination problem, emphasized by Schelling, that arises when *both* sides may try to commit.⁹

Most closely related to our paper is Ellingsen and Miettinen [14], who in turn build on Crawford [11]. In Crawford’s model both sides can commit,

⁹Schelling (p. 26, [25]) considered haggling over the price of a house: “If each party knows the other’s true reservation price, the object is to be first with a firm offer. Complete responsibility rests with the other, who can take it or leave it as he chooses (and who chooses to take it). Bargaining is all over; the commitment (that is, the first offer) wins.”

He went on to describe the coordination problem: “Interpose some communication difficulty. They must bargain by letter; the invocation becomes effective when signed but cannot be known to the other until its arrival. Now when one person writes such a letter the other may already have signed his own or may yet do so before the letter of the first arrives. There is then no sale; both are bound to incompatible positions.”

Intuition might suggest that such a game would have strategic substitutes, but as we will show, this is not necessarily the case.

which leads to multiple equilibria. Ellingsen and Mietinen showed that if commitments are costly then the number of equilibria is reduced. Specifically, they showed that if the cost of making a commitment is small enough, and a commitment attempt only succeeds with probability $q < 1$, then there is a unique equilibrium where both parties attempt to make the maximum commitment (demand all the territory) with probability 1. When $q = 1$, their model also has asymmetric equilibria where one player gets all the territory with probability one. In our model there is no such discontinuity – uniqueness is a consequence of private information, it is not dependent on an exogenous probability that commitments fail. Also, unlike them we do not assume an uncommitted player would always concede to any demand. Some parameter values do generate “corner solutions”, where demanding the whole territory is an optimal challenge, and the opponent concedes to this.¹⁰ In reality it sometimes happens that countries do take such extreme positions – China claims essentially *all* of the disputed South China Sea. However, a “reasonable compromise” may be proposed if a party fears that a more extreme proposal will be rejected.

In previous work (see Baliga and Sjöström [3]) we focussed on coordination with incomplete information. Chassang and Padro i Miquel [9] show that, under strategic uncertainty induced by incomplete information, increasing weapons stocks can increase the probability of conflict by increasing preemption rather than deterrence. Bueno de Mesquita [8] studies a coordination model of revolution where a vanguard group might try to mobilize citizens. Acharya and Ramsay [1] show that communication might not defuse tensions in a global game model of conflict. All these papers take the pay-off structure as exogenous, and assume actions are strategic complements. Ramsay [24] provides a recent survey. Finally, Meirowitz, Morelli, Ramsay and Squintani [20] adopt a rather different approach, where a private arming decision is followed by communication and bargaining.

3 The Bargaining Game

There are two players, A and B . In the status quo, player i controls a share $\omega_i \in (0, 1)$ of a disputed territory, his *endowment*, where $\omega_A + \omega_B = 1$. Player i 's utility of controlling a share x_i is $u_i(x_i)$, where u_i is an increasing, strictly

¹⁰This can be contrasted with standard bargaining models, such as Binmore, Rubinstein and Wolinsky [6], where the parties tend to always propose a “reasonable compromise”.

concave and differentiable function on $[0, 1]$. Without loss of generality we normalize so that $u_i(1) = 1$ and $u_i(0) = 0$. If a conflict occurs, then each player $i \in \{A, B\}$ suffers a cost $\phi_i > 0$.

The bargaining game has two stages. In stage 1, each player i can either make a claim x_i , where $\omega_i < x_i \leq 1$, or make no claim. A claim is a *challenge* (to the status quo) which incurs a cost c_i for the challenger. To make no claim incurs no cost. We interpret player i 's challenge as a non-revokable instruction to player i 's military to cross the status quo demarcation. Let λ_i be a parameter that represents player i 's relative military strength, where $\lambda_A + \lambda_B = 1$. Let θ be a parameter that measures the first-mover advantage (to be explained below).

The game ends after stage 1 if either no player makes a claim, or both make claims. Stage 2 is reached if only one player makes a claim, in which case the other player (having observed the claim) chooses to concede or not to concede. The final outcome is determined by three rules.

Rule 1. If nobody challenges in stage 1, then the status quo remains in place.

Rule 2. If only player i challenges, and claims $x_i > \omega_i$ in stage 1, then we move to stage 2. In stage 2, if player $j \neq i$ concedes to player i 's claim then player i gets x_i and player j gets $1 - x_i$. If player j does not concede, there is a conflict: with probability $\lambda_i + \theta$, player i (the challenger) wins and takes all of the territory; with probability $\lambda_j - \theta = 1 - (\lambda_i + \theta)$ player j wins and takes all of the territory.

Rule 3. If both players challenge the status quo in stage 1 then there is a conflict. Player i wins, and takes all of the territory, with probability λ_i .

We interpret these rules as follows. If neither player challenges the status quo, then there is no reason why either player should retreat from his initial position, and the status quo remains in place. If only player i challenges in stage 1 then he becomes the *first-mover* and player j the *second-mover*. The challenge is a strategic move in the sense of Schelling [25]: a commitment to start a conflict unless player j concedes to player i 's claim. If there is a concession then player i gets what he claimed, and thus increases his share of the territory. If player j does not concede, there is a conflict which player i wins with probability $\lambda_i + \theta$; there is no way to “gracefully back down” and avoid a conflict at this point. Finally, if *both* players challenge the status quo, a conflict occurs because they have made mutually incompatible

commitments. This conflict is won by player i with probability λ_i .¹¹

The parameter θ indicates the first-mover advantage: it is the increased chance of player i winning when he is a first-mover, compared to the “baseline” of λ_i . Since $\lambda_i + \theta$ is a probability, we assume $0 < \lambda_i + \theta < 1$ for $i \in \{A, B\}$. In particular, this implies $\theta < 1/2$. Note that if $\theta = 0$ then there is no first-mover advantage, and the probability that player i wins a conflict under Rule 2 is λ_i just as under Rule 3.

