Diffusion of Ideas in Networks with Endogenous Search

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ThReD

Motivation

- Vast differences in productivity across firms
- Creates possibilities for firms to learn from each other
- Firms learn from firms they interact with who in turn learn from their connections (Coleman, Katz and Menzel (1957), Conley and Udry (2010))
- $\Rightarrow$ **What is the effect of the network structure on diffusion?**

This Paper

- Study diffusion in a network with endogenous adoption and a continuum of productivity levels
- The higher the productivity of one's connections the higher the incentives to adopt
- Consistent with recent evidence that gains from learning increase in the productivity gap (Nix (2016), Jarosch et al. (forthcoming), Herkenhoff et al. (2018) and Akcigit et al. (2018))

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- Consistent with recent evidence that gains from learning increase in the productivity gap (Nix (2016), Jarosch et al. (forthcoming), Herkenhoff et al. (2018) and Akcigit et al. (2018))
- Find that the **more dense the network** the **higher aggregate productivity**
- This is because the **gains to learning** are higher in a dense network
- The **share of agents that learn in equilibrium is not affected by the network**

Related literature

- **Networks and diffusion** Bailey (1975), Jackson and Rogers (2007), Acemoglu, Ozdaglar andYildiz (2011) and Fogli and Veldkamp (forthcoming)

  $\Rightarrow$ Exogenous search and binary (productivity) state

- **Endogenous search and diffusion** Lucas and Moll (2014), Perla and Tonetti (2014) and Benhabib, Perla and Tonetti (2017)

  $\Rightarrow$ Endogenous search intensity depends on productivity distribution, but equally likely to learn from all firms

Model

Continuum of profit maximizing firms. **Productivity Z can improve in 2 ways**

Innovation (exogenous)

- Firms switch between two innovation states according to a Markov process λ
- In high innovation state productivity grows with γ

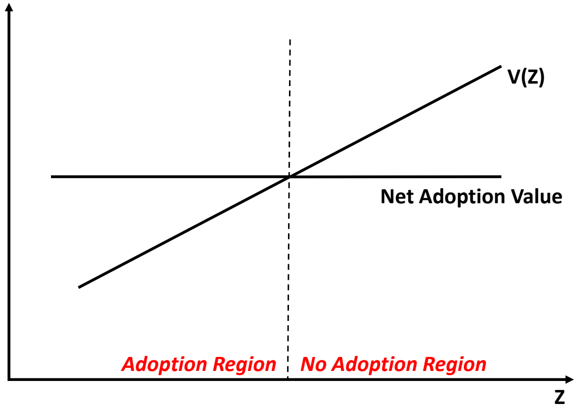
Adoption (endogenous)

- Firms can adopt an existing technology by paying a cost Θ
- Draw a firm from one of their first-degree connections
- Productivity will jump to that level instantaneous

Who Adopts?

Denote by $V(Z)$ the value of not adopting, and by $\hat{\Phi}(\cdot)$ the productivity distribution of the first-degree connections

$$\left.\begin{array}{ll}\text{Don't adopt if} & V(Z) > \\ \text{Adopt if} & V(Z) \leq \end{array}\right\} \underbrace{\int V(Z')d\hat{\Phi}(Z')}_{\text{Gross adoption value}} - \underbrace{\Theta}_{\text{Adoption cost}}$$



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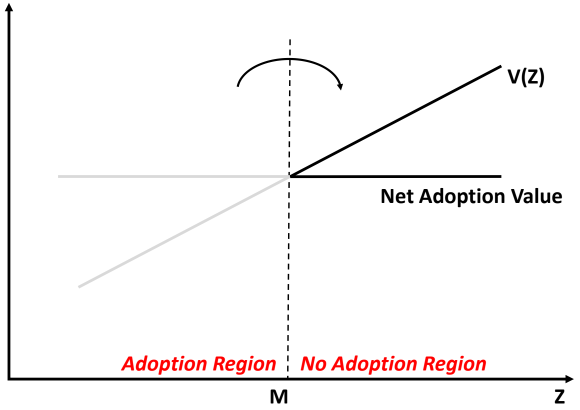

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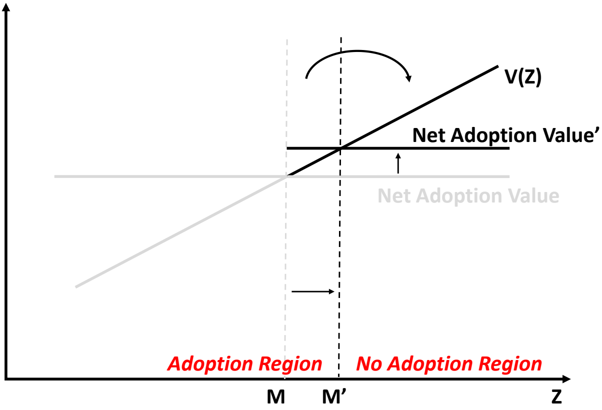


