Costly Screening and Categorical Inequality

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Categorical Inequality

Inequality in (income) distributions among social groups or categories

Examples: gender, race, ethnicity, religion

Theories of persistent categorical inequality

- Self-fulfilling negative stereotypes (Arrow)
- Differential observability of market-valued traits (Phelps)
- Prejudice (Becker)
- Segregation and social capital (Loury)

How do these mechanisms interact with costly information acquisition?

Model Overview

Candidates

- Candidates may be qualified or unqualified, depending on effort and chance
- Cost of effort may vary across candidates; cost distributions may vary across groups
- Candidates choose efforts based on cost and anticipated earnings in post-screening market

Screener

- Screener designs category-contingent tests, greater precision comes at greater cost
- Cost based on Bregman information (includes Shannon information as special case)
- Reward for qualified passing candidates, penalty for unqualified passing candidates
- Screening intensity based on (category-contingent) qualification rate and information cost

Post-screening market

- Earnings based on posterior beliefs about qualification conditional on screening outcome

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Main Results

Baseline Model

- There is a screening interval of prior beliefs within which active screening occurs
- Posteriors correspond to endpoints of screening interval (independent of priors)
- Candidate efforts are increasing in screening intensity and earnings differential
- One passive and (possibly) two active equilibria, passive and high active are stable

Categorical Inequality

- Higher screening cost $\rightarrow$ smaller difference in conditional posteriors $\rightarrow$ lower investment
- Prejudice $\rightarrow$ upward shift in conditional posteriors, ambiguous effect on investment
- Prejudice (higher penalty and Shannon information) $\rightarrow$ higher investment
- Less favorable cost distribution $\rightarrow$ lower investment, no change in conditional posteriors

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Candidates

Characteristics and Categories

- Continuum of candidates $i \in [0, 1]$
- Each has characteristic $\theta \in \Theta$ and belongs to category $\kappa \in K$; both sets finite
- θ is private information, κ observable without cost or error
- Characteristic distribution in category κ is μ_κ

Investment and Qualification

- All candidates initially unqualified; may move to the qualified state by investment of effort
- Let $S_i = 1$ if candidate i attains qualified state, $S_i = 0$ if not, state unobserved
- Probability that candidate who invests $x_i \geq 0$, achieves qualified state is $F(x_i)$
- $F' > 0$, $F(0) = 0$, $\lim_{x \to \infty} F(x) = 1$, $f = F'$ bounded and unimodal
- Cost of effort is θx_i

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Screening

- Each candidate is examined by screener and accepted $(Z_i = 1)$ or rejected $(Z_i = 0)$
- Utility of candidate with characteristic θ who exerts $x_i \geq 0$ is

$$U(x_i, Z_i, \theta) = Z_i v_1 + (1 - Z_i) v_0 - \theta x_i$$

where v_1 and v_0 are endogenous earnings obtained by passing and failing candidates

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- Screener reward $\beta > 0$ for qualified, penalty $\gamma > 0$ for unqualified candidates accepted
- Cannot observe state but can calibrate test to adjust conditional acceptance rates
- Acceptance rates are $y_0 = \Pr[Z = 1 \mid S = 0]$ and $y_1 = \Pr[Z = 1 \mid S = 1])$
- These decision variables can be modified by costly information acquisition
- Screening intensity based on prior p (category-coningent, correct in equilibrium)

Conditional Posteriors

Overall acceptance rate is

$$y = py_1 + (1-p)y_0$$

Conditional on acceptance, posterior probability that candidate is qualified

$$q_1 = \Pr\left[S=1 \mid Z=1\right] = \frac{py_1}{y}.$$

Conditional upon rejection, posterior probability that candidate is qualified is

$$q_0 = \Pr\left[S=1 \mid Z=0\right] = \frac{p(1-y_1)}{1-y}.$$

Screener's expected payoff is

$$\beta q_1 y - \gamma(1-q_1)y - I$$

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Information Cost

The Bregman information of random posterior q_Z, given prior p is

$$I = \mathbb{E}\left[G(q_Z)\right] - G(\mathbb{E}\left[q_Z\right]) = yG(q_1) + (1-y)G(q_0) - G(p)$$

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Special case is Shannon mutual information, where G is negative entropy:

$$G(q) = q \ln q + (1-q)\ln(1-q)$$

If screener chooses not to acquire information, $q_0 = q_1 = p$ and information cost is zero

