The Microfinance Disappointment: An Explanation based on Risk Aversion[*]

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Abstract

Recent research indicates that microcredit has not contributed significantly to poverty reduction. Take up of affordable credit by the poor for investment in businesses, education and health, turned out to be very low. We argue that this can be explained by risk aversion, when investment affects the probability of success of a risky project. Our model abstracts from fixed costs in the production technology, commonly assumed in the existing literature. There are no imperfections in the loan market, and we abstract from assumptions about false beliefs by the poor regarding the production function or other behavioral assumptions. We conclude that to facilitate investment and thereby reduce poverty, policy should be aimed at reducing the risk faced by the poor.

1 Introduction

A large body of research, as well as popular views regarding the persistence of poverty in developing countries, suggest that credit constraints prevent the poor from accessing profitable investment opportunities. This has led to the conclusion that a key policy intervention for alleviating poverty is the provision of affordable credit. This policy has been implemented on a large scale in the last few decades. Thousands of microcredit NGOs were established, offering the poor billions of dollars in loans at affordable interest rates, and offering hope for significant reduction in poverty. The UN declared 2005 the "Year of Microcredit," and the 2006 Nobel Peace Prize was awarded to Muhammad Yunus and the Grameen Bank for their contribution to the reduction in world poverty.

But the years that followed revealed disappointing results.[1] First, microcredit availability encouraged borrowing for consumption and thereby increased debt and poverty. Second, microcredit failed to deliver the anticipated levels of productive investment because the poor tended to turn down the opportunity to borrow at a reasonably low interest rate for the sake of investing in high return projects. Banerjee et al. (2015) find, in a randomized evaluation in India, low take up of microcredit, no increase in households who are business owners, and despite some increase in the size of existing businesses, the average business remained small and not very profitable. Low take up and limited impact on business creation is also found in other places (Banerjee et al. (2013), Crépon et al. (2015), Angelucci et al. (2015)). Tarozzi et al. (2015) find higher take up in Ethiopia, but, similarly to other regions, no significant impact on business creation.

In this paper, we show that risk aversion can explain why the poor do not take up affordable high return investment opportunities, and thus why poverty persists. In our model, investment increases the probability that a risky project will succeed. This structure of the model is the key for our theoretical contribution and its application. The model abstracts from any fixed costs (or more generally from any increasing marginal returns to investment), which are inconsistent with recent evidence, but nevertheless play a key role in the existing literature.

Existing explanations for the persistence of poverty are typically based on credit constraints combined with increasing marginal returns in the production function, typically in the form of a fixed cost. (E.g., Dasgupta and Ray (1987), Galor and Zeira (1993), and Banerjee and Newman (1993), Piketty (1997), Maoz and Moav (1999), Ghatak and Jiang (2002), and Mookherjee and Ray (2003), among many others).[2] The fixed cost prevents the poor from investing a modest amount that they can afford, so that they can gradually escape poverty. We address a related but different question: why do the poor leave high return investment opportunities unexploited when they do have access to credit?

When the fixed cost is coupled with risk, the risk averse poor might choose the safe

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[1]Banerjee et al. (2015). See also The Guardian https://www.theguardian.com/global-development-professionals-network/2013/nov/19/microcredit-south-africa-loans-disaster

[2]In Piketty (1997), the effort level, rather than capital investment, is indivisible. In Mookherjee and Ray (2003), the multiplicity of steady states requires indivisibilities in the return to education. Other scholars focused on group-level influences as a source of non-convexities in the production function in generating poverty traps (e.g., Benabou (1996), Durlauf (1996), Mookherjee et al. (2010)). For a summary of theories on poverty traps see Bowles et al. (2006)

alternative and avoid the investment, even if affordable credit is available. Indeed, Banerjee and Duflo (2011), argue that "[r]isk is a central fact of life for the poor, who often run small businesses or farms [...] with no assurance of regular employment. In such lives a bad break can have disastrous consequences." (p. 133). They further claim that investment is often equivalent to buying a lottery ticket (p. 87). For instance, the outcome of schooling is employment by the government or a large firm, if successful, or subsistence self-employment, if not. Thus, the downside outcome – if the investment fails to yield the expected high return – could be too painful: It leaves the poor with a low income, less assets, and often a debt to pay. The fixed cost, according to this explanation, plays a crucial role, as it prevents the poor from investing a modest amount that exposes them to a level of risk they are willing to take.[3]

However, Banerjee and Duflo (2011) claim that many investment opportunities, such as in education or health, offer a high expected return and do not have a significant fixed cost. The marginal return to investment in education, they argue, is high at low levels of investment: "... every little bit of education helps. People who are comfortable with reading are more likely to read newspapers and bulletin boards and to find out when there is a government program available for them. People who go on to secondary education are more likely to get a formal-sector job, but even those who don't are able to run their businesses better" (p. 82). Similarly, they argue that modest investments in health reduce significantly the risk of illness, and thus provide a high expected return. The cost of illness – treatment and forgone earnings – could be devastating for the poor.

The absence of any fixed cost in the production function isn't limited to education or health. Kraay and McKenzie (2014) survey the empirical literature and conclude that the evidence is inconsistent with technology-based poverty traps. Not much capital is needed to start a business in a developing country and returns to investment are very high: 5% to 20% per month, at investment levels as low as 100 dollars (McKenzie and Woodruff (2006), de Mel et al. (2008, 2009, 2012), Fafchamps et al. (2014)). Similarly, despite high returns, many farmers fail to invest in fertilizer that is availability in small quantities Duflo et al. (2011)), and many shopkeepers fail to make small inventory investments (Kremer et al. (2011)). Microfinance, in many cases, is available to finance these unexploited high-return small investments.

To explain why the poor repeatedly avoid small affordable investments with high expected return, Banerjee and Duflo (2011) suggest that the poor typically believe that the production function has an "S-shape" – the marginal return is low at low levels of investment and high at higher levels. Thus, the poor believe, despite the facts, that in order to enjoy a high return, the investment should be large. The combination of false beliefs and the risk associated with the investment push the poor to avoid it all together. That is, even if the poor could borrow the required funds at a low cost, they give up the opportunity because of the risk associated with a large investment. "In reality, there should not be an education-based poverty trap: Education is valuable at every level. But the fact that parents believe that education is S-shaped [...] create[s] one." (p. 89). This claim is

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[3]The argument that risk aversion leads to underinvestment isn't new, of course. It is proposed by Stiglitz (1969) and is further developed, with an emphasis on the poor, by many others (See the literature review in Banerjee (2000)).

supported by some evidence (e.g., Nguyen (2008)) but other evidence presented by the authors led them to conclude, as consistent with our approach, that the poor have a good understanding of the relevant economic environment: "the poor are no less rational than anyone else – quite the contrary. Precisely because they have so little, we often find them putting much careful thought into their choices: They have to be sophisticated economists just to survive." (p. ix).

Kremer et al. (2013) address the same facts but propose an alternative behavioral explanation. They argue that small businesses in developing countries reveal risk aversion in small-stakes gambles that cannot be explained by any reasonable degree of risk aversion within expected utility theory, and propose that loss aversion within prospect theory may play a role.

More recently, Banerjee et al. (2015) propose that the low take-up of loans for starting a business could be an outcome of the lack of complementary factors such as proper training or skills, and more generally, that there is less potential for high return businesses for the poor than anticipated by microcredit enthusiasts. However, Bandiera et al. (2017) show that when poor women receive a productive asset (a couple of cows) and some relevant training, they have the skills to run a simple yet successful business that alleviates poverty in the long run. These results seem consistent with our theory. Women who could borrow the funds for the required investment and training, avoid it despite the high return. Risk could be the main difference between borrowing for investment and receiving the asset with no debt.

Our contribution is to show that the assumption that the poor have false beliefs or loss aversion is not necessary in many important cases addressed by these theories. In particular, we show that investment projects with a binary outcome of success or failure, where investment increases the probability of success at a constant or diminishing rate (no fixed costs or any other s-shape in the production function), could lead the risk averse poor to avoid any investment, despite the high expected return at low levels of investment – which they are fully aware of. A simple illustrative calibration of our model, with parameter values estimated from Augsburg et al. (2015), demonstrates that reasonable values of the risk aversion coefficient are sufficient to prevent the poor from investing (see Appendix B.2).

We show that risk averse agents choose a corner solution in such investment projects: do not invest or invest a lot. For projects of this nature (investment increases the probability of success) the expected utility of a risk averse agent, as a function of investment, is typically U-shaped. Moreover, if risk aversion is diminishing with wealth, an increase in wealth would lead to a larger increase in expected utility at the high end of investment than at the low end. The u-shape of the expected utility could lead to corner solutions and the effect of wealth could lead to a shift from one corner to the other. The continuous decline in risk aversion with wealth therefore leads to a discontinuous rise in investment, as long as the risk cannot be fully alleviated within a reasonable cost. Thus, the observed investment behavior looks as if there is a fixed cost or a belief that there is such a cost, even when there is not.

