Wealth Dynamics in Communities

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Abstract

This paper develops a model to explore how favor exchange in communities influences wealth dynamics. We identify a key obstacle to wealth accumulation: wealth crowds out favor exchange. Therefore, low-wealth households are forced to choose between growing their wealth and accessing favor exchange within their communities. The outcome is that some communities are left behind, with wealth disparities that persist and sometimes even grow worse. Using numerical simulations, we show that place-based policies encourage both favor exchange and wealth accumulation and so have the potential to especially benefit such communities.

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1 Introduction

Rising economic tides do not lift all households equally. Many economies include communities that are left behind, with persistently lower wealth than surrounding areas (*The Economist* 2017, Austin et al. 2018). Such left-behind communities can be found in both rich and developing countries, and in both rural and urban areas.[1]

Faced with limited wealth, members of left-behind communities rely on one another for practical support. Community members engage in all kinds of favor exchange, from the trade of food, lodging, and childcare within poor neighborhoods in Milwaukee (Desmond 2012, 2016) and within immigrant groups in Miami (Portes and Sensenbrenner 1993), to the exchange of rice and kerosene among villagers in India (Jackson et al. 2012). However, in sharp contrast to its role in improving consumption, community support plays only a limited role in improving wealth (Stack 1975). Households struggle to "get by" (Warren et al. 2001), even in the presence of high-return savings opportunities like paying off short-term debts (Ananth et al. 2007, Stegman 2007, Mel et al. 2008). Given these opportunities, why doesn't community support translate into growing wealth?

This paper develops a model to explore how favor exchange shapes wealth accumulation. We show that wealth crowds out favor exchange. Therefore, to access favor exchange, households must keep their wealth artificially low. The outcome is a persistent wealth gap between left-behind communities and the rest of the economy. This result explains why community support need not translate into growing wealth, and in fact, may preclude it. In doing so, our framework sheds light on policies to help left-behind communities.

The foundation of our analysis is a model that introduces wealth dynamics into favor-exchange relationships. Households face a standard consumption-saving problem, with the twist that consumption can be "purchased" using not only money but also promises of future favors. Using this model, we show that wealth undermines the trust that is essential for favor exchange. The reason is that, even after losing access to favor exchange, households can use money to buy consumption. Wealthier households therefore have less to lose from not reciprocating favors; namely, they have better outside options to favor exchange. Recognizing this, community members are less willing to exchange favors with households that are either wealthy or expected to become so in the future. In short, households that accumulate wealth become "too big for their boots" and lose access to community support.

Our main result identifies how this "too-big-for-their-boots" effect shapes wealth dynamics. For low-wealth households, reliance on favor exchange results in sharply limited saving,

[1] Desmond [2012] and Hendrickson et al. [2018] document left-behind communities in rich countries. Hoff and Sen [2006], Jakiela and Ozier [2016], and Munshi and Rosenzweig [2016] document examples in developing countries.

and sometimes even strictly decreasing wealth. Wealthier households, in contrast, opt out of favor exchange and accumulate wealth until they exhaust all positive-return savings opportunities. Thus, rather than disappearing, initial wealth disparities persist and can even grow worse.

Anything that exacerbates the "too-big-for-their-boots" effect undermines both favor exchange and wealth accumulation within communities. For instance, over the past two decades, high-productivity metropolitan areas in the United States have seen further productivity gains relative to the rest of the country (Parilla and Muro 2017). In simulations, we model these gains as increasing consumption utility outside of a left-behind community. The consequence is that more households leave, while those that remain face better outside options, accumulate less wealth, and have substantially lower welfare. In this sense, and consistent with evidence in Alesina and Ferrara [2000, 2002], inequality can strain social relationships.

Public policies can help left-behind communities by counteracting the "too-big-for-their-boots" effect. Recently, researchers have emphasized an important role for "place-based" policies, which provide benefits that are localized to a community (Busso et al. 2013, Austin et al. 2018, Bartik 2020). We simulate "place-based" policies as increasing the payoff from staying in the community. We show that these polices mitigate the "too-big-for-their-boots" effect and so *simultaneously* encourage favor exchange and saving. In contrast, "mobility-based" policies, which increase the payoff from *leaving* the community, can have the unintended consequence of disrupting favor exchange and discouraging saving among households that remain.

Our analysis resonates with economic and ethnographic evidence on favor exchange in left-behind communities. Support for the "too-big-for-their-boots" effect dates back at least to Stack [1975]'s classic study of favor exchange in a low-wealth U.S. community. Stack [1975, p43] notes that the *wealthiest* members of the community are most at risk of being excluded from favor exchange:

> As people say, "The poorer you are, the more likely you are to pay back." This criterion often determines which kin and friends are actively recruited into exchange networks.

To explain why wealthier households are excluded, Stack [1975] suggests that everyone knows that these households can more easily leave the community and move to a nearby city. Briggs [1998] echoes this tension between favor exchange and mobility. We show how this tension impacts wealth accumulation. In doing so, we explain empirical studies that find limited long-term effects from one-time transfers (Ananth et al. 2007, Karlan et al. 2019, Balboni et al. 2020): such transfers are consumed rather than saved to avoid the "too-big-for-their-boots" effect.

The contribution of this paper is to study how favor exchange within communities affects wealth accumulation. Much of the work on favor exchange examines how community enforcement encourages cooperation (e.g., Kranton 1996, Hauser and Hopenhayn 2008, Jackson et al. 2012, Wolitzky 2013, Ambrus et al. 2014, Ali and Miller 2016 and 2020, Miller and Tan 2018). Our key departure from this literature is to study wealth dynamics in these relationships.