Equilibrium

For each category

- Screener sets screening intensity $r = y_1 - y_0$ based on beliefs about qualification rate p
- Candidates choose effort levels x_i based on screening intensity and private cost
- This determines aggregate qualification rate p for each category
- Beliefs about (p, r) are correct for all categories in equilibrium
- Candidate payoffs in post-screening markets equal conditional posteriors

$$v_0 = q_0, \quad v_i = q_1$$

Focus on best stable equilibrium for each group

Optimal Screening Interval and Conditional Posteriors

There exists a nonempty interval $[p_{\text{min}}, p_{\text{max}}]$ such that screener rejects all candidates when $p \leq p_{\text{min}}$ and accepts all when $p \geq p_{\text{max}}$

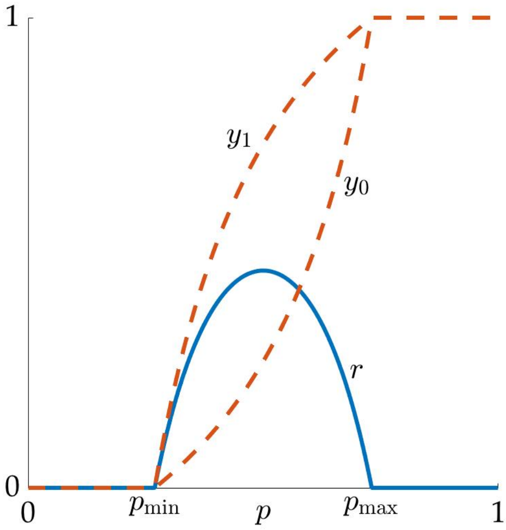
Posterior beliefs about those accepted and rejected are constant across screening interval:

$$\hat{q}_0(p) = p_{\text{min}}, \quad \hat{q}_1(p) = p_{\text{max}}$$

if $p \in (p_{\text{min}}, p_{\text{max}})$

On the screening interval $\hat{y}_0(p)$ is strictly convex and $\hat{y}_1(p)$ is strictly concave

Screening intensity, $\hat{r}(p)$ is continuous and strictly concave on active screening interval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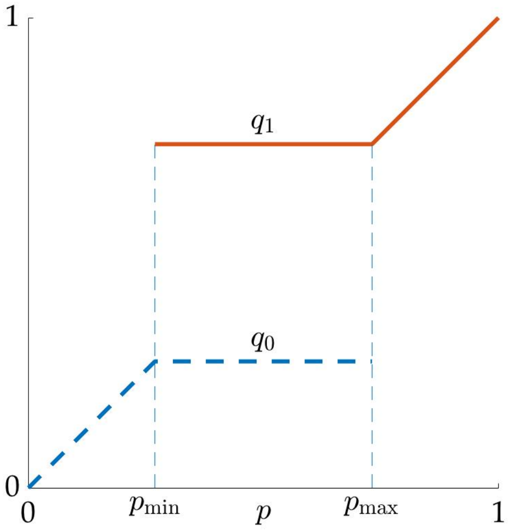
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Optimal Investments

Candidate payoffs in post-screening markets based on conditional posteriors

$$v_0 = p_{\min} \quad \text{and} \quad v_1 = p_{\max}$$

Candidate i with characteristic θ who makes effort $x_i \geq 0$ obtains:

$$\mathbb{E}\left[U(x, Z_i, \theta)\right] = F(x) \cdot [y_1 v_1 + (1 - y_1) v_0] + (1 - F(x)) \cdot [y_0 v_1 + (1 - y_0) v_0] - \theta x.$$

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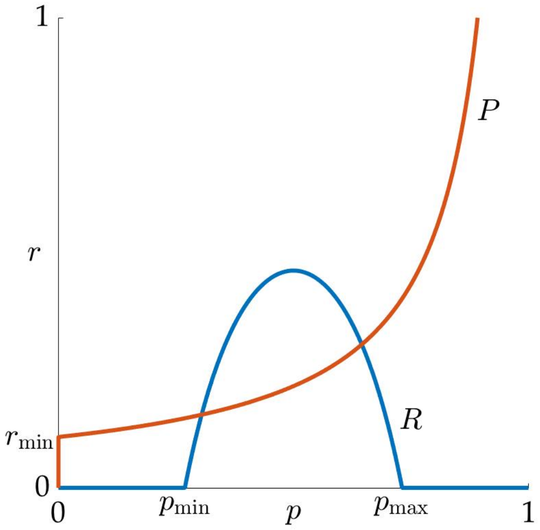
For p outside screening interval all optimal efforts are zero