To understand the reason for the U-shape of expected utility as a function of investment size and the resulting corner solutions, consider a lottery with zero expected return. With probability p the outcome of the lottery is a prize of one dollar, and with probability $1 - p$ the outcome is zero dollars (no prize). The probability p is equal to investment. That is,

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with no investment the probability of winning the prize is zero, and with investment of one dollar the probability is one, and p is one half if the investment is half a dollar, and so on. In this case, any individual is clearly indifferent between no investment and investing the maximum ($p = 1$), where with probability one the agent simply receives the dollar invested back. The outcome is certain and identical in both cases. For any investment strictly between the two corners, the expected return is the same as for the two corners, but the realization is uncertain. It follows, therefore (by definition of risk aversion) that any risk averse individual would strictly prefer the corners over any other investment strictly between zero and one.

To understand why the poor might avoid a high expected return investment that the wealthy would take, consider two changes to the lottery described above. First, the prize in case of a successful outcome is higher than one, so that the expected return of the lottery is positive. Second, there is a limit to investment that is strictly less than one. Risk averse agents, as previously explained, would typically choose between one of the two corners, now facing a tradeoff between avoiding risk (by not investing) and enjoying an expected positive return (by investing the maximum possible). If the prize is not too high, since risk aversion declines with wealth, the result would be a threshold wealth level above which individuals invest in the project and below which they don't.

This result doesn't depend on any constraints on borrowing. Moreover, even if agents could declare bankruptcy (in the case that they borrowed, were unsuccessful in their investment, and cannot repay the entire debt), results still hold as long as: (1) liability is limited and the wealth of individuals after bankruptcy is correlated with their wealth before bankruptcy (e.g., they can hide some wealth), and (2) bankruptcy, following an unsuccessful investment, leaves individuals worse off in comparison to the option of not investing.

Our model, as mentioned above, assumes no fixed costs, or more generally, that the production function is not S-shaped: the expected return on the first dollar invested is at least as large as that of any other dollar. However, the realized income has to be sufficiently large if the project is successful. Thus, we have an indivisibility in our model without any fixed costs. We interpret this structure as investment that increases the expected return from an existing asset. We propose that the large majority of investment opportunities that are relevant for the poor are consistent with this interpretation. The indivisible asset is the individual's labor, which is the main productive asset of the poor, and the productivity of this asset can be augmented by investment. This is clearly the case if the individual is seeking employment with a firm or the state: Investment in general education, specific skills, or health, increase the probability of finding a well-paying stable job.

When the poor run their own small business, it is still true that the main asset of the business is typically the owner's labor. An individual whose business is providing services (such as plumbing or electricity services, or even jut simple manual tasks) could invest in augmenting the relevant skills, health and physical abilities, or invest in marketing the services, purchase useful complementary tools, or anything else that increases the probability the business is successful. Any other small business is not that different. The owner has mainly her own time and could augment the expected income of the business with investment in complementary factors or intermediate goods, such as a larger stock and more shelves for storage in a shop, or more fertilizer, tools and irrigation equipment

in a field.

The view that risk plays a significant role, in particular among the poor, is supported by evidence. Morduch (1990) empirical findings suggest that the poor avoid profitable but risky technologies. Moreover, risk aversion declines with income and is significant among the poor (see Andrisani (1978), Hill et al. (1985), Cicchetti and Dubin (1994), and Shaw (1996). The World Bank (2001) report shows that "[the] poor are highly risk averse and reluctant to engage in the high-risk, high-return activities that could lift them out of poverty" (p. 138), and "[a]s households move closer to extreme poverty and destitution, they become very risk averse" (p. 145). Bryan et al. (2014) provide consistent evidence concerning seasonal migration. The poor avoid low cost migration that is highly rewarding because of the fear of failure. Finally, insurance against adverse weather shocks can induce farmers to invest more in high return risky production options (Karlan et al. (2014), Cole et al. (2017)).

Banerjee and Duflo (2011) provide further support for the claim that risk plays a central role in the decisions of the poor who understand their economic environment. They argue that the poor are constantly worrying about the future, particularly about imminent disasters, and take a variety of ingenious and costly precautionary measures to limit the risks they are subject to, such as managing their businesses conservatively and diversifying their portfolio of activities, including by marriage and temporarily migration (pp. 141 - 143).

We believe that our simple result – that risk aversion can lead to corner solutions – may have been overlooked by the existing literature because of the conventional strategy for modeling risky investment. In the existing models of investment under uncertainty, when fixed costs are absent, the probability of success is typically exogenous. The model then limits the optimization of the agent to the scale of the project: the investor decides how much to invest in an asset, but her investment has no effect on the rate of return (e.g., investment in a stock). In a project in which the rate of return is drawn from an exogenous distribution, the typical result is that the expected utility of a risk averse agent is a concave function of investment, leading to an interior solution. The optimal investment increases with wealth (if risk aversion is declining with wealth), but the change is continuous. In that framework, the S-shape of wealth dynamics is achieved by assuming a fixed cost (see, for instance, Acemoglu and Zilibotti (1997), and Banerjee (2000)).[4]

Our approach that investment affects the probability of success (rather than, or in addition to, the outcome conditional on success) is consistent with the evidence on education and health. The outcome in the former, as mentioned above, is a good job if successful and no such job otherwise. Of course, income could still be affected by education within the two options, but as long as a significant part of the return to investment is in the form of higher probability of success our results hold. In the latter, investment in health could increase productivity when not ill, but as argued by Banerjee and Duflo (2011), it has a significant effect on preventing illness. In addition, our approach is consistent with the findings of Banerjee et al. (2015), that businesses who borrowed and invested, have significantly more assets, and business profits are higher above the 85th percentile of profitabil-

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[4]Aghion and Bolton (1997) model the probability of success as an endogenous outcome of investment, but they do not show our main result of the "u-shape" utility.

ity, without a significant effect on profits for other businesses. Finally, we show that the data from Augsburg et al. (2015) is consistent with a positive correlation between investment and the probability of successes in a project (see Appendix B.3).

Our theory also sheds light on two additional related facts described by Banerjee and Duflo (2007). First, the poor often spend the little they have above their subsistence needs on consumption of goods that do not alleviate poverty and could most likely be avoided. Second, the modest savings they might have, are invested at low rates of returns. Thus, the poor avoid high return investment, even if this investment is possible, either by borrowing when affordable credit is available, or by a temporary cut in consumption (such as the consumption of "temptation" or "conspicuous" goods) or by reallocating some of their low return assets. We propose that as they are reluctant to take risky investments they prefer the short-term utility of consumption and low-risk/low-return savings.

A potential caveat should be acknowledged. Our theory explains why some high return investments are not exploited by the poor. These are investments that are mainly aimed at increasing the probability of success rather than the income if successful. These projects require large investment by risk averse agents, who therefore decline the opportunity. But why don't the poor invest in projects in which the probability of success is given and the investment increases the return conditional on the outcome? Perhaps, for most of the poor the available options are of the type we study here. Be that as it may, in Section 5 below, we describe circumstances under which agents would prefer to invest in "probability" than in "return conditional on success."

We are not the first to propose an explanation for persistence of poverty, which isn't based on a fixed cost in the production technology. A growing literature shift focus from the technology to the behavior of the poor. Moav (2002) shows that non-homothetic preferences, such that the marginal propensity to save increases with income, can lead to a poverty trap. Chakraborty and Das (2005) and Moav (2005) obtain similar results, without assuming non-homothetic preference, based on the interaction between health and human capital in the former and the trade-off between fertility and education in the latter. Banerjee and Mullainathan (2010) assume non-homothetic preferences with respect to "temptation goods" and Bernheim et al. (2015) focus on self-control problems. Moav and Neeman (2010, 2012) show that conspicuous consumption signaling equilibria could lead to persistence of poverty despite homothetic preferences. These papers propose different explanations for the low savings of the poor that prevent them from access to high return investment opportunities. Here we focus on risk aversion to explain why the poor choose not to invest even if they could.

In the next section, we present a simple version of our model that illustrates our main claim: in projects in which investment increases the probability of success, the expected utility of the risk averse agent leads to corner solutions. We further show that the degree of risk aversion has an effect on the optimal solution: the higher risk aversion is, the more likely it is that the agent would choose not to invest. Thus, risk aversion declining with wealth leads to a wealth threshold above which individuals invest a lot and below which they do not invest at all. We contrast this result with a model in which the probability of success is exogenous and the return for any given outcome increases proportionally with investment. This structure of investment leads to an internal solution.