Our model draws from the literature on relational contracting (e.g., Macaulay 1963, Bull 1987, Levin 2003, Malcomson 2013). We contribute to this literature by introducing wealth as an endogenous variable and by modeling market exchange as an endogenous outside option to favor exchange. This latter contribution builds on papers that study the role of outside options in relationships (e.g., Baker et al. 1994, Kovrijnykh 2013), as well as papers that study how favor and market exchange co-exist (Kranton 1996, Banerjee and Newman 1998, Gagnon and Goyal 2017, Banerjee et al. 2020, Jackson and Xing 2020).

In our model, saving can be reinterpreted as investing in capital, so our results speak to the literature on underinvestment. This literature has argued that underinvestment can result from fixed costs of investment (e.g., Nelson 1956, Advani 2019), monopolistic credit markets (e.g., Mookherjee and Ray 2002, Liu and Roth 2020), or behavioral preferences (e.g., Banerjee and Mullainathan 2010, Bernheim et al. 2015). We share individual modeling ingredients with several of these papers; for instance, Advani [2019] considers favor exchange, but in the context of fixed-cost investments, while Liu and Roth [2020] considers the role of outside options, but without focusing on favor exchange and with a monopolistic credit market. We depart from these explanations by combining favor exchange with investments that do not have fixed costs. This combination reveals a hidden cost of investment, which is that wealth undermines favor exchange. This cost leads to persistently low wealth, even in the absence of other frictions like fixed costs, monopolistic credit markets, or behavioral preferences.

2 Model

A long-lived **household** ("it") has initial wealth $w_0 \geqslant 0$ and discount factor $\delta \in (0, 1)$. The household starts in the community. At the beginning of each period $t \in \{0, 1, ...\}$, if the household still lives in the community, it can choose to either stay or move to a city. Once it moves to the city, it remains there forever.

If the household is in the community in period t, it plays the following **community**

game with a short-lived **neighbor** t ("she"), who is another member of the community:

1. The household requests a consumption level $c_t \geqslant 0$ and offers a payment $p_t \geqslant 0$ in exchange. The payment cannot exceed the household's wealth, $p_t \leqslant w_t$.

2. Neighbor t either accepts or rejects this exchange, $d_t \in \{1, 0\}$. If she accepts ($d_t = 1$), she receives p_t and incurs the cost of providing c_t. If she rejects ($d_t = 0$), no trade occurs.

3. The household decides how much of a favor, $f_t \geqslant 0$, to perform for neighbor t. The household incurs the cost of providing f_t.

4. The household invests its remaining wealth, $w_t - p_t d_t$, to generate w_{t+1}. Let $R(\cdot)$ give the return on investment, so that $w_{t+1} = R(w_t - p_t d_t)$.

Let $U(\cdot)$ be the household's consumption utility in the community. The household's period-t payoff is $\pi_t = U(c_t d_t) - f_t$. Neighbor t's payoff is $(p_t - c_t)\, d_t + f_t$. The community is tight-knit and so all actions are observed by all neighbors.

We assume that consumption utility $U(\cdot)$ and investment returns $R(\cdot)$ are strictly increasing, with $U''(\cdot)$ and $R'(\cdot)$ continuous, $U(0) = R(0) = 0$, $U(\cdot)$ strictly concave, $R(\cdot)$ concave, $\lim_{c \downarrow 0} U'(c) = \infty$, and $\lim_{c \to \infty} U'(c) = 0$. We say that investment *generates positive returns* at wealth w if $R'(w) \geqslant \frac{1}{\delta}$. We assume that $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$ and $R'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}$, so that investment generates positive returns at all wealth levels and strictly so below a threshold $\bar{w} > 0$.

If the household has moved to the city by period t, then it plays the **city game** with a short-lived **vendor** t ("she"), who has the same actions and payoff as neighbor t. The city game is identical to the community game in all but two ways. First, each vendor observes only her own interaction with the household, so that interactions are anonymous in the city. Second, the household's marginal utility of consumption is weakly higher in the city. Formally, the household's period-t payoff in the city is $\pi_t = \hat{U}(c_t d_t) - f_t$, where $\hat{U}(\cdot)$ satisfies the same regularity conditions as $U(\cdot)$, with $\hat{U}'(c) \geqslant U'(c)$ for all $c > 0$.

The household's continuation payoff at the beginning of period t is

$$\Pi_t \equiv (1 - \delta) \sum_{s=t}^{\infty} \delta^{s-t} \pi_s.$$

We characterize household-optimal equilibria, which are the perfect Bayesian equilibria that maximize the household's *ex ante* expected payoff. Without loss of generality, we assume that the household leaves the community if it is indifferent between staying and leaving.

The following assumption ensures that in equilibrium, households that stay in the community have access to strictly positive-return investments.

Assumption 1 *Define $\bar{c} > 0$ as the solution to $U'(\bar{c}) = 1$. Then, $R(\bar{w} - \bar{c}) > \bar{w}$.*

In the context of the low-income Midwestern community described in Stack [1975], the household and neighbors are members of a community called "the Flats." Members regularly exchange food, clothing, childcare, and other goods and services (c_t). To compensate one another, households can pay with money (p_t) and "pay" with future favors (f_t). For example, the recipient of childcare ($c_t > 0$) can reciprocate with future childcare ($f_t > 0$). As in our model, while these favors are costly to the provider, they are in-kind and so do not require money. Households accumulate wealth ($R(\cdot)$) by repaying high-interest debt or making other investments. The Flats is a tight-knit community and "everyone knows who is working, when welfare checks arrive, and when additional resources are available" (p. 37), as well as who has reneged on promised favors. Households in the Flats can move to a nearby city, Chicago, which harbors greater opportunities ($\hat{U}' \geqslant U'$) but separates them from their favor-exchange network.

In online appendix B, we show that our main findings are robust to relaxing either of the following two assumptions. First, rather than assuming that moving to the city is irreversible, we could allow the household to return to the community after leaving. Second, we could allow the household's cost of providing favor f_t to be convex rather than linear.