For p within screening interval, optimal effort, $\hat{x}_\theta$, is either zero or is increasing in screening intensity r and earnings differential $v_1 - v_0$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Equilibrium

- There exists a passive equilibrium with no investment, no screening, full rejection
- There may also exist active equilibria with positive investment and screening intensity

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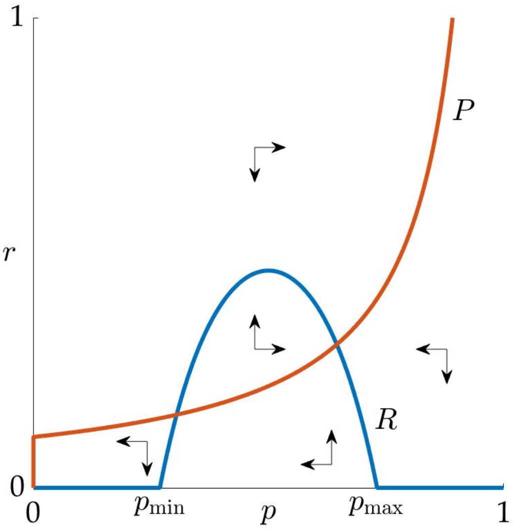
Stability

Suppose that system (p, r) is subject to a dynamic

$$\dot{p} = \phi(\hat{p}(r) - p)$$
$$\dot{r} = \psi(\hat{r}(p) - r)$$

where $\phi(0) = \psi(0) = 0$, $\phi', \psi' \geq 0$ and $\phi'(0), \psi'(0) > 0$.

Then passive equilibrium is asymptotically stable, the low active equilibrium is unstable, and the high active equilibrium is asymptotically stable



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Categorical Inequality

Consider two groups, A, B such that

- Screener bonuses and penalties (β_A, γ_A) and (β_B, γ_B)
- Cost distributions μ_A and μ_B of the individual characteristics θ
- Response curves for the two groups are given by $(\hat{p}_A, \hat{r}_A)$ and $(\hat{p}_B, \hat{r}_B)$
- Screening intervals are denoted $(p_{A\min}, p_{A\max})$ and $(p_{B\min}, p_{B\max})$ respectively
- Qualification rates at stable active equilibrium p_A^* and p_B^*

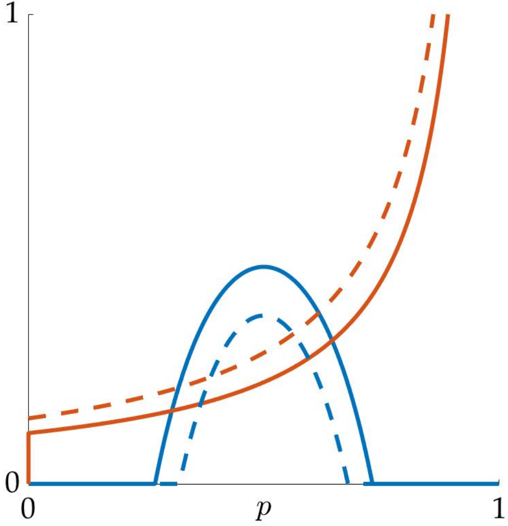
No capacity constraints, constant unit cost of information, so groups can be treated in isolation

Differential Screening Cost

Group A is costlier to screen, groups identical in all other respects

Then Group A

- faces a smaller screening interval: $p_{A\,\max} < p_{B\,\max}$ and $p_{A\,\min} > p_{B\,\min}$
- faces more pessimistic beliefs conditional on passing, more optimistic conditional on failing
- faces smaller earnings differential
- invests at lower rates for each value of screening intensity
- invests at lower rates in the stable active equilibrium: $p_A^* < p_B^*$