Suppose stage 2 is reached. If player i is the second-mover and concedes to the claim x_j he gets $u_i(1 - x_j)$. If he doesn't concede, he gets expected payoff

$$(\lambda_j + \theta)u_i(0) + (1 - (\lambda_j + \theta))u_i(1) - \phi_i = \lambda_i - \theta - \phi_i, \quad (1)$$

since $\lambda_i = 1 - \lambda_j$, $u_i(1) = 1$ and $u_i(0) = 0$. Thus, player i prefers to concede if

$$u_i(1 - x_j) \geq \lambda_i - \theta - \phi_i. \quad (2)$$

This is satisfied for $x_j = 1$ if

$$\phi_i \geq \lambda_i - \theta. \quad (3)$$

If (3) holds then player i would rather concede the whole territory than have a conflict. If (3) is violated, i.e., if

$$\phi_i < \lambda_i - \theta, \quad (4)$$

then the maximum claim x_j player i will concede to satisfies (2) with equality, or

$$1 - x_j = u_i^{-1} [\lambda_i - \theta - \phi_i]. \quad (5)$$

Thus, in general, the maximum claim player i would concede to in stage 2 is the claim $x_j = 1 - \eta_i$, where

$$\eta_i \equiv \begin{cases} u_i^{-1} [\lambda_i - \theta - \phi_i] & \text{if } \phi_i < \lambda_i - \theta, \\ 0 & \text{if } \phi_i \geq \lambda_i - \theta. \end{cases} \quad (6)$$

¹¹A more general formulation would be that if both decide to challenge, there is some probability $\alpha > 0$ that player $i \in \{A, B\}$ can commit first, in which case player j must decide whether or not to concede. Thus, each player would have a probability α of getting the first mover advantage. With probability $1 - 2\alpha$, they both become committed, and there is a conflict. Similarly, following Crawford [11] and Ellingsen and Mietinen [14], we could assume that a challenge only leads to a successful commitment with probability $q < 1$. But adding the generality does not seem to add any additional insights; we therefore focus on the case $\alpha = 0$ and $q = 1$ for the sake of exposition. (Unlike in Ellingsen and Mietinen [14], nothing dramatic happens to the set of equilibria at the point $q = 1$.)

Notice that if (4) holds then η_i is defined implicitly by

$$u_i(\eta_i) = \lambda_i - \theta - \phi_i \quad (7)$$

and satisfies $\eta_i > 0$. Equation (7) says that player i is indifferent between the share η_i and a conflict when he is the second-mover (c.f. equation (1)). Notice that η_i is decreasing in ϕ_i . The more costly a conflict would be, the more territory player i is willing to concede.

To make the problem interesting, we will assume:

Assumption 1 $\eta_i < \omega_i$ for $i \in \{A, B\}$.

Assumption 1 implies that if the first-mover's claim is sufficiently "modest", i.e., close to the status quo, then the second-mover prefers to concede. Assumption 1 rules out the less interesting case where the second-mover would never concede (even to an arbitrarily small change in the status quo). Assumption 1 is equivalent to the inequality

$$u_i(\omega_i) > \lambda_i - \theta - \phi_i. \quad (8)$$

The left-hand side of (8) is player i 's payoff from the status quo, and the right hand side is his expected payoff from Rule 2 when he is the second-mover and does not concede. Assumption 1 can also be re-written as

$$\lambda_j + \theta > 1 - u_i(\omega_i) - \phi_i. \quad (9)$$

This reveals that if the cost of conflict is high enough, specifically if $\phi_i > 1 - u_i(\omega_i)$, then Assumption 1 is automatically satisfied. Note also that strict concavity implies $u_i(\frac{1}{2}) > \frac{1}{2}u_i(1) + \frac{1}{2}u_i(0) = \frac{1}{2}$. Therefore, in the symmetric case where $\omega_i = 1/2$ the right-hand side of (9) is less than $1/2$, so Assumption 1 is satisfied whenever $\lambda_j + \theta \geq 1/2$, i.e., as long as the first-mover has at least an even chance of winning.

We assume all parameters of the game are commonly known, with one exception: neither player knows the opponent's cost of violating the status quo. For each $i \in \{A, B\}$, the cost c_i is independently drawn from a distribution F with support $[\underline{c}, \bar{c}]$ and density $f(c) = F'(c)$. Player $i \in \{A, B\}$ knows

c_i but not c_j . We refer to c_i as player i 's type.¹²

If either the support of F is very small, or the density of F is highly concentrated around one point, then the players are fairly certain about each others' types and private information is unimportant. To rule this out, we assume (i) that the support is not too small, and (ii) that the density is sufficiently "flat". Let Ω_i be defined as follows:

$$\Omega_i \equiv \lambda_i - \phi_i - u_i(\eta_i) - u_i(1 - \eta_j) + u_i(\omega_i). \quad (10)$$

Assumption 2 (Sufficient uncertainty about types) (i)

$$\underline{c} < \min\{u_i(1 - \eta_j) - u_i(\omega_i), \lambda_i - \phi_i - u_i(\eta_i)\}$$

and

$$\bar{c} > \max\{u_i(1 - \eta_j) - u_i(\omega_i), \lambda_i - \phi_i - u_i(\eta_i)\}$$

for $i \in \{A, B\}$. (ii)

$$f(c) < \frac{1}{|\Omega_i|}$$

for all $c \in [\underline{c}, \bar{c}]$ and $i \in \{A, B\}$.

If F is uniform, then (ii) is redundant because (i) implies (ii). Indeed, the uniform distribution is maximally "flat". However, we do not restrict attention to the uniform distribution. In the non-uniform case, (ii) guarantees that the density is not highly concentrated at one point.

4 Equilibrium, First-Mover Advantage and the Costs of War

We will show that there is a unique pure strategy perfect Bayesian equilibrium. Note first that sequential rationality implies that if player i challenges and player j doesn't, then player j concedes if player i 's claim satisfies

¹²The cost c_i may partly be due to physical resources and manpower needed to cross the status quo demarcation. But it could also include the disutility of being condemned by the international community, leading to a loss of reputation and goodwill, possible sanctions or embargoes, etc. It may be difficult for outsiders to assess these costs: how much do Russian leaders suffer from sanctions and international condemnation after Russia's invasion of Ukraine? The Russian economy is suffering but President Putin's popular support is high.

$x_i < 1 - \eta_j$, but not if $x_i > 1 - \eta_j$. If $x_i = 1 - \eta_j$ then player j is indifferent between conceding and not conceding – by a standard argument, in equilibrium he must concede in this case, for otherwise player i would not have a best response.

Let p_j denote the equilibrium probability that player j challenges.

Lemma 1 *Consider a perfect Bayesian equilibrium such that $p_j < 1$. If player i challenges in equilibrium, then he will claim $x_i = 1 - \eta_j$ and player j will concede.*

Proof. The exact size of player i 's claim only matters if player j does not challenge. Therefore, to find player i 's optimal claim we may restrict attention to this event, which by hypothesis happens with probability $1 - p_j > 0$.