In our model, the city is an alternative to the community. We include the city to match our applications, which typically include mobility across locations, and to compare the effects of place-based and mobility-based policies. However, similar wealth dynamics would arise without a city; wealth would still crowd out favor exchange, because even within the community, wealthier households rely less on favor exchange. In particular, our main result would hold if we eliminated the city and instead assumed that any deviation was punished by reversion to a Markov perfect equilibrium in the community.

3 Life in the City

We first characterize wealth dynamics in the city. In the city, the household faces a standard consumption-saving problem. It takes full advantage of investment opportunities and accumulates wealth.

Interactions are anonymous in the city, so $f_t = 0$ in equilibrium. Vendor t is therefore willing to accept an offer only if the payment covers her costs (i.e., $p_t \geqslant c_t$), and strictly prefers to do so if $p_t > c_t$. Consequently, every equilibrium entails $p_t = c_t$ in every $t \geqslant 0$, so that $w_{t+1} = R(w_t - c_t)$. For a household with wealth w, the resulting optimal payoff and consumption are given, respectively, by:

$$\hat{\Pi}(w) = \max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big) \text{ and}$$
$$\hat{C}(w) \in \arg\max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big).$$

Our first result shows that $\hat{\Pi}(w)$ and $\hat{C}(w)$ are the unique equilibrium outcome in the city.

Proposition 1 *Both $\hat{\Pi}(\cdot)$ and $\hat{C}(\cdot)$ are strictly increasing, with $\hat{\Pi}(\cdot)$ continuous. In any equilibrium, $\Pi_t = \hat{\Pi}(w_t)$ and $c_t = \hat{C}(w_t)$ in any $t \geqslant 0$, with $(w_t)_{t=0}^{\infty}$ increasing and*

$$\lim_{t\to\infty} w_t > \bar{w}$$

on the equilibrium path.

The proof of Proposition 1 is routine and relegated to online appendix A. Household consumption and wealth increase over time. Since $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, the standard Euler equation,

$$\hat{U}'(\hat{C}(w_t)) = \delta R'(w_t - \hat{C}(w_t))\hat{U}'(\hat{C}(w_{t+1})),\ \forall\, t, \tag{Euler}$$

implies that the household's long-term wealth is strictly above $\bar{w}$ in the city. Figure 1 simulates equilibrium outcomes in the city.[2]

4 Household-Optimal Wealth Dynamics

We now characterize household-optimal equilibria for a household starting in the community. Section 4.1 states our main result and discusses its intuition. Section 4.2 explores its implications with numerical comparative statics. Section 4.3 gives the proof.

[2]Parameters in Figure 1 to 4 are $\delta = \frac{8}{10}, U(c) = \frac{\sqrt{c}}{2}, \hat{U}(c) = \frac{13\sqrt{c}}{25}$, and

$$R(w) = \begin{cases} 3\left(\sqrt{w+1}-1\right) & w \leqslant \frac{11}{25}; \\ \frac{5w}{4} + \frac{1}{20} & \text{otherwise.} \end{cases}$$

In Figure 3, the higher $R(w)$ is

$$R(w) = \begin{cases} \frac{31}{10}\left(\sqrt{w+1}-1\right) & w \leqslant \frac{336}{625}; \\ \frac{5w}{4} + \frac{9}{125} & \text{otherwise;} \end{cases}$$

and the higher $\hat{U}(c)$ is $\frac{27\sqrt{c}}{50}$. In Figure 4, we augment the per-period payoff by 1/250 in the community, the city, or both locations.

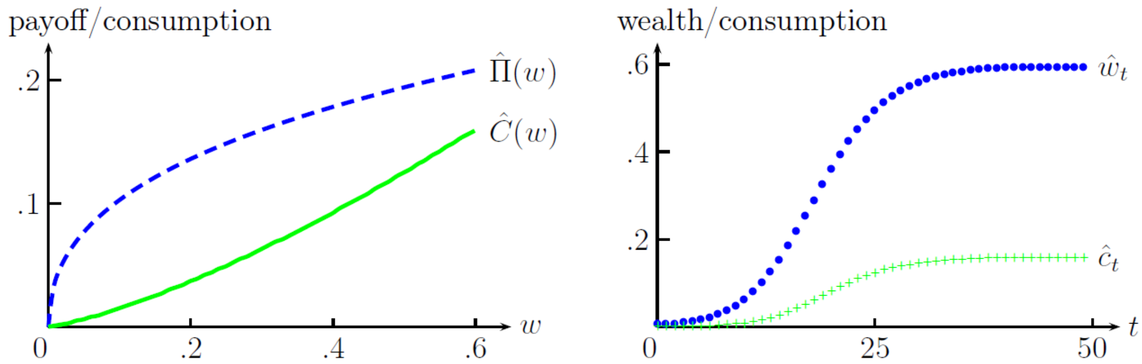

Figure 1: Left panel: the household's equilibrium payoff and consumption as a function of w. Right panel: consumption and wealth over time, starting at $w_0 = 0.006$.

4.1 Life Starting in the Community

Our main result identifies two reasons why wealth in the community remains substantially below wealth in the city. First, there is a *selection margin:* sufficiently wealthy households leave the community, whereas poorer households remain. Second, there is a *treatment effect:* the "too-big-for-their-boots" effect constitutes an extra cost of investment for households in the community, resulting in sharply limited long-term wealth.

To understand this result, note that the community's sole advantage over the city is that neighbors can observe and punish a household which reneges on $f_t > 0$. Consequently, the household can credibly promise $f_t > 0$ to repay neighbor t for providing a consumption level c_t that strictly exceeds the payment p_t. Favor exchange can therefore augment consumption only within the community.

The opportunity to engage in favor exchange is most attractive to low-wealth households, which would otherwise have low consumption and a high marginal utility of consumption. Conversely, wealthy households already consume a lot, so their marginal utility from further increasing c_t is low. Thus, wealthy households leave the community while low-wealth households stay, leading to our selection margin.