Network and Learning Distribution

- Firms differ in innovation intensity (λ)
- N types indexed by n
- Define the the learning matrix $A = \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix}$

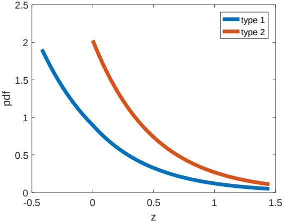
  where a_{ij} is the probability that an adopting firm of type i meets a firm of type j. $\sum_j a_{ij} = 1$
- 'Learning distribution' $\hat{\Phi}(\cdot)$ is a mixture of the productivity distributions of the first-degree connections:
  $\mathbf{1} - \hat{\Phi}(\hat{Z}) = X^{-1} A \left(\mathbf{1} - \Phi(\hat{Z}) \right)$
- $\Rightarrow$ solve for balanced growth path (quantiles of productivity distribution grow at the same rate)

Theoretical results

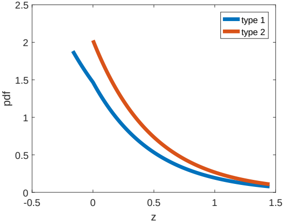
- Growth rate equals γ (exogenous innovation) but TFP depends on network
- Get an analytic solution for the distribution of productivity $\Phi(Z)$ but this depends on two variables that cannot be solved by pencil and paper
- The productivity distribution has a right Pareto tail of which the shape parameter is the same for each type and does not depend on the network
- The network has an effect on the left tail
- The normalization factor is not affected by the network

Example with two nodes

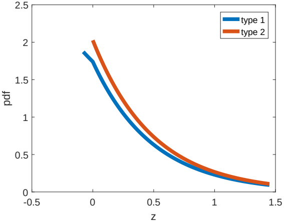
Red type is more likely to be in the high innovation state than the blue type



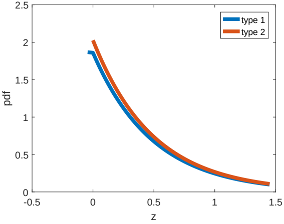
(a) Probability that a type-I agent learns from a type-II agent is (a) 5%



(b) 20%



(c) 50%



(d) 95%

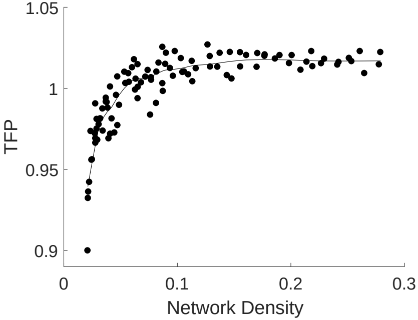
Simulations with many nodes

- To study effect of network properties on diffusion use simulations

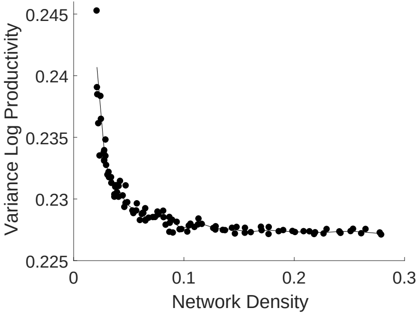
- Estimate innovation process (λ) using firm-level data

- Vary the network across simulations

Erdös-Rényi random network

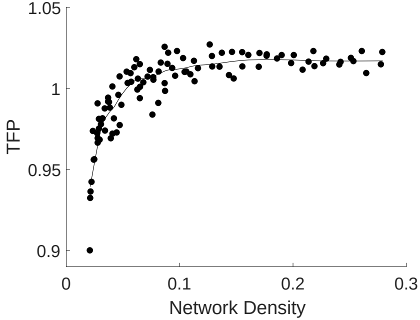


(a) Productivity



(b) Inequality

Erdös-Rényi random network



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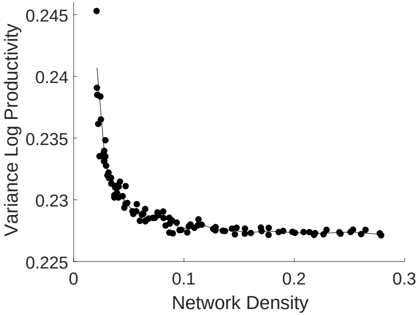
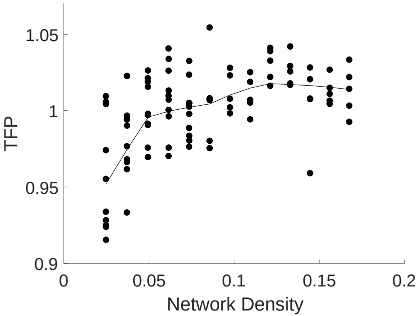