Suppose player i claims $x_i > 1 - \eta_j$. Then we know that player j rejects the claim and there is conflict. By Rule 2, this conflict gives player i expected payoff

$$(\lambda_i + \theta)u_i(1) + (1 - (\lambda_i + \theta))u_i(0) - \phi_i = \lambda_i + \theta - \phi_i \quad (11)$$

using our normalizations. If instead player i claims $x_i < 1 - \eta_j$ then we know that player j will concede, and player i gets $u_i(x_i)$. If player i claims just slightly less than $1 - \eta_j$ then his payoff is very close to

$$u_i(1 - \eta_j). \quad (12)$$

But (12) is strictly greater than (11). To see this, note that, by definition of η_j , if player i claims $1 - \eta_j$ then player j 's payoff is $u_j(\eta_j)$ whether there is a conflict or not (see (7)). But conflicts are inefficient (since the players are risk-averse and $\phi_i > 0$), so player i strictly prefers to not have a conflict and get $1 - \eta_j$ for sure. Thus, (11) is strictly smaller than (12), so claiming $x_i < 1 - \eta_j$ sufficiently close to $1 - \eta_j$ will be strictly better than claiming $x_i > 1 - \eta_j$. This means player i 's equilibrium claim cannot be strictly above $1 - \eta_j$.

Since the payoff from $x_i < 1 - \eta_j$ is $u_i(x_i)$, which is strictly increasing in x_i , the equilibrium claim cannot be $x_i < 1 - \eta_j$, since a small increase would be a strict improvement. Hence, if player i makes a challenge in equilibrium, he must claim $x_i = 1 - \eta_j$.

Finally, player j must concede to the claim $x_i = 1 - \eta_j$ with probability one. For if he does not, $x_i = 1 - \eta_j$ would not maximize player i 's expected

payoff: player i would do better by claiming $x_i < 1 - \eta_j$ sufficiently close to $1 - \eta_j$, since player j is sure to concede to this. ■

We now show that the probability that player i challenges must be strictly less than one, because he will not challenge if c_i is too high.

Lemma 2 *In equilibrium, $p_i < 1$.*

Proof. By challenging, player i can get at most

$$-c_i + p_j (\lambda_i - \phi_i) + (1 - p_j) u_i(1 - \eta_j), \quad (13)$$

using the fact that if $p_j < 1$ then his optimal challenge is $1 - \eta_j$. His expected payoff from not challenging is at least

$$p_j u_i(\eta_i) + (1 - p_j) u_i(\omega_i), \quad (14)$$

using the fact that if player j challenges then player i gets at least $u_i(\eta_i)$. Assumption 2 implies that (14) is strictly greater than (13) when $c_i = \bar{c}$. Thus, if c_i is sufficiently close to \bar{c} then player i will not challenge. ■

Combining Lemmas 1 and 2, we find that in perfect Bayesian equilibrium, player i will either not challenge, or choose the *optimal challenge* with $x_i = 1 - \eta_j$. For convenience, we will label the optimal challenge *Hawk* (or H). To not make any challenge is to choose *Dove* (or D). Thus, the multi-stage bargaining game can be reduced to the following 2×2 payoff matrix. Player i chooses a row, player j a column, and only player i 's payoff is indicated.

	Hawk (claim $x_j = 1 - \eta_j$)	Dove (no challenge)
Hawk (claim $x_i = 1 - \eta_j$)	$\lambda_i - \phi_i - c_i$	$u_i(1 - \eta_j) - c_i$
Dove (no challenge)	$u_i(\eta_i)$	$u_i(\omega_i)$

(15)

Lemmas 1 and 2 imply that the set of perfect Bayesian equilibria of the multi-stage game of Section 3 is isomorphic to the set of Bayesian equilibria (henceforth “equilibria”) of the simultaneous-move Hawk-Dove game (where player i is privately informed about c_i). Hence, we turn to an analysis of the Hawk-Dove game. A *strategy* for player i is a function $g_i : [\underline{c}, \bar{c}] \rightarrow \{H, D\}$ which specifies an action $g_i(c_i) \in \{H, D\}$ for each type $c_i \in [\underline{c}, \bar{c}]$. In equilibrium, all types maximize their expected payoff. Therefore, $g_i(c_i) = H$ if (17) is positive, and $g_i(c_i) = D$ if (17) is negative. If (17) is zero then type c_i is indifferent, and for convenience we assume he chooses H in this case.

Lemma 3 *If either $\theta \geq 0$ or $\omega_i \geq 1/2$ (or both) then player i is better off when player j chooses Dove, whatever action player i himself chooses.*

Proof. First note that $\omega_i > \eta_i$ implies $u_i(\omega_i) > u_i(\eta_i)$. Second, we showed above that $u_i(1 - \eta_j) > \lambda_i + \theta - \phi_i$ so if $\theta \geq 0$ then

$$u_i(1 - \eta_j) - c_i > \lambda_i - \phi_i - c_i. \quad (16)$$

If $\omega_i \geq 1/2$ then the inequality (16) follows from $1 - \eta_j > 1 - \omega_j \geq 1/2$ and concavity of u_i . ■

Player i is a *dominant strategy hawk* if Hawk (H) is his dominant strategy.¹³ Player i is a *dominant strategy dove* if Dove (D) is his dominant strategy.¹⁴ Assumption 2(i) implies that the support of F is big enough to include dominant strategy types of both kinds.

Suppose player i thinks player j will choose H with probability p_j . Player i 's expected payoff from playing H is

$$-c_i + p_j(\lambda_i - \phi_i) + (1 - p_j)u_i(1 - \eta_j),$$

while his expected payoff from D is

$$p_j u_i(\eta_i) + (1 - p_j)u_i(\omega_i).$$

Thus, if he chooses H instead of D , his *net gain* is

$$-c_i + p_j(\lambda_i - \phi_i - u_i(\eta_i)) + (1 - p_j)(u_i(1 - \eta_j) - u_i(\omega_i)). \quad (17)$$

Player i uses a *cutoff strategy* if there is a *cutoff point* $x \in [\underline{c}, \bar{c}]$ such that $g_i(c_i) = H$ if and only if $c_i \leq x$. Because the expression in (17) is monotone in c_i , all equilibria must be in cutoff strategies. Therefore, we can without loss of generality restrict attention to cutoff strategies. Any such strategy is identified with its cutoff point $x \in [\underline{c}, \bar{c}]$. If player j uses cutoff point x_j , the probability he plays H is $p_j = F(x_j)$. Therefore, using (17), player i 's best response to player j 's cutoff x_j is the cutoff $x_i = \Gamma_i(x_j)$, where

$$\Gamma_i(x) \equiv F(x)(\lambda_i - \phi_i - u_i(\eta_i)) + (1 - F(x))(u_i(1 - \eta_j) - u_i(\omega_i)). \quad (18)$$

¹³Formally, $c_i \leq u_i(1 - \eta_j) - u_i(\omega_i)$ and $c_i \leq \lambda_i - \phi_i - u_i(\eta_i)$ with at least one strict inequality.