To understand the treatment effect, consider a household with wealth w_t that stays in the community, consumes c_t, and invests $I_t \equiv w_t - p_t$. As in the standard Euler equation (Euler), increasing investment I_t allows for higher future consumption, which has marginal benefit $U'(c_{t+1})R'(I_t)$, at the cost of lower current consumption, which has marginal cost $U'(c_t)$.

In the community, this familiar trade-off is augmented by a second, indirect cost of investment. Since the household can renege on f_t, leave the community, and earn $\hat{\Pi}(R(I_t))$ in the city, the *maximum* favor that can be sustained in equilibrium depends on I_t. Denote a household's maximum continuation payoff if it starts in the community and has wealth $R(I_t)$ by $\Pi^*(R(I_t))$. Then f_t must satisfy the following **dynamic enforcement constraint***:*

$$f_t \leqslant \bar{f}(I_t) \equiv \frac{\delta}{1-\delta}(\Pi^*(R(I_t)) - \hat{\Pi}(R(I_t))). \tag{DE}$$

This constraint ensures that the household prefers to do favor f_t and earn continuation payoff $\Pi^*(R(I_t))$, rather than reneging on f_t and earning punishment payoff $\hat{\Pi}(R(I_t))$. If (DE) binds, so $f_t = \bar{f}(I_t)$, then changing I_t affects the size of the favor, which affects period-t consumption because $c_t = p_t + f_t$. Assuming that $\bar{f}(\cdot)$ is differentiable and that (DE) binds, we can write the *indirect* marginal cost of investment as $-\bar{f}'(I_t)\,(U'(c_t) - 1)$. The first term, $\bar{f}'(I_t)$, represents how changing investment I_t affects the favor f_t, and the second term, $(U'(c_t) - 1)$, represents how changing f_t affects the household's period-t payoff, $U(c_t) - f_t$.

In a household-optimal equilibrium, the marginal costs of investment must equal the marginal benefit, resulting in the following *modified Euler equation:*

$$U'(c_t) - \bar{f}'(I_t)\,(U'(c_t) - 1) = \delta R'(I_t) U'(c_{t+1}). \tag{modEuler}$$

The "too-big-for-their-boots" effect holds whenever $\bar{f}'(I_t) < 0$, so that investment crowds out favor exchange. We will show that $U'(c_t) - 1 > 0$ for any household in the community. Consequently, whenever the "too-big-for-their-boots" effect holds, the investment that satisfies (modEuler) is strictly below the investment that satisfies (Euler). This is the sense in which a household in the community underinvests.

The intuition provided above elides a key complication: $\bar{f}(\cdot)$ depends on both $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$, which in turn depend on the household's future decisions about consumption, favor exchange, and investment. Wealth affects all of these decisions, rendering a full characterization of household-optimal equilibria intractable.

Our main result, Proposition 2, focuses on the selection and treatment effects. *Selection* is summarized by a wealth threshold, $w^{se} < \bar{w}$, and a set $\mathcal{W} \subseteq [0, w^{se}]$. The household stays forever if $w_0 \in \mathcal{W}$ and leaves the community otherwise. *Treatment* is summarized by a wealth level, $w^{tr} < w^{se}$, such that the long-term wealth of a household that stays is below w^{tr}.

Proposition 2 *Impose Assumption 1. There exist wealth levels* $w^{tr} < w^{se} \in (0, \bar{w})$ *and a positive-measure set* $\mathcal{W} \subseteq [0, w^{se}]$ *with* $\sup \mathcal{W} = w^{se}$ *such that in any household-optimal equilibrium:*

1. ***Selection.*** *The household stays in the community forever if* $w_0 \in \mathcal{W}$*, and otherwise leaves in period* 0.

2. ***Treatment.*** *If the household stays in the community, then* $(w_t)_{t=0}^{\infty}$ *is monotone, with*

$$\lim_{t\to\infty} w_t \leqslant w^{tr}.$$

Moreover, $\mathcal{W} \cap [w^{tr}, w^{se}]$ *has a positive measure.*

Section 4.3 gives the proof. We have already argued that wealthy households leave the community, giving us the selection threshold w^{se}, while some poorer households stay, giving us the set $\mathcal{W}$. Moreover, any household that stays in $t = 0$ stays forever. A household that leaves in period $t > 0$ cannot engage in favor exchange during period $t - 1$, so it might as well leave in period $t - 1$. Iterating this argument, such a household might as well leave in period 0.

To understand why long-term wealth in the community is below w^{tr}, consider a household that stays with wealth *just below* w^{se}. Such a household is close to indifferent between leaving and staying. Thus, if the household's wealth always remains near w^{se}, then the right-hand side of (DE) is close to zero and $f_t \approx 0$ in every $t \geqslant 0$. Since staying implies that wealth remains below w^{se} and $w^{se} < \bar{w}$, it is optimal only if $f_t \gg 0$ in some t. Thus, this household prefers to stay only if it underinvests so severely that its wealth decreases. The proof of Proposition 2 strengthens this result by showing that $(w_t)_{t=0}^{\infty}$ is monotone and that a positive measure of households, $\mathcal{W} \cap [w^{tr}, w^{se}]$, stay in the community. These households have decreasing wealth, with long-term wealth below w^{tr}.