In section 3 we show that results hold when a competitive financial market and possible

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bankruptcy are included in the model. We assume that individuals' risk aversion declines with wealth and initial wealth varies between individuals. As in the simple version, individuals can invest in the probability of success of a risky project with an exogenous binary outcome of high or low income. They can augment their investment with a loan and pay the competitive risk-adjusted interest rate in a perfect loan market, which takes into account the probability of a low-income outcome and that of bankruptcy. We assume that individuals' wealth is unobserved by the financial intermediary, and the interest rate is determined by a zero-profit condition, under limited liability: only the income from the investment project can be used to pay back the debt. Individuals could also lend their wealth to the financial market and enjoy the risk-free return.

We show that an individual's choice of investment is discontinuous in risk aversion, and therefore in wealth. This implies that individuals face a tradeoff between a safe, low return option, and a risky, high expected return option. If an individual chooses the safe option, her initial wealth is augmented by low income. If she invests in the project, the end outcome in case of failure is that she is left with her initial wealth and the low income from the unsuccessful project, net of the investment cost. If this initial wealth is low then risk aversion is high: the disutility from losing the low income in the risk-free option is high. This leads to our main result: despite the absence a fixed cost in the production technology, optimization prevents the poor from investing in a high return project even if credit is available at a competitive rate. We remove any frictions from the credit market, but one market friction is crucial: full insurance against risk (at an affordable cost) isn't available in our model. We believe that this is a reasonable assumption. For many reasons related to moral hazard and adverse selection, such insurance doesn't exist for many profitable investments – in human capital in particular.[5]

In section 4 we address the question of insurance against a bad outcome and show that the poor are willing to pay more than the wealthy for insurance. More importantly, the option to insure increases the likelihood that the poor would exploit high return investment projects. This leads to the policy conclusion that providing microinsurance alongside microcredit could help to reduce poverty. Section 5 offers some concluding remarks.

2   A Simple Model

In this section, we illustrate our main claim: in projects with a binary outcome of either a high or a low income, in which investment increases the probability of the realization of high income ("success"), the maximization of the expected utility of a risk averse agent yields a corner solution. We further show that the degree of risk aversion has an effect on the optimal solution: for a project with a positive expected return, there is a threshold

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[5]Udry (1990), Townsend (1995), and Morduch (1995) provide evidence that the poor are often insured against the risks they take, but as suggested by Morduch (1990) and Banerjee (2000), these studies only observe the risky activities people have chosen to take. The poor may have foregone other investment opportunities to limit the risk they bear. Moreover, Townsend (1995) shows that full insurance is limited to some risks. In addition, the World Bank (2001) reports that "poor people, even though they need insurance most, are more likely to drop out of informal [insurance] arrangements." (p. 144). Finally, Banerjee and Duflo (2011) show that the poor avoid insurance, in particular health insurance, because "[c]redibility is always a problem with insurance products." (p.153).

level of risk aversion above which agents choose zero investment and below which maximal investment. We contrast this result with the one obtained in the "standard" model in which the probability of success is exogenous and the return for any given outcome increases proportionally with investment. In such a model the optimal level of investment is obtained at an interior solution.

An agent has a CARA utility function $u(x) = -e^{-\lambda x}$ where the parameter $\lambda > 0$ describes the agent's Arrow-Pratt coefficient of risk aversion. A higher value of λ indicates that the agent is more risk averse. As λ decreases to zero, the utility function converges to the risk neutral identity function $u(x) = x$.[6]

The agent's income is determined by the success or failure of an investment project: it is high, H, with probability p and low, L, with probability $1 - p$, where $H > L \geq 0$. The agent controls the probability of success of the project $p \in [0, \overline{p}]$, $0 < \overline{p} < 1$, at a linear cost $c(p) = \alpha p$ for some $\alpha > 0$. We focus on the case in which investment in the probability of success of the project p generates a positive expected return, or $pH + (1 - p)L - \alpha p > L$ for any $p > 0$. This assumption is equivalent to the assumption that $\alpha < H - L$.

The objective of the agent is to choose the probability $p \in [0, \overline{p}]$ that maximizes its expected utility from the project, which is given by

$$U(p) \equiv pu(H - \alpha p) + (1 - p)u(L - \alpha p).$$

Proposition 1. *The agent's expected utility function $U(p)$ is U-shaped in p. Namely, it is nonincreasing on an interval $[0, \hat{p}]$ and increasing on the interval $[\hat{p}, \overline{p}]$ for some $\hat{p} \in [0, \overline{p}]$. Notice that $U(p)$ may be either nonincreasing or nondecreasing on the entire range.*

Proof. We show that if U is increasing at some p, then it is increasing for all $p' > p$. The agent's expected utility is equal to

$$U(p) \equiv -pe^{-\lambda(H - \alpha p)} - (1 - p)e^{-\lambda(L - \alpha p)}$$

and its derivative with respect to p is equal to

$$U'(p) = (1 + \lambda\alpha p)\, e^{\lambda\alpha p} \left(e^{-\lambda L} - e^{-\lambda H} \right) - \lambda\alpha e^{\lambda\alpha p} e^{-\lambda L}.$$

Suppose that U is increasing at p, or $U'(p) > 0$. It follows that

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\begin{aligned}
U''(p) &= \lambda\alpha \left[(2 + \lambda\alpha p)\, e^{\lambda\alpha p} \left(e^{-\lambda L} - e^{-\lambda H} \right) - \lambda\alpha e^{\lambda\alpha p} e^{-\lambda L} \right] \\
&= \lambda\alpha \left[U'(p) + e^{\lambda\alpha p} \left(e^{-\lambda L} - e^{-\lambda H} \right) \right] \\
&> 0,
\end{aligned}
$$

which implies that $U'(p') > U'(p)$ for all $p' > p$ or that U continues to increase throughout the remainder of its range. ∎

The fact that $U(p)$ is U-shaped implies that the agent would choose either the smallest or the largest probability $p \in [0, \overline{p}]$. The realized utility of an agent who chooses the

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[6]To see this, recall that von Neumann-Morgenstern utility functions are only unique up to affine transformations. Hence, the utility function $u(x) = -e^{-\lambda x}$ is equivalent to the utility function $u(x) = \frac{1 - e^{-\lambda x}}{\lambda}$, which converges to the identity function as λ tends to zero by L'Hôpital's Rule.

minimal probability $p = 0$ is certain and is equal to $U(0) = u(L)$. The realized utility of an agent who chooses the maximal probability $p = \bar{p}$ is uncertain; it is equal to $u(H - \alpha\bar{p})$ with probability $\bar{p}$, and to $u(L - \alpha\bar{p})$ with probability $1 - \bar{p}$. We denote this lottery by $(H - \alpha\bar{p}, L - \alpha\bar{p}; \bar{p}, 1 - \bar{p})$. The assumption that investment in p generates a positive expected return ($0 < \alpha < H - L$) implies that all sufficiently risk neutral agents would choose $p = \bar{p}$. On the other hand, all sufficiently risk averse agents would choose $p = 0$. The fact that the expected utility $U(p)$ is U-shaped implies that the agent's choice is discontinuous in its degree of risk aversion as described in the next proposition.

Proposition 2. *The agent's choice of p is discontinuous in its level of risk aversion λ. There exists a threshold level of risk aversion $\lambda^o > 0$ such that more risk averse agents with $\lambda > \lambda^o$ choose the minimal probability $p = 0$, and less risk averse agents with $\lambda < \lambda^o$ choose the maximal probability $p = \bar{p}$.*

Proof. As noted in footnote 6, as λ tends to zero, the agent's utility function converges to a linear function, which implies that the agent becomes risk neutral and so prefers the lottery $(H - \alpha\bar{p}, L - \alpha\bar{p}; \bar{p}, 1 - \bar{p})$ over the certain outcome L. By continuity, this is also the case for all agents with small enough level of risk aversion λ.

An agent prefers the certain outcome L over the lottery $(H - \alpha\bar{p}, L - \alpha\bar{p}; \bar{p}, 1 - \bar{p})$ if and only if

$$-pe^{-\lambda(H - \alpha p)} - (1 - p)e^{-\lambda(L - \alpha p)} < -e^{-\lambda L}$$

if and only if

$$pe^{\lambda(\alpha p + L - H)} + (1 - p)e^{\lambda \alpha p} > 1.$$

As λ increases to infinity, $e^{\lambda(\alpha p + L - H)}$ tends to zero, but $e^{\lambda \alpha p}$ tends to infinity, which implies that the last inequality is satisfied for all λ large enough.

Finally, the fact that an agent with a smaller λ is less risk averse than an agent with a larger λ implies that any lottery that is preferred over a certain outcome by the former is also preferred by the latter. It therefore follows that there exists a threshold level of risk aversion $\lambda^o > 0$ such that more risk averse agents with $\lambda > \lambda^o$ choose the minimal probability $p = 0$, and less risk averse agents with $\lambda < \lambda^o$ choose the maximal probability $p = \bar{p}$. ∎

Thus, Proposition 1 shows that the agent's expected utility is U-shaped, which induces corner solutions; Proposition 2 shows how the optimally chosen corner solutions varies depending on the agent's level of risk aversion.