¹⁴Formally, $c_i \geq u_i(1 - \eta_j) - u_i(\omega_i)$ and $c_i \geq \lambda_i - \phi_i - u_i(\eta_i)$ with at least one strict inequality.

The function Γ_i is the best-response function for cutoff strategies.

Assumption 2(i) rules out corner solutions, where all types do the same thing. Indeed, Assumption 2(i) implies that

$$\Gamma_i(\underline{c}) = u_i(1 - \eta_j) - u_i(\omega_i) > \underline{c}$$

and

$$\Gamma_i(\bar{c}) = \lambda_i - \phi_i - u_i(\eta_i) < \bar{c}$$

so the equilibrium cutoff point will lie strictly between \underline{c} and \bar{c} .

Since the function $(\Gamma_A(x_B), \Gamma_B(x_A)) : [\underline{c}, \bar{c}]^2 \rightarrow (\underline{c}, \bar{c})^2$ is continuous, a fixed-point $(\hat{x}_A, \hat{x}_B) \in (\underline{c}, \bar{c})^2$ exists. This is an equilibrium (where player i uses cutoff \hat{x}_i). Thus, an equilibrium exists. The slope of the best response function is $\Gamma'_i(x) = \Omega_i f(x)$, where Ω_i is defined by (10). A standard sufficient condition for the existence of a *unique* equilibrium is that the absolute value of the slope of each player's best response function is less than 1. Assumption 2(ii) guarantees this. Thus, while Assumption 2(i) guarantees that any equilibrium is interior, Assumption 2(ii) guarantees that there is a unique equilibrium.¹⁵

We turn to comparative statics results for the symmetric case: the players are *ex ante symmetric* if they have the same utility function, $u_A = u_B = u$, the same cost of a conflict, $\phi_A = \phi_B = \phi$, the same military strength, $\lambda_A = \lambda_B = 1/2$, and the same initial endowment of territory, $\omega_A = \omega_B = 1/2$. In this case, the unique equilibrium must be symmetric, and the equilibrium cutoff \hat{x} is the same for both players and implicitly defined by the equation

$$\hat{x} - \Omega F(\hat{x}) = u(1 - \eta) - u(1/2) \tag{19}$$

where

$$\Omega \equiv \frac{1}{2} - \phi - u(\eta) - u(1 - \eta) + u(1/2). \tag{20}$$

Consider how η depends on ϕ and θ . If $\phi + \theta > 1/2$ then $\eta = 0$ and $d\eta/d\theta = d\eta/d\phi = 0$. But if $\phi + \theta < 1/2$ then (7) holds, and the second-mover

¹⁵Without Assumption 2(ii), there would be multiple interior equilibria. We could then use the techniques in Milgrom and Roberts [21] to study the comparative statics of the “highest” and “lowest” equilibria. Assumption 2(i) implies all equilibria are interior. For our comparative statics results to have bite, either the highest or the lowest equilibrium must be interior. Hence, at least one of the parameter regions of dominant strategy types must be in the support of the distribution of costs.

concedes more if θ or ϕ increases:

$$\frac{d\eta}{d\theta} = \frac{d\eta}{d\phi} = -\frac{1}{u'(\eta)} < 0. \quad (21)$$

Suppose there is an increase in θ , representing an increased first-mover advantage. By definition, the magnitude of θ matters when one player chooses Hawk and the other Dove. Now if θ increases from an initially low level, the the dove will lose more territory to a hawk, $d\eta/d\theta < 0$, and hence hawk will extract more territory from the dove. This makes it more tempting to choose Hawk. The effect vanishes once θ becomes so high that $\eta = 0$. Thus, we have the following result.¹⁶

Proposition 4 *Suppose the players are ex ante symmetric. An increase in first-mover advantage θ increases the probability of conflict if $\phi + \theta < 1/2$. It has no effect on the probability of conflict if $\phi + \theta > 1/2$.*

Proof. Totally differentiating (19) we obtain

$$(1 - \Omega f(\hat{x})) \frac{d\hat{x}}{d\theta} = - [u'(\eta)F(\hat{x}) + u'(1 - \eta)(1 - F(\hat{x}))] \frac{d\eta}{d\theta} \quad (22)$$

where $1 - \Omega f(\hat{x}) > 0$ from Assumption 2(ii). From (6), the expression in (22) vanishes if $\phi + \theta > 1/2$. In this case, the second-mover concedes everything, so an increased θ has no effect on behavior. But if $\phi + \theta < 1/2$ then (21) holds. From (22), the equilibrium cutoff increases, so each player becomes more likely to choose H when θ increases. ■

The effect of an increase in ϕ , the cost of conflict, is more subtle. On the one hand, it lowers the payoff from Hawk-Hawk, which reduces the incentive to choose Hawk. But on the other hand, an increased cost of conflict also affects the payoffs when one player chooses Hawk and the other Dove, since the former can now extract a larger concession from the latter. This increases the incentive to choose Hawk. When ϕ is low, the second effect (due to $d\eta/d\phi < 0$) dominates, and as both players become more hawkish the probability of conflict increases with ϕ . When ϕ becomes high enough, however, $\eta = 0$ and the second effect vanishes. Any further increases in ϕ will make the players more dovish. This non-monotonicity is consistent with

¹⁶Note that a conflict occurs only when *both* players challenge the status quo (because if only one player challenges, the other will concede).

the stability-instability paradox (Hart [18]). Increasing ϕ from an initially low level causes “instability” at first, as each player tries to exploit the first-mover advantage, but then “stability” as ϕ becomes sufficiently large to make the players more cautious.

Proposition 5 *Suppose the players are ex ante symmetric. An increase in the cost of conflict ϕ increases the probability of conflict if $\phi + \theta < 1/2$, but reduces the probability of conflict if $\phi + \theta > 1/2$.*

Proof. Suppose $\phi + \theta < 1/2$. Totally differentiate (19) with respect to ϕ and use (21) to obtain

$$\frac{d\hat{x}}{d\phi} = \frac{1}{1 - \Omega f(\hat{x})} \frac{u'(1 - \eta)}{u'(\eta)} (1 - F(\hat{x})) > 0.$$

Thus, when ϕ increases \hat{x} increases, making conflicts more likely.