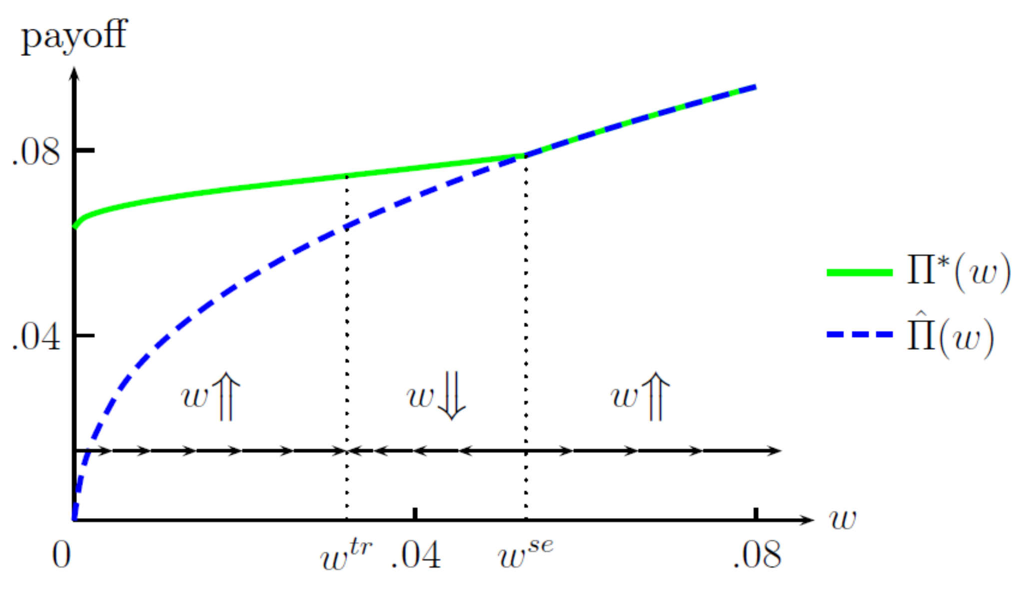

Figure 2: Simulated household-optimal equilibrium payoffs and wealth dynamics.

Figure 2 summarizes Proposition 2. In this simulation, the household moves to the city if

$w_0 \geqslant w^{se}$ and otherwise stays. Among those that stay, households with $w_0 \leqslant w^{tr}$ grow their wealth, but only up to w^{tr}. Those with $w_0 \in (w^{tr}, w^{se})$ have declining wealth over time. One empirical implication of this result is that within the community, underinvestment is most severe for the wealthiest households. Intuitively, wealthier households have better outside options from moving to the city and so face tighter dynamic enforcement constraints.

A second implication is that one-time transfers need not improve long-term wealth. Consider a one-time transfer to a household with initial wealth $w_0 < w^{se}$. If the household's post-transfer wealth is still below w^{se}, then long-term wealth remains below w^{tr}. This implication resonates with Karlan et al. [2019], which finds that temporary debt relief tends not to improve long-term solvency. In contrast, a transfer that brings wealth above w^{se} induces further investment, but only by spurring the household to leave the community.

4.2 Numerical Comparative Statics

This section explores how changes in the economic context impact wealth and welfare.

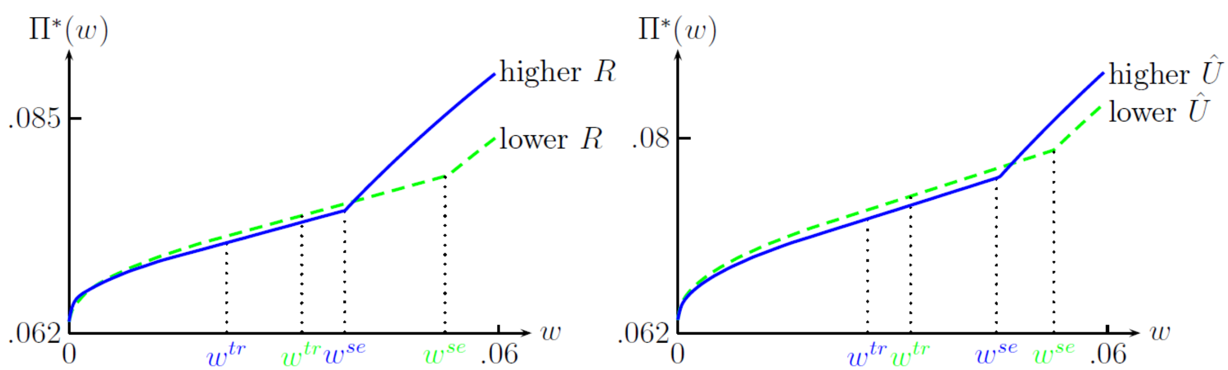

Figure 3: Simulated comparative statics with respect to $R(\cdot)$ and $\hat{U}(\cdot)$.

The left panel of Figure 3 illustrates the effect of an increase in investment returns, $R(\cdot)$. In the city, a household benefits from higher $R(\cdot)$ regardless of its initial wealth. In the community, increasing $R(\cdot)$ increases the household's payoff from leaving, which tightens the dynamic enforcement constraint, (DE), and so undermines favor exchange in the community. Among households that stay, this negative impact is larger for wealthier households, since they face a tighter (DE). It can even outweigh the ability to grow wealth faster. Thus, while poorer members of the community benefit from an increase in $R(\cdot)$, wealthier members can be harmed and long-term wealth in the community can decrease.

The right panel of Figure 3 considers increasing productivity in the city, which we model by increasing $\hat{U}(\cdot)$. Over the past two decades, productivity has diverged across locations in the United States, with the most productive metropolitan areas growing even more productive relative to the rest (Parilla and Muro 2017). While nothing material has changed within the community, increased productivity in the city increases outside options and so tightens (DE). Hence, it can disrupt favor exchange, decrease long-term wealth, and decrease welfare in the community. By the same logic, the "too-big-for-their-boots" effect is less severe for households with worse outside options. In their study of the Dominican immigrant community in New York City, Portes and Sensenbrenner [1993] suggest that one reason this community was able to accumulate wealth was that they faced limited outside opportunities. Figure 3 shows precisely how worse outside options can lead to higher long-term wealth in the community.