Remark. The U-shape of the agent's utility as a function of investment stands in sharp contrast with its shape if the probability of success p is exogenously given and is independent of the agent's level of investment. Specifically, suppose that the payoff to the agent upon success $H(c) \geq L$ is increasing and weakly concave in the agent's cost of investment $c \geq 0$. In this case, the objective of the agent is to choose the cost of investment c so as to maximize its expected utility

$$V(c) \equiv pu(B + H(c) - c) + (1 - p)u(B + L - c)$$

where the utility function u is increasing and concave (not necessarily CARA).

Proposition 3. *The agent's expected utility function $V(c)$ is concave in its cost of investment c. It follows that it is either increasing throughout its range, decreasing throughout its range, or is inverse U-shaped in c.*

Proof. The first and second derivatives of $V(c)$ are given by

$$V'(c) = pu'(B + H(c) - c)\left(H'(c) - 1\right) - (1 - p)u'(B + L - c)$$

and

$$V''(c) = pu''(B + H(c) - c)\left(H'(c) - 1\right)^2 + pu'(B + H(c) - c)H''(c) + (1 - p)u''(B + L - c),$$

respectively. The conclusion follows from the concavity of the functions $u(\cdot)$ and $H(\cdot)$. ■

Notably, the agent's choice of level of investment is still decreasing in its level of risk aversion, but unlike in the case with an endogenous choice of probability, it is *continuous* in the level of risk aversion.[7]

Finally, one wonders to what extent our result that when the agent controls the probability of success investment is discontinuous in the agent's degree of risk aversion generalizes to other utility functions beyond CARA. In Appendix B.1 we provide a numerical example that shows that our result extends also to the case of a CRRA function, albeit with an "approximate" rather than an exact U-shape result as is the case for CARA functions. It should be emphasized that the agent's investment is still discontinuous in its level of risk aversion.

Moreover, as explained in the introduction, the intuition for our U-shape result doesn't depend on the specific utility function used. Consider the following specific case for our lottery: $L = 0$, $H = 1$, and $\alpha = 1$. In this case any investment $p \in [0, 1]$ has an expected return of zero. Clearly any individual is indifferent between the two corners: invest zero and receive zero and invest one and receive one back. The outcome is certain and identical in both cases. For any investment strictly between the two corners, the expected return is the same as for the two corners, but the realization is uncertain. It therefore follows that any risk averse individual would strictly prefer the corners over any other investment strictly between zero and one.

One concern regarding our theory is, of course, its relevance to reality. In particular, can the theory explain the facts with reasonable parameter values? Following Rabin (2000), Kremer et al. (2013) argue that risk aversion cannot explain the fact that many poor households neglect small, high expected return investment opportunities. They claim that the required coefficient of risk aversion is too high, and propose instead that loss aversion provides a better explanation. In Appendix B we use data from Augsburg et al. (2015)[8] and find, as consistent with our framework, that wealthier households invest more and that there is a positive correlation between investment and the probability a project is successful. Using the same data, we present a simple illustrative calibration of our model,

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[7]Specifically, it is possible to show that if $\{u_n\}$ is a sequence of utility functions that converges to a utility function u and $\{c_n\}$ and c are the associated costs of investment, then if u_n exhibits more/less risk aversion than u_{n+1} then c_n is smaller/larger than c_{n+1} and the sequence $\{c_n\}$ converges to c.

[8]See Appendix A for data description.

and show that under reasonable values of relative risk aversion, sufficiently poor agents do not invest. Finally, we estimate the risk aversion parameter from the data and find that for the average project, under a relative risk aversion coefficient of 1.82, the model has a good fit with the data.

3 A Model with a Resource Constrained Agent

In this section, we show that the results of the simple model continue to hold when the model is embedded in a competitive financial market with the possibility of bankruptcy (when the investment project fails and the agent pays only part of the debt). As in the simple model, individuals can invest in the probability of success of a risky project with an exogenous binary outcome of high or low income. They can augment their investment with a loan and pay the competitive risk-adjusted interest rate in a perfect loan market, which takes into account the probability of a low-income outcome and bankruptcy. We assume that individuals' wealth is unobserved by the financial intermediary, and the interest rate is determined by a zero-profit condition, under limited liability: only the income from the investment project can be used to pay back the debt.

We show that an individual's choice of investment is discontinuous in its degree of risk aversion, and therefore in wealth, assuming that risk aversion declines with wealth. This implies that individuals face a tradeoff between a safe, low return option, and a risky, high expected return option. If an individual chooses the safe option, her initial wealth is augmented by low income. If she invests in the project, the end outcome in case of failure is that she is left with her initial wealth and the low income from the unsuccessful project, net of the investment cost. If this initial wealth is low then risk aversion is high: the disutility from losing the low income in the risk free option is high. This leads to our main result: despite the absence of non-convexities in the production technology, optimization implies that the poor behave as if there is a fixed cost which prevents them from investing in a high return project despite the fact that credit is available at a competitive rate.

Suppose that the agent has an initial income of $B \geq 0$ that it can use in order invest in a risky binary project as described in the previous section. An investment of size $c(p) = \alpha p$ generates an additional income H with probability p and an additional income L, $0 < L < H$, with probability $1 - p$, where $0 < \alpha < H - L$.

An investment that is larger than B requires the agent to borrow. Suppose that the agent has access to a competitive credit market in which the riskless interest rate is normalized to zero. A loan of size $b \geq 0$ can be obtained at the interest rate $r(b)$ that allows lenders to break even. We assume that B is non-verifiable to lenders so that an individual who borrows any amount return a maximum amount L if its additional income is realized to be L, and a maximum amount H if it is realized to be H. We assume that the success of the project as well as the agent's choice of probability p are verifiable (for example, because the project requires investment in observable physical capital) so that lenders are able to asses the correct interest rate to charge the loan, and would refuse loans that are larger than what is needed in order to finance the agent's investment.[9] Finally, we assume that

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[9]Note that the agent may want to borrow a larger amount than the amount necessary to finance its in-

in case of indifference, the agent prefers a larger to a smaller loan (this last assumption is not necessary for our results, but it simplifies the analysis below).

Suppose that the individual has a CARA utility function and chooses the probability p at cost $c(p) = \alpha p$ as described above.

Proposition 4. *The agent finances its entire investment $c(p) = \alpha p$ through a loan.*

Proof. If $c(p) \leq L$ then the individual is indifferent with respect to how it finances its investment because regardless of whether it finances the investment from its own funds or through a loan, its income is $B + H - c(p)$ and $B + L - c(p)$ following success and failure of the project, respectively. It therefore follows that if $c(p) \leq L$ then the individual will finance its investment entirely through a loan.

If $c(p) > L$ then borrowing allows the individual to insure itself against risk because an individual who borrows the entire amount necessary for investment $c(p)$ returns only L if the project fails and so enjoys an income of B in that case, whereas an individual who borrows a smaller amount and relies on its own funds (but still chooses the same probability p) still has to pay back L if the project fails so only enjoys a smaller income than B in that case (note that a same probability p produces equal expected incomes in the two cases and that lenders earn zero profits). A bigger loan implies that the induced lottery second-order-stochastically-dominates the lottery induced by a smaller loan. So every risk averse individual would prefer a bigger loan over a smaller loan. It therefore follows that in this case the agent would borrow the largest amount possible, which is equal to $c(p)$.[10,11] ∎

We show that, as in the case described in the simple model, the agent's expected utility is U-shaped in p. The fact that individuals borrow the entire amount needed to finance their investment implies that the individual's induced expected utility function $U(p)$ is given by:

$$U(p) = \begin{cases} pu(B + H - \alpha p) + (1-p)u(B + L - \alpha p) & \text{if } p < \frac{L}{\alpha} \\ pu(B + H - L - \alpha + \frac{L}{p}) + (1-p)u(B) & \text{if } \frac{L}{\alpha} \leq p \end{cases} \tag{1}$$

because an individual who borrows an amount $c(p) = \alpha p < L$ returns αp in both states of the world, and an individual who borrows an amount $c(p) = \alpha p \geq L$ returns L if the project fails, and $L + \alpha - \frac{L}{p}$ if the project succeeds, so that $p\left(\frac{\alpha p - (1-p)L}{p}\right) + (1-p)L = \alpha p$ overall.[12]

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vestment because such a loan provides insurance to the agent: an agent who borrows such a larger amount enjoys a certain income that is paid back only upon success. Lenders may be reluctant to lend larger sums because of moral hazard considerations, and in any case, the point of this paper is that poor agents cannot reduce or eliminate their exposure to risk, which such larger loans would facilitate.