When $\phi + \theta > 1/2$, an increase in ϕ will have no effect on η , and therefore it will reduce the probability of conflict. Indeed, when η is fixed at 0 we get

$$\frac{d\hat{x}}{d\phi} = -\frac{1}{1 - \Omega f(\hat{x})} F(\hat{x}) < 0.$$

■

5 Power Imbalances and Conflict

The hawkishness of a player – how likely he is to challenge the status quo – depends not only on his military strength, but also on how much he benefits from the status quo. If player B is militarily slightly weaker than player A , then player B may still be very hawkish if the status quo greatly favors player A . Conversely, player A may be quite dovish in this situation, since he would have little to gain and much to lose from a conflict. Now we consider two questions. First, what makes a situation balanced, in the sense that neither player has more reason than the other to challenge the status quo? Second, if a balanced situation becomes unbalanced because the weaker player B catches up with the stronger player A (in terms of military strength), will this increase the risk of conflict? This is relevant to the discussion of whether a shift of relative military strength in favor of a “rising power” is likely to be destabilizing.

Restricting attention in this section to the interesting case where $\eta_A > 0$ and $\eta_B > 0$, a natural measure of player i 's military capability is $1 - \eta_j$, the amount of territory player i can get by a strategic move.¹⁷ Using (7) and the fact that $\lambda_j = 1 - \lambda_i$ we can write the best-response functions as

$$\Gamma_i(x) = F(x)\theta + (1 - F(x))(u_i(1 - \eta_j) - u_i(\omega_i)) \quad (23)$$

An increase in λ_i will make player i “tougher” (i.e., shift $\Gamma_i(x)$ up), because it reduces η_j and so make challenges more profitable. An increase in ω_i would have the opposite effect of making player i “softer”. The equilibrium is *balanced* if both players are equally hawkish, i.e., the cutoff points satisfy $x_A = x_B$. It follows from (23) that $\Gamma_A(x) = \Gamma_B(x)$ for any x when the following condition holds:

$$u_A(\omega_A) - u_B(\omega_B) = u_A(1 - \eta_B) - u_B(1 - \eta_A). \quad (24)$$

But if the players have the same best-response functions, clearly $x_A = x_B$ in equilibrium. Therefore, the equilibrium is balanced if (24) holds. Conversely, suppose (24) is violated, say

$$u_B(1 - \eta_A) - u_B(\omega_B) > u_A(1 - \eta_B) - u_A(\omega_A). \quad (25)$$

Then $\Gamma_B(x) > \Gamma_A(x)$ for any x , so the equilibrium cannot be balanced: player B must in equilibrium be more hawkish than player A . Equation (23) reveals that it is precisely the difference between what player i can extort by a strategic move, $1 - \eta_j$, and his status quo allocation ω_i , which determines his incentive to make a challenge. If (25) holds then player B will be the hawkish one, because the status quo does not adequately reflect his military capability.

Suppose player A controls most of the territory under the status quo ($\omega_A > \omega_B$), but this unequal split is “legitimate” because player A is militarily stronger ($1 - \eta_B > 1 - \eta_A$), so that (24) holds. Now suppose λ_B increases. Then, from (7),

$$\frac{d\eta_B}{d\lambda_B} = \frac{1}{u'_B(\eta_B)} \quad (26)$$

and

$$\frac{d\eta_A}{d\lambda_B} = \frac{-1}{u'_A(\eta_A)} \quad (27)$$

¹⁷If $\eta_i = 0$ then we are at a corner solution where a change in military might will not correspond to a change in η_i .

so (25) holds. From (18), player B becomes more hawkish and player A less hawkish, Hence, the effect on each player's aggressiveness and the net effect on the probability of conflict is ambiguous. However, if the two players have the same utility function, $u_A = u_B = u$, then it is sure that a conflict becomes more likely.

Proposition 6 *Suppose each player has the same utility function. If starting from an initially balanced equilibrium, there is a shift in military power in favor of the weaker player, then the probability of conflict strictly increases.*

Proof. The equilibrium probability of a conflict is $F(x_A)F(x_B)$, where x_A and x_B are the equilibrium cut-off points. Suppose there is a small increase in λ_B (and a corresponding decrease in $\lambda_A \equiv 1 - \lambda_B$). This causes the probability of conflict to change by

$$F'(x_A)F(x_B)\frac{dx_A}{d\lambda_B} + F(x_A)F'(x_B)\frac{dx_B}{d\lambda_B}. \quad (28)$$

If the initial situation is balanced then $x_A = x_B = x^*$ so (28) equals

$$F'(x^*)F(x^*) (dx_A/d\lambda_B + dx_B/d\lambda_B).$$

We therefore need to show that

$$\frac{dx_A}{d\lambda_B} + \frac{dx_B}{d\lambda_B} > 0.$$

The equilibrium conditions are

$$x_A = \Gamma_A(x_B) = F(x_B)\theta + (1 - F(x_B))(u_A(1 - \eta_B) - u_A(\omega_A))$$

and

$$x_B = \Gamma_B(x_A) = F(x_A)\theta + (1 - F(x_A))(u_B(1 - \eta_A) - u_B(\omega_B))$$

Totally differentiating, we find that

$$\frac{dx_A}{d\lambda_B} = F'(x_B)\frac{dx_B}{d\lambda_B} (\theta - (u_A(1 - \eta_B) - u_A(\omega_A))) - (1 - F(x_B)) u'_A(1 - \eta_B) \frac{d\eta_B}{d\lambda_B}$$

and

$$\frac{dx_B}{d\lambda_B} = F'(x_A)\frac{dx_A}{d\lambda_B} (\theta - (u_B(1 - \eta_A) - u_B(\omega_B))) - (1 - F(x_A)) u'_B(1 - \eta_A) \frac{d\eta_A}{d\lambda_B}$$

Adding these two equations and using the fact that (24) holds, so that $x_A = x_B = x^*$, yields

$$\frac{dx_A}{d\lambda_B} + \frac{dx_B}{d\lambda_B} = -(1 - F(x^*)) \frac{u'_A(1 - \eta_B) \frac{d\eta_B}{d\lambda_B} + u'_B(1 - \eta_A) \frac{d\eta_A}{d\lambda_B}}{1 - F'(x^*) (\theta - u_i(1 - \eta_j) + u_i(\omega_i))}. \quad (29)$$