4.3 The Proof of Proposition 2

Let $\Pi^*(w)$ be the maximum equilibrium payoff of a household with wealth w. Define

$$\begin{aligned}\Pi_c(w) \equiv \quad & \max_{c\geqslant 0, f\geqslant 0} \quad \{(1-\delta)(U(c)-f)+\delta\Pi^*(R(w+f-c))\} \\ & \text{s.t.} \qquad 0 \leqslant c-f \leqslant w \qquad (1) \\ & \qquad\qquad f \leqslant \frac{\delta}{1-\delta}\Big(\Pi^*(R(w+f-c))-\hat{\Pi}(R(w+f-c))\Big). \qquad (2)\end{aligned}$$

We show that $\Pi_c(w)$ is the household's maximum payoff conditional on staying in the community in the current period. Hence, the household's maximum equilibrium payoff, $\Pi^*(w)$, is the maximum of $\hat{\Pi}(w)$ and $\Pi_c(w)$.

Lemma 1 *The household's maximum equilibrium payoff is* $\Pi^*(w_0) = \max\left\{\hat{\Pi}(w_0), \Pi_c(w_0)\right\}$, *where* $\Pi_c(\cdot)$ *and* $\Pi^*(\cdot)$ *are strictly increasing.*

Proof of Lemma 1: In any equilibrium, neighbor 0 accepts only if $c_0 \leqslant p_0 + f_0$. The household's continuation payoff is at most $\Pi^*(R(w_0-p_0))$ and at least $\hat{\Pi}(R(w_0-p_0))$. Hence, it is willing to do favor f_0 only if

$$f_0 \leqslant \frac{\delta}{1-\delta}\Big(\Pi^*(R(w_0-p_0))-\hat{\Pi}(R(w_0-p_0))\Big).$$

Setting $c_0 = p_0 + f_0$ yields $\Pi_c(w_0)$ as an upper bound on the household's payoff from staying.

This bound is tight. For any (c_0, f_0) that satisfies (1) and (2), it is an equilibrium to set $p_0 = c_0 - f_0 \geqslant 0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the household leaving and $f_t = 0$ in all future periods.[3] Thus,

[3] If $f_t = 0$ in all $t \geq 0$, then the household is willing to leave because $U \leq \hat{U}$.

$\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying. It follows that $\Pi^*(w) = \max\{\hat{\Pi}(w), \Pi_c(w)\}$. Since $\Pi_c(\cdot)$ is strictly increasing by inspection and $\hat{\Pi}(\cdot)$ is strictly increasing by Proposition 1, $\Pi^*(\cdot)$ is strictly increasing. $\square$

The next three lemmas characterize household-optimal equilibria in the community. First, we show that households that stay in the community, stay forever.

Lemma 2 *If $w_0 \geqslant 0$ is such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then in any $t \geqslant 0$ of any household-optimal equilibrium, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ on the equilibrium path.*

Proof of Lemma 2: Suppose $t > 0$ is the first period in which $\Pi^*(w_t) = \hat{\Pi}(w_t)$, so $\Pi^*(w_{t-1}) = \Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1})$. Let $\{c_{t-1}, f_{t-1}\}$ achieve $\Pi_c(w_{t-1})$. Since $\Pi^*(w_t) = \hat{\Pi}(w_t)$, (2) implies $f_{t-1} = 0$. Therefore, $\hat{\Pi}(w_{t-1}) \geqslant \Pi_c(w_{t-1})$, since if the household exits in $t-1$, it could choose the same p_{t-1} and c_{t-1} and earn continuation payoff $\hat{\Pi}(w_t) = \Pi^*(w_t)$. This contradicts the presumption that $\Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1})$. $\square$

Second, we show that wealthy households leave the community, while poorer households stay.

Lemma 3 *The set $\mathcal{W} \equiv \left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure. Moreover, $w^{se} \equiv \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ satisfies $0 < w^{se} < \bar{w}$.*

Proof of Lemma 3: First, we show that $\Pi^*(0) > \hat{\Pi}(0) = 0$. Because $\lim_{c\downarrow 0} U'(c) = \infty$, there exists a $c > 0$ such that $c \leqslant \delta U(c)$. Suppose that in all $t \geqslant 0$, $f_t = c_t = c$ and $p_t = 0$ on the equilibrium path, so the household's equilibrium payoff is $U(c) - c$. Any deviation is punished by $f_t = 0$ in all future periods and the household immediately exiting. This strategy delivers a strictly positive payoff. It is an equilibrium because $c \leqslant \delta U(c)$ implies (2). Thus, $\Pi^*(0) > 0$. Since $\Pi^*(w)$ is increasing and $\hat{\Pi}(w)$ is continuous, there exists an interval around 0 such that $\Pi^*(w) > \hat{\Pi}(w)$. So $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure.

Next, we show that $w^{se} < \bar{w}$. Let $\bar{c}$ satisfy $U'(\bar{c}) = 1$, and let w_0 be such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$. By Lemma 2, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ in any $t \geqslant 0$ of any household-optimal equilibrium. Suppose that $c_t > \bar{c}$ in period $t \geqslant 0$. If $f_t > 0$, then we can perturb the equilibrium by decreasing c_t and f_t by $\epsilon > 0$, which increases the household's payoff at rate $1 - U'(c_t) > 0$ as $\epsilon \to 0$. So, $f_t = 0$.

Let $\tau > t$ be the first period after t such that $f_\tau > 0$. Consider decreasing p_t and c_t by $\epsilon > 0$, increasing p_τ by $\chi(\epsilon)$, and decreasing f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geqslant \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geqslant \frac{1}{\delta}$. As $\epsilon \to 0$, this perturbation increases the household's payoff by at least $\delta^{\tau-t}\frac{1}{\delta^{\tau-t}} - U'(c_t) > 0$. It is an equilibrium because $f_s = 0$ for all $s \in [t, \tau-1]$, so (2) still holds in these periods.