[10]The reasoning above implies that the agent would like to borrow possibly even more than $c(p)$, but we assume that lenders would refuse larger loans (see the discussion in footnote 6).

[11]Lenders will obviously not lend more than $\min\{c(p), (1-p)L + pH\}$. However, the assumption that $(1-p)L + pH - c(p) > L$ ensures that this minimum is obtained on $c(p) = \alpha p$.

[12]Observe that $\frac{\alpha p - (1-p)L}{p} \leq \frac{pH + (1-p)L - (1-p)L}{p} = H$ so the individual can indeed return the loan if the project succeeds.

Proposition 5. *The individual's induced expected utility function $U(p)$ that is described in (1) is U-shaped in p.*

Proof. For values of p that are such that $c(p) < L$ or $p < \frac{L}{\alpha}$, it is possible to show that the function $U(p) = pu(B + H - \alpha p) + (1 - p)u(B + L - \alpha p)$ is U-shaped using a similar argument to the one used in the proof of Proposition 1.

For values of p that are such that $c(p) \geq L$ or $p \geq \frac{L}{\alpha}$, we need to show that the function $U(p) = pu(B + H - L - \alpha + \frac{L}{p}) + (1 - p)u(B)$ is increasing in p. This implies that $U(p)$ is U-shaped over its entire range because $U(p)$ is continuous so the argument is valid regardless whether $U(p)$ is decreasing, increasing, or U-shaped for $p \in [0, \frac{L}{\alpha}]$.

The derivative of $U(p)$ with respect to $p \geq \frac{L}{\alpha}$ is equal to

$$U'(p) = -pu'\left(B + H - L - \alpha + \frac{L}{p}\right)\frac{L}{p^2} + u\left(B + H - L - \alpha + \frac{L}{p}\right) - u(B).$$

For our CARA utility function, $u(x) = -e^{-\lambda x}$ and $u'(x) = \lambda e^{-\lambda x}$. So, $U'(p) > 0$ if and only if:

$$\lambda\left(H - L - \alpha + \frac{L}{p}\right) > \ln\left(1 + \frac{\lambda L}{p}\right).$$

The conclusion follows from our assumption that $H - L > \alpha$ together with the fact that $x > \ln(1 + x)$ for all $x > 0$. ∎

It follows that like in the simple model, also in this model we have:

Proposition 6. *The agent's choice of p is discontinuous in its level of risk aversion λ. There exists a threshold level of risk aversion λ^o such that more risk averse agents with $\lambda > \lambda^o$ choose the minimal probability $p = \underline{p}$, and less risk averse agents with $\lambda < \lambda^o$ choose the maximal probability $p = \overline{p}$.*

Proof. Follows from the same argument as in the proof of Proposition 2. ∎

If agents with a smaller initial income (bequest) B are also more risk averse in the sense that their CARA utility function has a larger risk coefficient parameter λ then we have:

Corollary. *The agent's choice of p is discontinuous in its initial income (bequest) B. There exists a threshold level of income B^o such that poorer agents who have a smaller initial income $B < B^o$ choose the minimal probability $p = \underline{p}$, and richer agents who have a larger initial income $B > B^o$ choose the maximal probability $p = \overline{\overline{p}}$.*

4   The Role of Insurance

We assumed that investment in the success of the project generates a positive expected net return for any probability $p > 0$, or that $H - L > \alpha$. This assumption implies that if the maximum probability of success $\overline{p} \leq 1$ is in fact equal to 1 then it is optimal for all agents to set $p = 1$ regardless of their level of risk aversion or initial income.

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In fact, the ability to set $p = 1$ is more valuable for risk averse or low income agents than for other agents. To see this, suppose that the probability p has the following convex cost function

$$c(p) = \begin{cases} \alpha p, & 0 \leq p < \tilde{p} \\ \alpha\tilde{p} + \alpha'(p - \tilde{p}), & \tilde{p} \leq p \leq 1 \end{cases}, \tag{2}$$

for some $\alpha' > \alpha$. This cost function implies that elimination of the per-unit risk of failure becomes more expensive beyond the point $p = \tilde{p}$.

We ask, are the poor willing to pay more than the rich for reducing risk ?

Figure 4 depicts the expected utility $U(p)$ in the case where the cost function is given by 2. The left panel depicts the expected utility for a risk averse or low income agent ($\lambda = 1$), while the right panel draws the expected utility for a high income or less risk averse agent ($\lambda' = .2$).
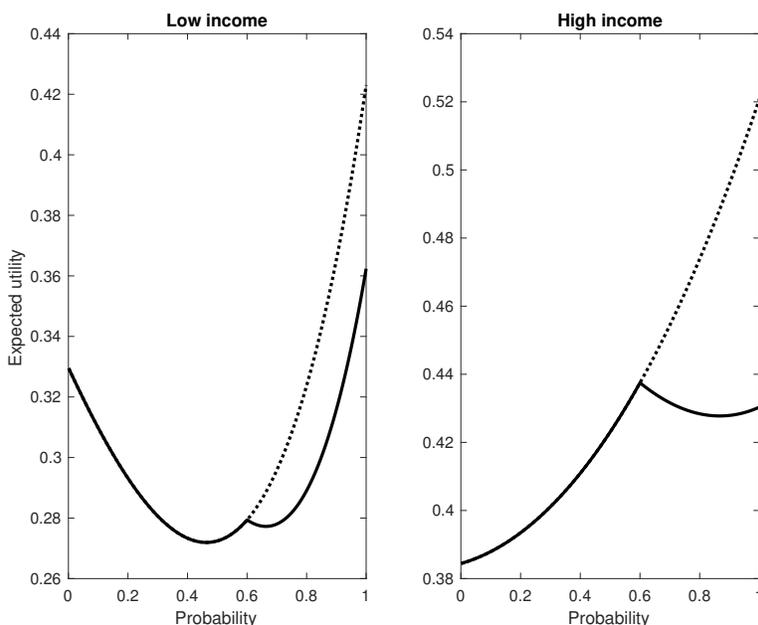

Figure 1: Expected utility. $H = 1.55; L = 0.4; \tilde{p} = 0.6; \alpha = 1; \alpha' = 1.25; \lambda = 1; \lambda' = .2$

As can be seen from the figure, for the low income agent $U(1) > U(0) > U(\tilde{p})$, while for the high income agent $U(\tilde{p}) > U(1) > U(0)$. Namely, the rich agent is less concerned about the risk associated with failure of the project and chooses optimally $p = \tilde{p} < 1$. On the other hand, a poor agent chooses optimally to reduce the risk associated with failure to zero by choosing $p = 1$ and is willing to pay more for it.

5 Investment in Probability vs. Investment in Return Upon Success

In certain circumstances, an individual who has an opportunity to invest either in increasing the probability of success or in the reward upon success, with equal expected return, would prefer to invest in probability.

To see this, consider the case of an agent who faces a binary project. The project either succeeds with probability $p \geq p_0 > 0$ and yields $H(p)$, or fails and yields L, where $H(p) > L \geq 0$ for all $p \geq p_0$.

Suppose that the expected return of the project is constant so that $pH(p) + (1-p)L \equiv C > L$ is fixed.[13] Hence, the individual faces the dilemma of whether to invest in the probability p at the expense of the reward upon success $H(p) = \frac{C-(1-p)L}{p}$, which is decreasing in p, or maximize the reward upon success but minimize the probability of success.

Suppose that $p > p' \geq p_0$. We compare the p-lottery in which the individual receives $H(p)$ with probability p and L with probability $1-p$ with the p'-lottery in which the individual receives $H(p') > H(p)$ with probability p' and L with probability $1-p'$. Our assumption that $pH(p) + (1-p)L \equiv C > L$ is fixed implies that these two lotteries generate the same expected income to the agent.

Proposition 7. *A p-lottery that pays $H(p)$ and L with probabilities p and $1-p$, respectively, generates a higher expected utility to a risk averse individual than a p'-lottery that pays $H(p') > H(p)$ and L with probabilities p' and $1-p'$, respectively, if $p > p' \geq p_0$.*

Proof. Follows immediately from the fact that the p-lottery second-order-stochastically-dominates the p'-lottery. ∎

Hence, under the circumstances described in this section, any risk averse individual would prefer to invest in the probability of success over the reward upon success.