Since $\theta = \lambda_i - u_i(\eta_i) - \phi_i$ by (7), Assumption 2(ii) implies the term in the denominator on the right-hand side of (29) is strictly positive. Using (26) and (27), the numerator on the right-hand side of (29) equals

$$\frac{u'_A(1 - \eta_B)}{u'_B(\eta_B)} - \frac{u'_B(1 - \eta_A)}{u'_A(\eta_A)} \quad (30)$$

Now if $u_A = u_B$, then strict concavity and the fact that $\eta_B < \eta_A$ implies that (30) is strictly negative. Then (29) is indeed strictly positive. ■

It is natural that the status quo allocation should favor the stronger side. But when the weaker side catches up in terms of military power, the status quo is no longer legitimate. The concavity of the utility function means that the “rising power” will have a big incentive to challenge the status quo. This is consistent the idea of the Thucydides trap. But the “declining power” will have an incentive to concede more. If utility functions are identical then, as we have shown, the first effect dominates the second. But if the contested territory is intrinsically less valuable to the “rising power” then (30) could be positive, in which case the shifting balance of power would actually reduce the probability of conflict.

6 Escalation and Deterrence: Strategic Complements and Substitutes

Strategic complements (substitutes) means that the incentive to choose Hawk is greater (smaller), the more likely it is that the opponent chooses Hawk. If player j chooses Hawk, then if player i switches from Dove to Hawk player i 's net gain, from the payoff matrix (15), is

$$\lambda_i - \phi_i - c_i - u_i(\eta_i). \quad (31)$$

If instead player j chooses Dove, then if player i switches from Dove to Hawk player i 's net gain is

$$u_i(1 - \eta_j) - c_i - u(\omega_i). \quad (32)$$

Actions are *strategic complements for player i* if (31) is strictly greater than (32), which is equivalent to $\Omega_i > 0$, where Ω_i is defined by (10). They are *strategic substitutes for player i* if $\Omega_i < 0$. The game has strategic substitutes (resp. complements) if the actions are strategic substitutes (resp. complements) for both players.

We begin by showing that actions are strategic substitutes if there is no military advantage to being the first-mover.

Proposition 7 *The game has strategic substitutes if $\theta \leq 0$.*

Proof. If $\eta_i > 0$ then (7) holds so

$$\Omega_i = \theta - u_i(1 - \eta_j) + u_i(\omega_i) < \theta$$

since $1 - \eta_j > \omega_i$. If $\eta_i = 0$ then (3) holds so

$$\Omega_i = \lambda_i - \phi_i - u_i(1 - \eta_j) + u_i(\omega_i) < \lambda_i - \phi_i \leq \theta$$

where the first inequality is due to $1 - \eta_j > \omega_i$. ■

To simplify the exposition, for the remainder of this section we assume the two players are symmetric ex ante (before they draw their types) in terms of preferences, military strength and endowments. We therefore drop the subscripts on u_i , ϕ_i and η_i and observe that $\omega_A = \omega_B = 1/2$ and $\lambda_A = \lambda_B = 1/2$. We have $\Omega_A = \Omega_B = \Omega$ as defined by (20). The game has strategic substitutes if $\Omega < 0$ and strategic complements if $\Omega > 0$.

Under the symmetry assumption, the payoff matrix (15) becomes

	Hawk	Dove	
Hawk	$\frac{1}{2} - \phi - c_i$	$u(1 - \eta) - c_i$	(33)
Dove	$u(\eta)$	$u(1/2)$	

Totally differentiating Ω yields

$$\frac{d\Omega}{d\theta} = - (u'(\eta) - u'(1 - \eta)) \frac{d\eta}{d\theta} \geq 0 \quad (34)$$

with strict inequality when $\phi + \theta < 1/2$, in view of (21) and strict concavity. Also,

$$\frac{d\Omega}{d\phi} = - (u'(\eta) - u'(1 - \eta)) \frac{d\eta}{d\phi} - 1 < 0. \quad (35)$$

If $\phi + \theta > 1/2$ then (35) follows from $d\eta/d\phi = 0$. If $\phi + \theta < 1/2$ then (21) implies

$$\frac{d\Omega}{d\phi} = (u'(\eta) - u'(1 - \eta)) \frac{1}{u'(\eta)} - 1 = -\frac{u'(1 - \eta)}{u'(\eta)} < 0.$$

Thus, actions are more likely to be strategic complements the bigger is θ and the smaller is ϕ .

It is intuitive that if ϕ is large, then the most important consideration is to avoid a conflict – just as in the classic Chicken game, where a collision would be disastrous. Thus, we have the following result.

Proposition 8 *Suppose the players are symmetric ex ante. If $\phi > u(1/2) - 1/2$ then the game has strategic substitutes.*

Proof. By concavity,

$$u(\eta) + u(1 - \eta) \geq u(0) + u(1) = 1.$$

Therefore, $\phi > u(1/2) - 1/2$ implies

$$\Omega = \frac{1}{2} - u(\eta) - u(1 - \eta) + u(1/2) - \phi < 0.$$

■

If ϕ is small, however, then the players will be more concerned about territorial gains and losses than about avoiding a conflict. Actions then become strategic complements if the first-mover advantage is large enough. In fact, a large θ has two effects: it will be costly to be caught out and play Dove against Hawk, but it will be very beneficial to play Hawk against Dove. The first effect tends to make actions strategic complements, while the second effect does the opposite. The first effect dominates because of strict concavity – it is more important to preserve your own territory than to acquire the opponent’s territory. Thus, we have:

Proposition 9 *Suppose the players are symmetric ex ante and $\phi < u(1/2) - 1/2$. There exists $\theta^* \in (0, 1/2)$ such that the game has strategic substitutes if $\theta < \theta^*$ and strategic complements if $\theta > \theta^*$.*

Proof. Fix ϕ such that $\phi < u(1/2) - 1/2$. Then $\phi < 1/2$ since $u(1/2) < u(1) \equiv 1$. From Proposition 7, we have $\Omega < 0$ if $\theta \leq 0$. Now define

$$\bar{\theta} \equiv \frac{1}{2} - \phi \in (0, 1/2).$$

From (6) it follows that $\eta = 0$ if and only if $\theta \geq \bar{\theta}$. When $\eta = 0$ we have

$$\Omega = u\left(\frac{1}{2}\right) - \frac{1}{2} - \phi > 0$$

so that $\Omega > 0$ when $\theta \geq \bar{\theta}$. Thus, there exists $\theta^* \in (0, \bar{\theta})$ such that if $\theta = \theta^*$ then $\Omega = 0$ and $\eta > 0$. It follows from (34) that $\Omega < 0$ if $\theta < \theta^*$ and $\Omega > 0$ if $\theta > \theta^*$. ■

The assumption of decreasing marginal utility of land is required for Proposition 9. If instead the marginal utility is increasing, perhaps because the territory is considered more or less indivisible, there is not much scope for compromise. The “all or nothing” flavor of such a conflict, as in a game of Chicken, would be exacerbated by large first-mover advantages ($d\Omega/d\theta \leq 0$ from (34)). The game would then have strategic substitutes even when θ is large.