(b) Inequality

More dense networks lead to higher productivity and lower inequality

Preferential attachment (fat tail)



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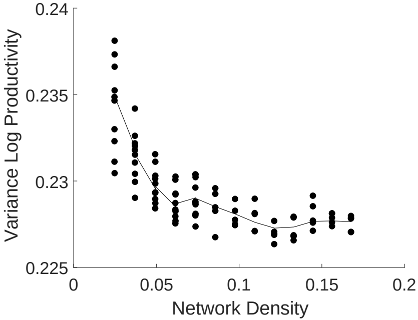
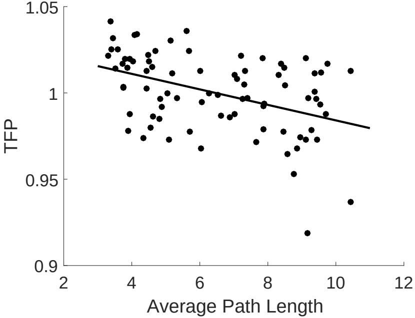


(b) Inequality

Average path length (Watts-Strogatz)



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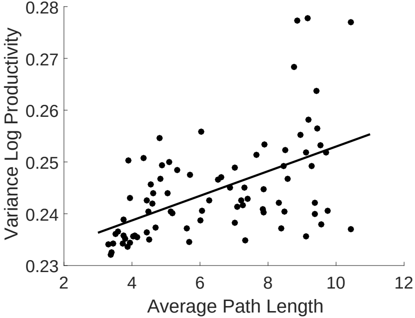

(b) Inequality

Both high network density and low average path length are beneficial for diffusion

Table: Effect of network properties on TFP and inequality (variance of log productivity).

	(1) TFP	(2) Inequality
Network Density	0.24^{***}	-0.0068
	(0.048)	(0.0046)
Average path length	-0.013^{***}	0.0032^{***}
	(0.0011)	(0.00031)
Mean dependent variable	1.54	0.23
Observations	271	271
R^2	0.53	0.68

Robust standard errors in parentheses. Based on simulations of Erdös-Rényi, Watts-Strogatz and preferential attachment networks.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Effect of centrality measures on TFP of a type

	(1) TFP	(2) TFP
Degree centrality	-0.0040***	-0.0040***
	(0.00068)	(0.00065)
Closeness centrality	0.045***	0.045***
	(0.0010)	(0.00096)
Eigenvector centrality	-0.00045	-0.00036
	(0.00044)	(0.00043)
Bonacich centrality	0.00024	0.00024
	(0.00022)	(0.00022)
Eigenvector centrality ΛA		0.041***
		(0.00085)
Type fixed effects	X	X
Mean dependent variable	1.54	1.54
Observations	43360	43360
R^2	0.50	0.56

Robust standard errors in parentheses. Point estimates denote the effect of a one standard deviation increase. Based on simulations of Erdös-Rényi, Watts-Strogatz and preferential attachment networks.

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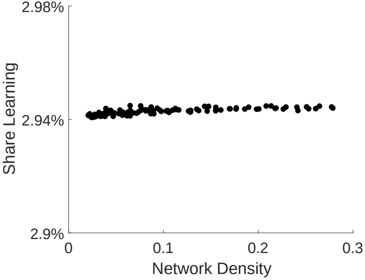
Share of agents learning unaffected by network



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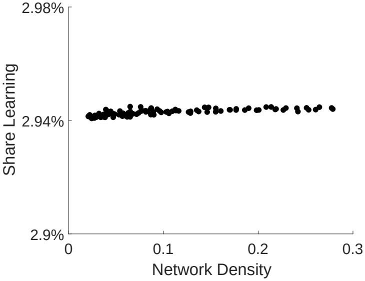

- Has implications for empirics
- Regressing the share of agents that is adopting on different (exogenous) network properties will lead to the, incorrect, conclusion that the network has no effect on adoption/search/productivity
- Instead could regress productivity on the network (but the network might be correlated with productivity for other reasons than diffusion)
- Should look at the effect on adoption after a change to the network

Conclusions

- Developed a model in which the network affects the learning decision of firms
- Increasing the number of links and decreasing the average path length increases TFP and decreases inequality
- Effects especially large for sparse networks
- $\Rightarrow$ Societies with low social capital (= sparse network) benefit from increasing interactions (especially between distant agents)
- $\Rightarrow$ In societies with high social capital the benefit from adding links is low
- No effect on equilibrium share of agents learning

Thank you!