6 Conclusion

High hopes of significantly reducing poverty through microfinance were countered by disappointing results. In this paper, we address the fact that take up of affordable credit by the poor for investment in businesses, education and health, turned out to be very low. We propose that this can be explained by risk aversion. Our key assumption is that investment affects the probability of success of a risky project. With this structure, we can abstract from any fixed costs (or more generally an S-shape) in the production function, which is consistent with the evidence. Our focus is on the short run: we offer an explanation for why risk averse poor leave high return investment projects unexploited, despite affordable credit. An extension of our model, which in the spirit of existing models illustrates persistence of poverty over generations, is straightforward to develop. The key

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[13]We require that $p \geq p_0 > 0$ because we want to consider projects that all have the same expected return that is strictly higher than the expected return when $p = 0$, which is L.

element for the persistence of poverty is an "S-shape" relationship between current income and future income, which crucially depends on our model's result of discontinuity of investment in income. This discontinuity, unlike in models in the existing literature, isn't a result of fixed costs in the production technology, non-homothetic preferences, "behavioral" elements, or credit constraints.

We conclude that to facilitate investment, policy should be aimed at reducing the risk faced by the poor. Microfinance is probably important, perhaps necessary, for the reduction of poverty, but it is not sufficient. We propose, therefore, that future randomized controlled trials would offer the poor access to credit coupled with some form of insurance. A possible direction, derived from our model, is to condition repayment of debt on outcomes, with a higher interest rate when investment yields successful outcomes, and forgiveness of most of the debt in case of failure.

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Appendices

A Data description

We use data from Augsburg et al. (2015) who studied the effect of microcredits in Bosnia. They conducted an experiment by providing randomly loans to those who were rejected by MFI.[14] To facilitate their study, the authors collected data on various socioeconomic variables, ranging from household consumption and assets to income and savings choices.

The data we use includes:

- **assetvalue** which corresponds to Endowment B in the model
  Source: Baseline & Follow-up surveys, Section 4

- **y_max** which corresponds to the return H in the good state of the world
  Source: Baseline survey, Section 5
  Question asked: *"Imagine that you do receive the loan from EKI and have a very good month/year, economic conditions are flourishing and stable and there is great demand for your product/service... What would be the **maximum** amount of profit this business of yours receives in such a situation over the next month/year?"*

- **bm_expenses** which corresponds to investment μ
  Source: Baseline & Follow-up surveys, Section 6
  Question: *"average yearly expenses of main business"*

- **past_success** which will be used to compute probability of success P
  Source: Baseline & Follow-up surveys, Section 6
  Question: *"Please respond to the following statements on a scale of 1 (Disagree) 2 (Neutral) 3 (Strongly Agree)*
  Previous year was successful financially"

The summary statistics of these variables are given in Table 1.

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[14]One may argue that this is a selected sample. Indeed the authors write "[w]e can also compare the average marginal client to the population of Bosnia and Herzegovina as a whole and to regular first-time clients of our MFI. We [...] find the average marginal client is younger and more likely to be male and married.[...] We also use data from the MFI's management information system to compare the marginal clients to regular first-time borrowers. This shows that marginal clients are younger, less likely to be married, and have less education. They are also less likely to be employed full-time." Since wealth is a sufficient statistic in our model this may be a selected sample as younger, unmarried and less educated are typically poorer individuals. However since our focus in our quantitative exercise is on those who have actually made investment decisions this selection is of less importance.

Table 1: Summary Statistics

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	count	mean	s.d.	min	max
H: max expected income, yearly	1105	61992.77	125618.14	100	1320000
B: value of all assets owned	988	109817.00	133953.59	100	1785700
μ: avg. yearly expenses of main business	295	6647.80	15840.73	120	180000
p: last year was successful financially	540	0.63	0.48	0	1

To calculate the probability of success, p, we use the question *previous year was successful financially*. We assume that those who strongly agreed with the statement could be considered successful. Then, we create a dummy variable that takes value 1 if a person was successful and 0 – otherwise. Next, we sort the sample based on the asset value B, from smallest to the largest. Then, we divide this sorted sample into 30 income groups by the value of total assets that they own: the first group contains 18 poorest respondents, the second – the next 18 poorest poorest, etc. Finally, for each group we compute the share of people within a group who were successful. Such share is an estimate of the probability of success p for a given income group. We notice that the minimal value of p is $0.(22)$, which corresponds to the first income group and the maximal value is 0.88, which corresponds to the 28th income group.

Similarly, we aggregate μ and B, by taking the mean of individual μ and B, respectively, for a given income group.

B Quantitative Analysis

In this section we show that the discontinuity result may also hold for a CRRA utility function. However, the optimal investment is not always a corner solution and may change with the initial wealth. Specifically, richer people may make higher investments. Then we provide a simple calibration, based on Augsburg et al. (2015), which suggests that, under reasonable values of relative risk aversion, poor agents do not invest. Then we use the same data to show some observations that are consistent with our framework. Finally, we estimate the risk aversion parameter and show that under reasonable values our model has a good fit with the data.

B.1 CRRA Example

Assume a CRRA utility function of the form $u(x) = \frac{x^{1-\sigma}-1}{1-\sigma}$. In this case, our expected utility function that corresponds to the one in section 2 becomes[15]

$$U(p) = p\frac{(B+H-\mu)^{1-\sigma}-1}{1-\sigma} + (1-p)\frac{(B+L-\mu)^{1-\sigma}-1}{1-\sigma} \qquad \text{(B.1)}$$

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[15]We denote by μ the cost of investment and keep, in this subsection, the assumption made in our model, $\mu = c(p) = \alpha p$.

Figure B.1 draws the expected utility for different levels of wealth , which corresponds to the values $\sigma = 1$, $H = 5.5, L = 0.6875, \alpha = 3, \bar{p} = 0.8$.[16]
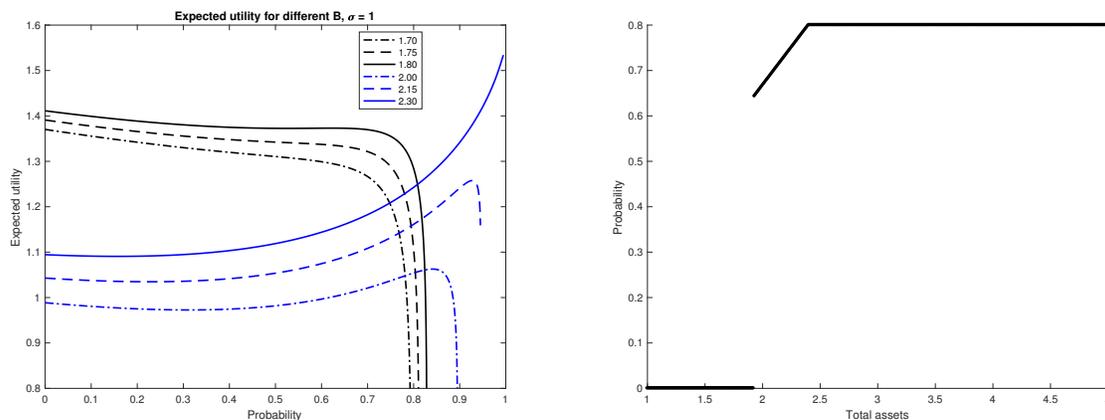

Figure B.1:   Left panel: Expected utility as a function of probability (levels adjusted by constant for convenience of presentation on one figure). Right panel: Optimal probability.

The left panel of Figure B.1 shows that while for a relatively low initial wealth agents (black curves) the optimum is achieved at $p = 0$, for a relatively high initial wealth agents (blue curves) the optimum is achieved at strictly positive and relatively high level of P. More importantly, the U-shaped pattern still exists but it does not have to occur allover the range $[0, 1]$ (blue curves). The right panel of Figure B.1 shows that the discontinuity in optimal choice of probability still holds. Specifically, for a relatively low initial wealth, $B < 1.93$ agents choose not to invest. Finally, throughout the range $1.93 < B < 2.40$, the optimal value of p increases with wealth.[17]

B.2   A Simple Calibration

We assume that the probability is a linear function of investment $p = \gamma\mu$, $L = 0$ and take $\bar{p} = 0.88$, $H = 62 \times 10^3$ from the data.[18] Then, we assume that the maximum μ from the data corresponds to $\bar{p}$ and use it to calculate γ.

We plot the probability of success as a function of total assets for different levels of σ in the range considered acceptable in the literature. The figure illustrates our main results: for reasonably low levels risk aversion poor individuals choose to avoid investment, and a continuous rise in wealth leads to a discontinuous jump in investment.

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[16]This corresponds to $u(x) = \ln(x)$.

[17]Remember that in the CARA case the U-shaped pattern holds for the whole range between 0 and 1. This implies that if it is optimal for the agent to invest, it occurs at $\bar{p}$. This made our analytical problem tractable.