Note that θ^* from Proposition 9 depends on ϕ , so we can write $\theta^* = \theta^*(\phi)$. It is easy to check that the function $\theta^*(\phi)$ is decreasing in ϕ .¹⁸ Hence, the parameter range where the game has strategic complements is decreasing in the cost of conflict.

¹⁸Since η depends on ϕ and θ , we can write $\eta = \eta(\phi, \theta)$. The function $\theta^* = \theta^*(\phi)$ identified in Proposition 9 is such that $\Omega = 0$ when $\eta = \eta(\phi, \theta^*)$. Substitute $\eta = \eta(\phi, \theta^*(\phi))$ in (10) to get

$$\frac{1}{2} - u(\eta(\phi, \theta^*(\phi))) - u(1 - \eta(\phi, \theta^*(\phi))) + u\left(\frac{1}{2}\right) - \phi \equiv 0 \quad (36)$$

for all $\phi < u(1/2) - 1/2$. The proof of Proposition 9 implies that $\theta^* < 1 - \phi$, so $\eta(\phi, \theta^*(\phi)) > 0$ satisfies (7). Using this fact, totally differentiating (36) yields

$$\frac{d\theta^*(\phi)}{d\phi} = \frac{-u'(1 - \eta)}{u'(\eta) - u'(1 - \eta)} < 0.$$

7 Commitment Tactics

Suppose player A can make an *ex ante* move (before types are drawn) that changes the parameters of the bargaining game. As in Fudenberg and Tirole [15], player A 's move may shift his own best response function $\Gamma_A(x)$, which in turn will have a strategic (equilibrium) effect on player B 's decision to choose H or D . Player A 's *ex ante* move makes him tougher (softer) if his best response curve $\Gamma_A(x)$ shifts up (down). The familiar logic of Fudenberg and Tirole [15] suggests that player A benefits by becoming tougher (softer) if the game has strategic substitutes (complements).¹⁹ There is, however, an additional effect in our model: player A 's *ex ante* move may shift player B 's best-response function $\Gamma_B(x)$, making player B either softer or tougher. By Claim 3, player A wants player B to become softer, whether the game has strategic substitutes or complements.

To simplify the exposition, assume $\lambda_A = \lambda_B = 1/2$ and $\omega_A = \omega_B = 1/2$. Moreover, assume utility functions are piecewise linear. There are constants $v_i > 0$ and $g_i > 0$ such that each unit of player i 's own endowment is worth v_i to him, but each unit of player j 's endowment is worth only $g_i < v_i$ to player i . Normalizing the status quo utility to zero,²⁰ we get the utility function

$$u_i(x_i) = \begin{cases} v_i(x_i - 1/2) & \text{if } x_i - 1/2 \leq 0 \\ g_i(x_i - 1/2) & \text{if } x_i - 1/2 \geq 0 \end{cases}$$

This utility function is concave but not strictly concave. However, strict concavity is only important in as much as it guarantees that each unit of player i 's own endowment is strictly more valuable to him than each unit of player j 's endowment.²¹ This property holds here, because $g_i < v_i$. Player i 's best-response function is now

$$\Gamma_i(x) = F(x) \left(\frac{v_i + g_i}{4} - \phi_i - v_i \eta_i \right) + (1 - F(x)) g_i \left(\frac{1}{2} - \eta_j \right). \quad (37)$$

¹⁹Following Fudenberg and Tirole [15], we focus on the strategic effect player A 's *ex ante* move has on player B 's behavior in the bargaining game. Specifically, player A derives a strategic benefit if player B becomes more likely to choose D . Tirole [28] has recently expanded the scope of this type of analysis to include two-stage games with *ex ante* information acquisition.

²⁰Thus, to simplify formulas, here we do not normalize $u_i(0) = 0$ and $u_i(1) = 1$.

²¹That is, all the results of the paper go through if strict concavity of u_i is replaced by the weaker assumption: if $0 < x < \omega_i < y < 1$ then $u'_i(x) > u'_i(y)$.

We will contrast an ex ante move that lowers ϕ_A with one that increases v_A . Borrowing the terminology of Fudenberg and Tirole [15], player A “invests” in either cost-reduction (reducing ϕ_A) or in his own endowment (raising v_A). The latter investment could be interpreted, for example, as building settlements on the territory player A controls.²² Another interpretation of these “investments” is that player A delegates decision-making to an agent who is less conflict-averse than he is, or to someone who values player A ’s territory more. Lowering ϕ_A or increasing v_A will have *direct effect* on player A ’s payoff. (We keep the cost of these investments implicit to save notation and as they will play no role in the analysis.) But the change in player A ’s best-response function $\Gamma_A(x)$ and any external impact of player A ’s investment on player B ’s payoff induces a change in player B ’s best-response and hence a change in player A ’s payoff. This is the *strategic effect* of player A ’s investment. If the strategic effect is positive, player A should *over-invest* over and above the level implied by the direct effect. If it is negative, he should *under-invest*.

We can compute

$$\eta_A = \begin{cases} (1 - 2\theta) \frac{v_A + g_A}{4v_A} - \frac{\phi_A}{v_A} & \text{if } \phi_A < (1 - 2\theta) \frac{v_A + g_A}{4} \\ 0 & \text{if } \phi_A \geq (1 - 2\theta) \frac{v_A + g_A}{4} \end{cases}. \quad (38)$$

First suppose $\phi_A > (1 - 2\theta)(v_A + g_A)/4$. Then equations (37) and (38) show that small changes in ϕ_A or v_A have no effect on $\Gamma_B(x)$, and the analysis becomes similar to Fudenberg and Tirole [15]. From (37), reducing ϕ_A or increasing v_A shifts $\Gamma_A(x)$ up. If the game has strategic substitutes,²³ then this is a *strategic advantage* for player A as player B becomes less aggressive and this is good for player A from Claim 3. In the terminology of Fudenberg and Tirole [15], player A should use a “top dog” strategy of over-investing in cost-reduction and building settlements. If, however, the game has strategic complements, then the investment is a *strategic disadvantage* for player A as player B is more likely to play H . Player A should therefore employ a

²²Alternatively, since v_A can be interpreted as player A ’s cost of giving up territory, he might signal that his endowment is sacred to him. Giving up territory then becomes more costly in as much as it implies a loss of face.