The above argument implies that if $c_t > \bar{c}$, then $f_\tau = 0$ for all $\tau \geqslant t$. But then $\Pi^*(w_t) \leqslant \hat{\Pi}(w_t)$, contradicting Lemma 2. Therefore, if $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then $c_t \leqslant \bar{c}$ in every $t \geq 0$ and so $\Pi^*(w_0) \leqslant U(\bar{c})$. Since $R(\bar{w} - \bar{c}) > \bar{w}$ by Assumption 1, it follows that $\Pi^*(\bar{w}) \geqslant \hat{\Pi}(\bar{w}) > \hat{U}(\bar{c}) \geqslant U(\bar{c})$. By the definition of w^{se}, there exists a sequence of initial wealth levels in $\mathcal{W}$ which are arbitrarily close to w^{se} such that the household strictly prefers to stay in the community with those initial wealth levels. If $w^{se} \geqslant \bar{w}$, the equilibrium payoffs at those initial wealth levels would be strictly above $U(\bar{c})$ due to the continuity of $\hat{\Pi}(w)$. This leads to a contradiction, so $w^{se} < \bar{w}$. □

Finally, we show that household-optimal equilibria exhibit monotone wealth dynamics.

Lemma 4 *In any household-optimal equilibrium, $(w_t)_{t=0}^{\infty}$ is monotone.*

The key step of this proof shows that household-optimal investment, $w_t - p_t$, increases in w_t. Thus, if $w_1 \geqslant w_0$, then $w_2 = R(w_1 - p_1) \geqslant R(w_0 - p_0) = w_1$ and so on, and similarly if $w_1 \leqslant w_0$. The proof of Lemma 4 is in appendix A.

We can now prove Proposition 2. **Selection** is implied by Lemma 2 and Lemma 3. For **treatment**, define

$$\tilde{c}(w) = w - R^{-1}(w)$$

and $\tilde{\Pi}(w) = U(\tilde{c}(w))$. Since $w^{se} < \bar{w}$ by Lemma 3, consumption $\tilde{c}(w^{se})$ does not satisfy (Euler). Thus, there exists $K > 0$ such that

$$\tilde{\Pi}(w^{se}) + K < \hat{\Pi}(w^{se}).$$

Define

$$\bar{f}(w) = \frac{\delta}{1-\delta}\left(\hat{\Pi}(w^{se}) - \hat{\Pi}(w)\right)$$

and

$$\bar{p}(w) = w^{se} - R^{-1}(w).$$

Consider $w_0 < w^{se}$ such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$, and suppose that there exists an equilibrium in which $(w_t)_{t=0}^{\infty}$ is increasing on the equilibrium path. We claim that $p_t \leqslant \bar{p}(w_0)$ and $f_t \leqslant \bar{f}(w_0)$ in every $t \geq 0$. Indeed,

$$f_t \leqslant \frac{\delta}{1-\delta}\left(\Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1})\right) \leqslant \frac{\delta}{1-\delta}\left(\Pi^*(w^{se}) - \hat{\Pi}(w_0)\right) = \bar{f}(w_0),$$

and

$$p_t = w_t - R^{-1}(w_{t+1}) \leqslant w^{se} - R^{-1}(w_0) = \bar{p}(w_0),$$

where the inequalities hold because (i) $w_t, w_{t+1} \leqslant w^{se}$ by Lemma 2, and (ii) $w_{t+1} \geq w_0$ by our presumption that $(w_t)_{t=0}^{\infty}$ is increasing.

Since $c_t \leqslant p_t + f_t$, the household's payoff satisfies

$$\Pi^*(w_0) \leqslant U\left(\bar{p}(w_0) + \bar{f}(w_0)\right) \equiv H(w_0).$$

The function $H(w_0)$ is continuous and decreasing in w_0, with $H(w^{se}) = \tilde{\Pi}(w^{se})$. Since $\tilde{\Pi}(w^{se}) + K < \hat{\Pi}(w^{se})$, there exists $w^{tr} < w^{se}$ such that for any $w_0 \in (w^{tr}, w^{se})$,

$$H(w_0) < \hat{\Pi}(w_0).$$

Therefore, for any $w_0 \in (w^{tr}, w^{se})$, if $w_0 \in \mathcal{W}$, then $(w_t)_{t=0}^{\infty}$ must be strictly decreasing, with $\lim_{t\to\infty} w_t \leqslant w^{tr}$.

By definition of w^{se}, there exists $w_0 \in \mathcal{W} \cap (w^{tr}, w^{se})$ such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$. Since $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$ are increasing, with $\hat{\Pi}(\cdot)$ continuous, we conclude that $\Pi^*(w) > \hat{\Pi}(w)$ on a neighborhood around w_0. ■

5 Policy Simulations

This section considers policies to help left-behind communities. Our main takeaway is that **place-based policies**, which include local grants, tax incentives, infrastructure investments, and other policies that benefit those who stay in a community (Busso et al. 2013, Austin et al. 2018, Bartik 2020), mitigate the "too-big-for-their-boots" effect and so facilitate *both* favor exchange and wealth accumulation within the community.

We contrast place-based policies with two alternative approaches. **Income-based policies** provide benefits that depend only on a household's income. Many assistance programs in the United States, including the Temporary Assistance for Needy Families program, are essentially income-based. **Mobility-based policies** encourage households to leave left-behind communities. One example is the "Moving to Opportunity" program in the United States, which provides housing subsidies in higher-opportunity areas (Katz et al. 2001, Chetty et al. 2016). Unlike place-based policies, these alternatives do not mitigate the "too-big-for-their-boots" effect and may even exacerbate it.