[18]We take H to be the average in the sample. Thus, for simplicity, we assume that all individuals face the same project.
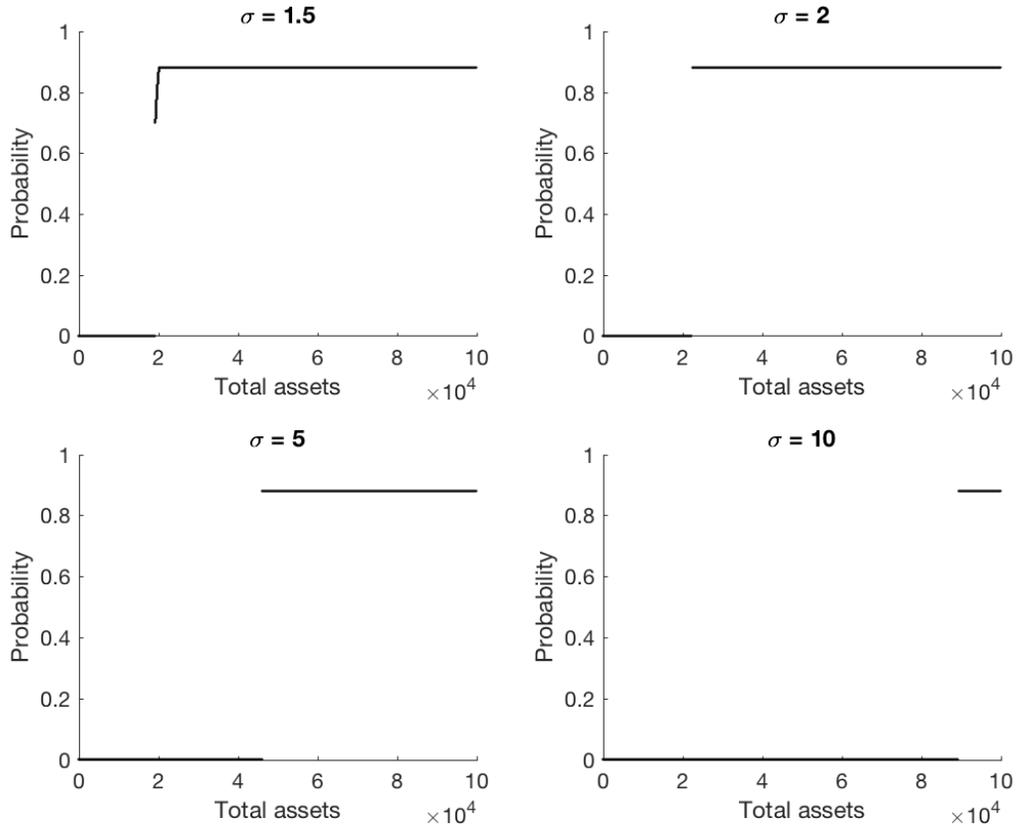
Figure B.2: $H = 62000$, $L = 0$, $\underline{p} = 0$, $\bar{p} = 0.88$

B.3 Some Observations from the Data

As described in Appendix A, we take the investments in projects, μ, probability of success, p, wealth, B and the high return, H from the data.[19] We divide the wealth distribution into 30 equal groups. For each group, i we calculate the average B_i, p_i and μ_i and treat this observation as the one describing a representative agent of the corresponding group.

Observation 1. μ increases in B.

Our first observation is that the data is consistent with the result that richer agents make larger investments. Figure B.3 shows a positive correlation between wealth B and investment μ. We run a regression of a quadratic from to allow for flexibility: $\mu_i = \beta_0 + \beta_1 B_i + \beta_2 B_i^2 + \epsilon_i$. The estimation yields $\hat{\beta}_1 = 0.039$ (significant at the 5% level), and $\beta_2 = -0.00$ (insignificant). This implies positive relationship between μ and B.

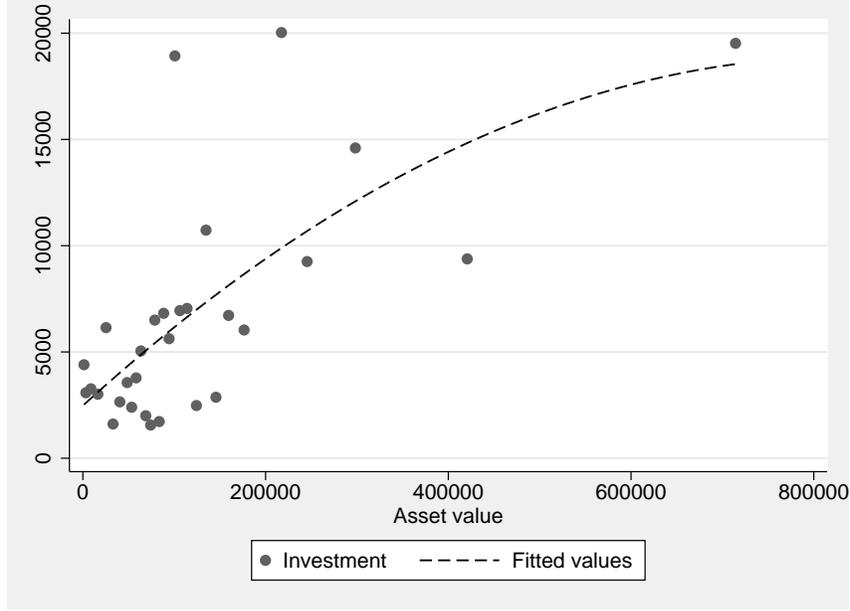[19]We will keep the assumption that $L = 0$.

Figure B.3: Investment (vertical axis) vs. wealth (horizontal axis).

Although this result is not unique to our model but also true in the exogenous probability model among other ones, it is reassuring to find that richer agents make larger investments.

Observation 2. p increases in μ.
One clear distinction between our model and the standard one is that we assume that agents invest, at least partially, in order to increase their probability of success. We check whether there exists any correlation between p and μ in the data. Here we deviate from our model's assumption of linear relationship between the probability and the investment, and assume a general relationship of the form $p = \underline{p} + \gamma\mu^\beta$, which allows for flexibility as β and γ can take any values.[20] We, thus, estimate the empirical specification:

$$\ln(p_i - \underline{p}) = \beta_0 + \beta_1 \ln(\mu_i) + \epsilon_i.^{[21]} \tag{B.2}$$

[20]Notice that this functional form implies that $\ln(p - \underline{p}) = \ln(\gamma) + \beta \ln(\mu)$. We take $\underline{p} = 0.22$ from the data as explained in Appendix A.

[21]As can be seen from Figure B.3 the data on μ is very noisy. We, thus, use the fitted value of μ_i from Observation 1. above.
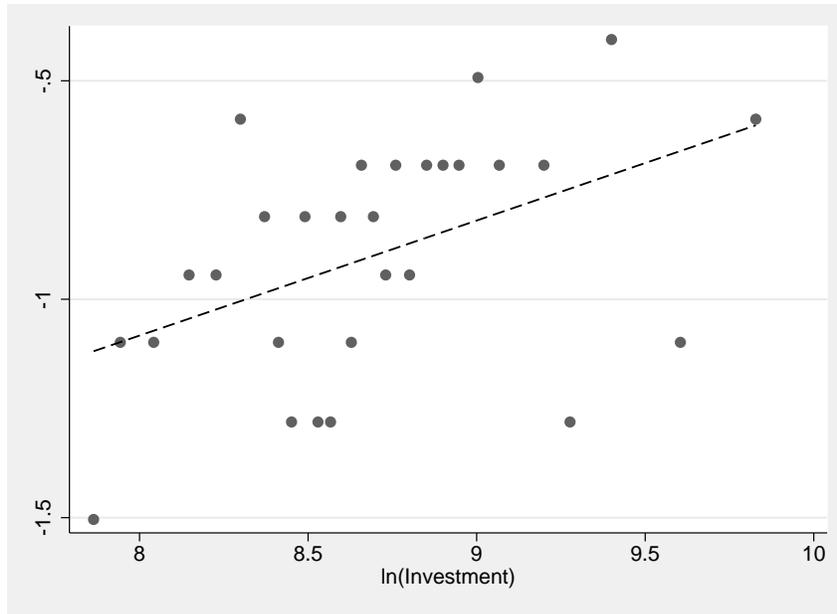
Figure B.4: $\ln(p_i - \underline{p})$ (vertical axis) vs. investment (horizontal axis)

Figure B.4 shows the data and the fitted line. Interestingly, we get that $\hat{\beta}_1 = 0.26$ (which is significant at the 5% level) and $\hat{\beta}_0 = -3.18$ (significant at the 1% level), which implies $\gamma = 0.04$. Thus, the relationship between p and μ is increasing and concave. This result stand in contrast to the exogenous probability model and support our endogenous probability framework.

Observation 3. p as a function of B.