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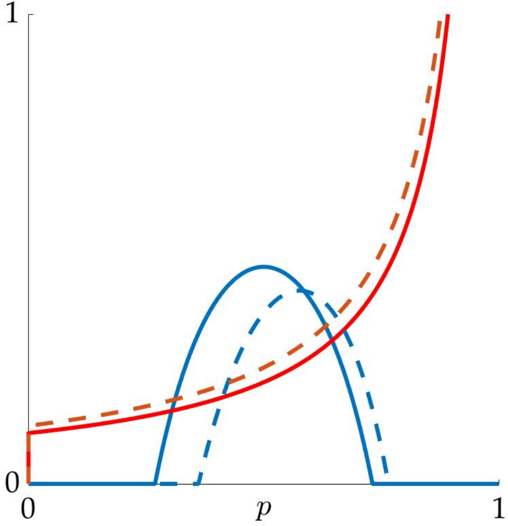
Prejudice (Lower Rewards)

Suppose $\beta_A < \beta_B$, groups otherwise identical

Then Group A

- faces a more demanding screening interval $p_{A\min} > p_{B\min}$ and $p_{A\max} > p_{B\max}$
- faces more optimistic beliefs conditional on both passing and failing
- In Shannon case, invests at lower rates for each value of screening intensity
- Effect on equilibrium investment rates is ambiguous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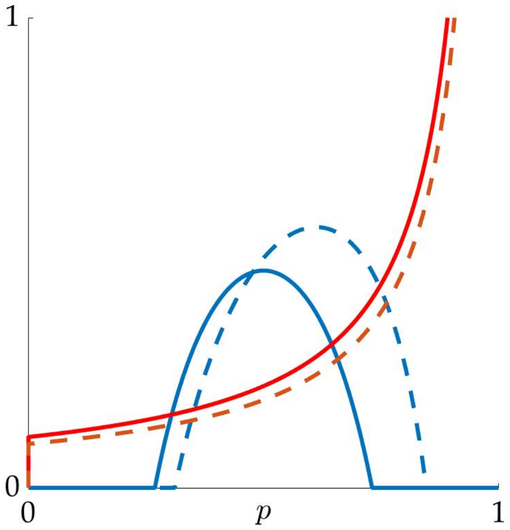
Prejudice (Higher Penalty)

Suppose $\gamma_A > \gamma_B$, groups otherwise identical

Then Group A

- faces a more demanding screening interval $p_{A\,\min} > p_{B\,\min}$ and $p_{A\,\max} > p_{B\,\max}$
- faces more optimistic beliefs conditional on both passing and failing
- In Shannon case, invests at higher rates for each value of screening intensity
- In Shannon case invests at higher rates in the stable active equilibrium: $p_A^* > p_B^*$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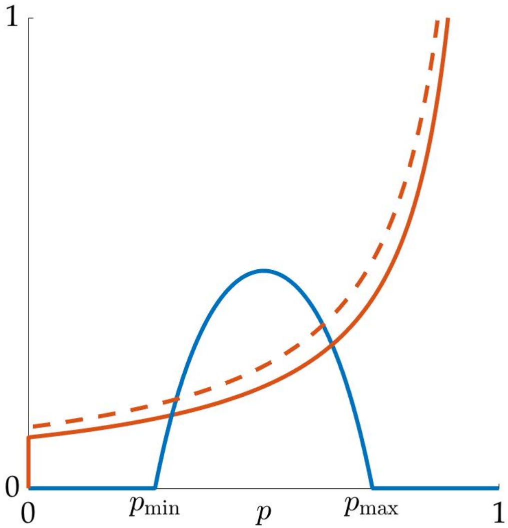
Social Capital

Suppose μ_A first-order stochastically dominates the distribution μ_B

Then Group A

- invests at lower rates for each value of screening intensity
- invests at lower rates in the stable active equilibrium: $p_A^* < p_B^*$
- faces the same screening interval: $(p_{A\,\min}, p_{A\,\max}) = (p_{B\,\min}, p_{B\,\max})$
- No difference in posteriors conditional on passing or failing
- Screening intensity adjusts to eliminate negative stereotype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Conclusions

- Allowing for endogenous information acquisition leads to some new insights
- Mechanisms generating categorical inequality have implications for screening intensity
- Affect equilibrium investment rates and conditional posteriors
- Negative stereotypes can be worsened or mitigated, depending on nature of disadvantage
- Extensions: cross-group spillovers, affirmative action, post-screening discrimination