²³It is easy to check that actions are strategic complements for player i if

$$(v_i - g_i)/4 - v_i\eta_i + g_i\eta_j - \phi_i > 0,$$

and strategic substitutes if the inequality is reversed.

“puppy dog” strategy of *under-investing* – that is, for strategic reasons he should raise his own cost of conflict and demolish settlements relative to the direct effect benchmark.

Now suppose $\phi_A < (1 - 2\theta)(v_A + g_A)/4$. In this case, (37) and (38) show that the two kinds of investments shift $\Gamma_B(x)$, and they do so in opposite directions. Consider first a small reduction in ϕ_A . Substituting from (38) into $\Gamma_A(x)$, we find

$$\Gamma_A(x) = F(x)\theta\frac{v_A + g_A}{2} + (1 - F(x))g_A\left(\frac{1}{2} - \eta_B\right) \quad (39)$$

Thus, reducing ϕ_A has no effect on $\Gamma_A(x)$, but $\Gamma_B(x)$ shifts down because η_A increases. There is an unambiguous strategic advantage for player A in making player B softer, whether actions are strategic complements or substitutes. Hence, if $\phi_A < (1 - 2\theta)(v_A + g_A)/4$ then player A should definitely over-invest in cost-reduction.

Now consider a small increase in v_A . From (38),

$$\frac{\partial\eta_A}{\partial v_A} = -\frac{(1 - 2\theta)\frac{g_A}{4} - \phi_A}{v_A^2} \quad (40)$$

so an increase in v_A makes player B softer (shifts $\Gamma_B(x)$ down) if $\phi_A > (1 - 2\theta)g_A/4$ but tougher if $\phi_A < (1 - 2\theta)g_A/4$. To understand this, consider that a small change $dv_A > 0$ has two effects on player A 's response to a challenge. If player A concedes to player B 's demand $1 - \eta_A$, then player A must give $\frac{1}{2} - \eta_A$ units of his endowment to player B . Thus, increasing v_A by dv_A increases player A 's cost of conceding by $(\frac{1}{2} - \eta_A) dv_A$. On the other hand, if player A does *not* concede, he will lose *all* of his endowment (which is of size $1/2$) with probability $\theta + 1/2$. Thus, increasing v_A by dv_A increases player A 's expected cost of *not* conceding by $(1/2)(\theta + 1/2)dv_A$. The first effect dominates, making player A less willing to concede ($\partial\eta_A/\partial v_A > 0$), when $\eta_A < (1 - 2\theta)/4$. From (40) this inequality is equivalent to $\phi_A > (1 - 2\theta)g_A/4$. But if $\phi_A < (1 - 2\theta)g_A/4$ then the second effect dominates and therefore $\partial\eta_A/\partial v_A < 0$.

Intuitively, since a conflict may lead to the loss of *all* the contested territory, having valuable settlements on his territory can make player A *more* willing to give up *some part* of his territory in order to avoid a conflict. Since this makes player B more inclined to challenge (shifts $\Gamma_B(x)$ up), for this reason player A 's settlements would tend to be a strategic disadvantage. However, we must also take into account the standard Fudenberg-Tirole strategic

effect on $\Gamma_A(x)$: an increase in v_A makes player A tougher by (39), which is advantageous if the game has strategic substitutes, but not if it has strategic complements.²⁴

We summarize our findings:

Proposition 10 *(i) Suppose $\phi_A > (1 - 2\theta)(v_A + g_A)/4$. Then if the game has strategic substitutes, player A should over-invest in reducing ϕ_A and in increasing v_A . If the game has strategic complements, he should instead underinvest. (ii) Suppose $\phi_A < (1 - 2\theta)(v_A + g_A)/4$. Then player A should definitely overinvest in reducing ϕ_A . He should overinvest in increasing v_A if $\phi_A > (1 - 2\theta)g_A/4$ and the game has strategic substitutes, but underinvest if $\phi_A < (1 - 2\theta)g_A/4$ and the game has strategic complements.*

8 Concluding Comments

A key distinction in the theory of conflict is whether actions are strategic substitutes or complements. Our simple bargaining game has strategic complements when first-mover advantages are big and conflicts are not too costly, and strategic substitutes otherwise. Among other implications, this helps clarify the strategic logic of *ex ante* moves (“investments”). But as shown in Section 7, the logic is more subtle than simply a recommendation to “appear tough” with strategic substitutes and “appear soft” with strategic complements. We have to take into account not only how a player’s investment shifts his own best-response function, but also how it shifts the opponent’s best response function. For example, a player who makes himself tougher by reducing his cost of conflict would, according to the standard logic of Fudenberg and Tirole [15], experience a strategic disadvantage if actions are strategic complements. However, if the investor becomes less inclined to make concessions, the opponent will become softer, which is a strategic advantage.

Our analysis focussed on “small” changes in parameters, but large changes can also be analyzed using this framework. Suppose a “Star Wars” defensive technology to destroy incoming nuclear missiles leads to a large reduction in

²⁴This implies cases are ambiguous. For example, suppose $\phi_A > (1 - 2\theta)g_A/4$ but the game has strategic complements. Although player B becomes softer via the shift in $\Gamma_B(x)$, he also becomes more aggressive as the game has strategic complements. Hence, the strategic effect might be positive or negative.

the cost of conflict. Actions may now go from being strategic substitutes to strategic complements, which could have dramatic implications. With strategic complements, confidence-building diplomacy can create cooperation; with strategic substitutes, one sees conflict through the lens of deterrence, and looking weak is dangerous. On the other hand, if the main effect of the new defensive technology is to reduce first-mover advantages, then the investment may have the reverse effect of turning strategic complements into strategic substitutes.

It is noteworthy that in our model, not only does a large cost of conflict generate strategic substitutes, but the fact that challengers expect large concessions will make it more tempting to challenge the status quo (Proposition 5). A new technology which increases the cost of conflict may be dangerous for many reasons.

Finally, we considered whether the model gives rise to a Thucydides trap. Suppose one country is militarily weaker than the other, but its military strength grows faster, causing the military balance to shift in its favor. In our model there is no reason why, in general, this shift should be destabilizing. However, it may be so if the status quo can no longer be reconciled with the new military reality. If the rising power is unsatisfied with the status quo allocation – in our model, this corresponds to a high marginal utility of land – then it will become inclined to challenge it.

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