From the two observations above it is clear that p increases with wealth. The dots in Figure B.5 illustrate the actual probability for each level of wealth given in log scale across the 30 wealth groups.
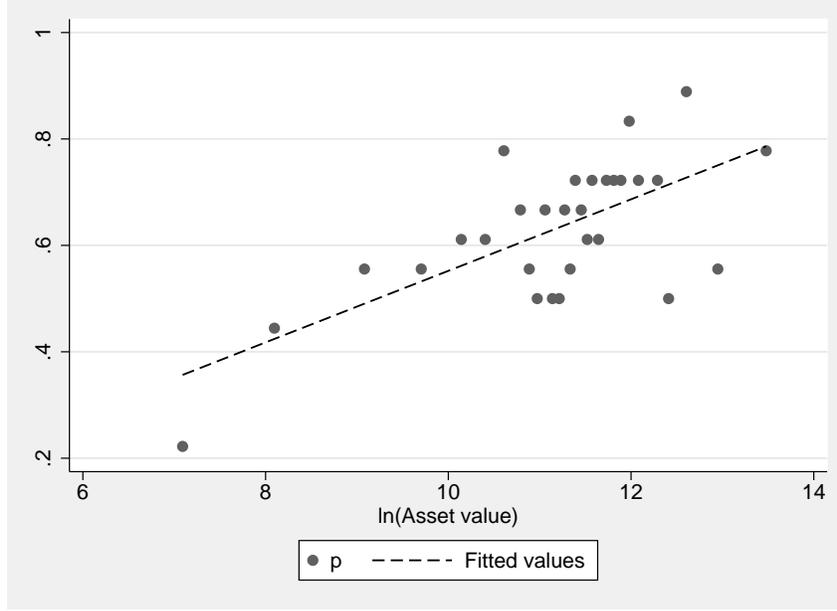
Figure B.5: p_i (vertical axis) vs. $\ln(B_i)$ (horizontal axis)

Figure B.5 verifies the positive correlation between p and B. We estimate

$$p_i = \beta_0 + \beta_1 \ln(B_i) + \epsilon_i \tag{B.3}$$

and get $\hat{\beta}_1 = 0.067$, which is significant at the 1% level.

B.4 Estimating the relative risk aversion parameter

In this subsection we discuss the estimation procedure of the relative risk aversion parameter, σ, and the fit of the model. In our model agents choose the size of investment μ. Given the structural relationship between p and μ, this pins down the probability of success. We, thus, match the probabilities chosen by agents that possess different levels of total assets in the model, with the probabilities observed in the data.

Our goal is to find out whether the model is capable of fitting the data on probabilities given a reasonable value of the relative risk aversion parameter.

It is important to notice that we match only individuals who made positive investments as we observe only them. Thus, in this quantitative exercise we won't capture the discontinuity in investment, but rather find that positive investment choices, according to our model, are consistent with reasonable values of the relative risk aversion.[22]

One necessary assumption that we make is that agents are homogeneous in all characteristics except for the level of the total assets. We feed in exogenously the value of the

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[22]Thus, in this estimation procedure, we are targeting the range, which is qualitatively consistent with $1.93 < B$ in our numerical example described in section B.1.

good outcome to be the average in the data, $H = 61922.77$, and assume that the bad outcome is $L = 0$. We also use the relationship $p(\mu)$ estimated in equation B.2 and set $\bar{p} = 0.8$ from the data.[23] We then calculate the optimal probability by maximizing the expected utility given in equation B.1.[24]

Formally, we pick parameters to minimize the following loss function:

$$\hat{\theta} = \underset{\theta}{\mathrm{argmin}} \quad \sum_{i=1}^{30} \left(1 - \frac{p(\theta, \bar{\theta}, B_i)}{\hat{p}_i} \right)^2$$

where $\hat{p}_i$ is the probability from the data,[25] $p(\theta, \bar{\theta}, B_i)$ is the model-predicted probability; $\bar{\theta}$ is the vector of calibrated parameters ($\bar{\theta} = [H, L, \gamma, \beta, \underline{p}, \bar{p}]$), and $\hat{\theta}$ is the vector of estimated parameters (consists of σ only in this case). $p(\theta, \bar{\theta}, B_i)$ is obtained from the maximization of the expected utility given B_i and the values of the parameters.

Note that the loss function we are using is an inverse probability-weighted distance, where a larger weight is assigned to lower probability:

$$\sum_{i=1}^{30} \frac{1}{\hat{p}_i^2} \left(\hat{p}_i - p(\theta, \bar{\theta}, B_i) \right)^2$$

This is a natural way to assign larger weights to the low-income groups of population, given our previous observation of the positive relation between the probability of the good outcome and total assets. Also note that in the dataset that we are using all the agents invested $\mu > 0$. Thus, we are matching the increasing segment of $p(B)$ within the range $1.92 < B$ (see Figure B.1).

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[23]The maximum value of p in the data is 0.88, which corresponds to the 28th wealth group. the value of p for the 10th wealth group is 0.7.

[24]As discussed above, the estimates of γ and β are inferred from the regression $\ln(p_i - \underline{p}) = \beta_0 + \beta_1 \ln(\mu_i) + \epsilon_i$ are $\hat{\beta} = 0.26$ and $\hat{\gamma} = 0.04$.

[25]Since the data on p is noisy, we smooth the data and target the fitted value from regression B.3, $\hat{p}_i$.
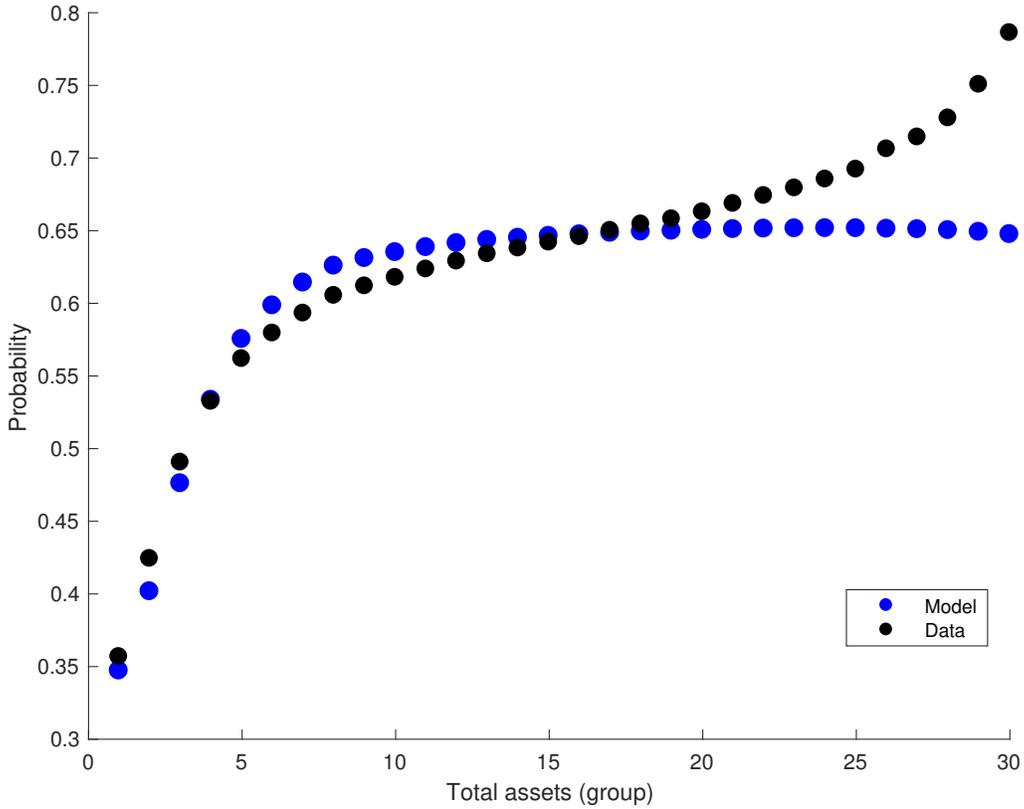
Figure B.6: Probabilities from the data and from the model for estimated $\hat{\sigma}$.

Figure B.6 illustrates how the model can match the data from Augsburg et al. (2015) described above. As can be seen from the figure, the fit is very good although we are missing the upper tail of the wealth distribution.

We obtain an estimate $\hat{\sigma} = 1.82$. The macroeconomic literature usually considers the values of the RRA parameter below or equal to 10 following the discussion provided in Mehra and Prescott (1985) (e.g. the baseline results of Bansal and Yaron (2004) are obtained for RRA = 10). Weil (1989) points out that the parameter is likely to be closer to 1, according to numerous studies. Thus, our estimate is consistent with the literature.[26]

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[26]To make sure that the estimate of σ we obtain corresponds to the global maximum, we used different initial points for the minimization algorithm, and we chose the result with the lowest value of the loss function. For robustness we check whether the estimates change significantly if we use the regular Euclidean distance as a loss function, and a weighted sum of these two loss functions. The obtained estimates do not differ significantly from the baseline 1.82 and do not exceed 2.