Figure 4 presents simulations of these three policy approaches. To facilitate comparison, we model each policy as increasing the household's per-period payoff, where place-, mobility-,

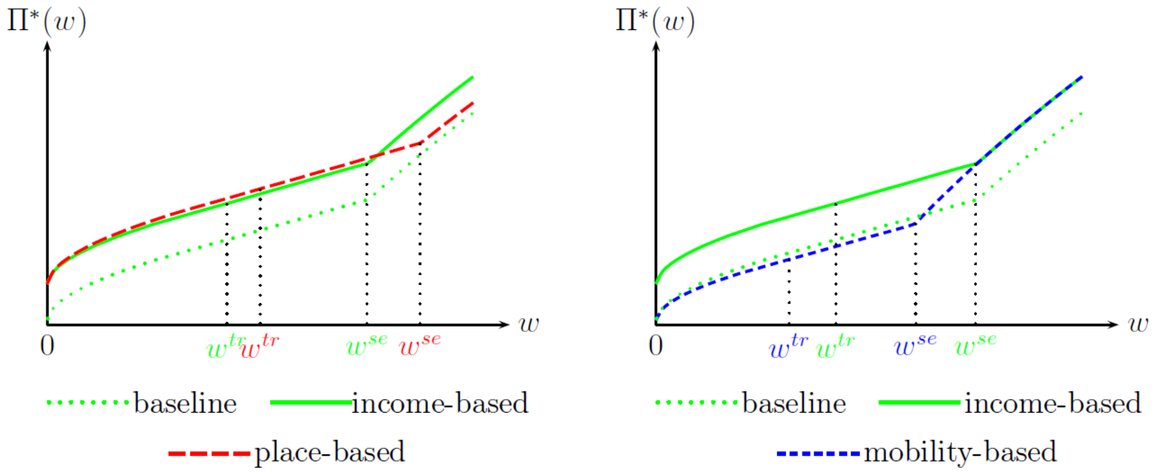

Figure 4: Simulated effects of income-, place-, and mobility-based policies.

and income-based policies increase payoffs in the community, in the city, or in both locations, respectively. These simulations assume that following a deviation, players play a Markov perfect equilibrium.[4]

Income-based policies do not affect the household's relative payoff from the city versus that from the community, resulting in a uniform increase in equilibrium payoffs. Thus, such policies affect neither selection nor treatment effects.

Mobility-based policies increase equilibrium payoffs in the city, encouraging households to leave and so decreasing w^{se}. Households that remain in the community are strictly worse off; they have a better outside option, which tightens the dynamic enforcement constraint (DE), exacerbates the "too-big-for-their-boots" effect, and decreases the long-term wealth limit w^{tr} in the community. Note that mobility-based policies have similar effects as increasing productivity in the city, as in the right panel of Figure 3.

Place-based policies, in contrast, increase the relative value of staying in the community, which relaxes (DE) and so mitigates the "too-big-for-their-boots" effect. Thus, place-based policies encourage the coexistence of favor exchange and investment, leading to higher w^{se} and w^{tr}, and therefore less long-term wealth inequality. These policies are thus especially effective at helping left-behind communities. Other resources in the community, such as family, social, or religious ties, have similar effects as place-based policies, so can similarly encourage both favor exchange and wealth accumulation.

[4]For income- and mobility-based policies, the optimal penal code is indeed Markovian. For place-based policies, the restriction to a Markov perfect equilibrium off-path gives us an upper bound on punishment payoffs and hence a lower bound on the household's optimal equilibrium payoff in the community.

6 Conclusion

Helping left-behind communities requires understanding the social constraints faced by those experiencing poverty. This paper argues that wealth separates households from community support. Thus, while favor exchange is an essential source of support in left-behind communities, it imposes hidden costs that can constrain wealth accumulation and deepen long-term inequality.

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A Online Appendix: Routine Proofs

A.1 Proof of Proposition 1

Suppose that the household lives in the city. In any period t, since future vendors don't observe f_t, the household always chooses $f_t = 0$. Hence, vendor t accepts only if $p_t \geq c_t$. This means that $c_t \in [0, w_t]$ are the feasible consumptions, so that the household's equilibrium continuation payoff is at most $\hat{\Pi}(w_t)$ given wealth w_t.

The following equilibrium gives the household an equilibrium continuation payoff of $\hat{\Pi}(w_t)$. In period t, (i) the household proposes $(c_t, p_t) = (\hat{C}(w_t), \hat{C}(w_t))$; (ii) vendor t accepts if and only if $p_t \geq c_t$. Vendor t has no profitable deviation. This strategy attains $\hat{\Pi}$, so the household has no profitable deviation either.

Let $\{c_t^*\}_{t=0}^{\infty}$ be the consumption sequence in the equilibrium above, given initial wealth w. If $w = 0$, then $c_t^* = 0$ in all $t \geq 0$, so $\hat{\Pi}(0) = \hat{U}(0) = 0$ is the unique equilibrium payoff. If $w > 0$, then it must be true that $c_t^* > 0$ in every $t \geq 0$. Suppose otherwise. Let $\tau \geq 0$ be the first period in which $\min\{c_\tau^*, c_{\tau+1}^*\} = 0$ and $\max\{c_\tau^*, c_{\tau+1}^*\} > 0$. If $c_\tau^* > 0$ and $c_{\tau+1}^* = 0$, consider the perturbation $c_\tau = c_\tau^* - \epsilon_1$, $c_{\tau+1} = c_{\tau+1}^* + \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. If $c_\tau^* = 0$ and $c_{\tau+1}^* > 0$, consider the perturbation $c_\tau = c_\tau^* + \epsilon_1$, $c_{\tau+1} = c_{\tau+1}^* - \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. In either case, the perturbation gives a strictly higher payoff, since $\lim_{c \downarrow 0} \hat{U}'(c) = \infty$.

Next, we show that $\hat{\Pi}(w)$ is the household's *unique* equilibrium payoff. At $w = 0$, $\hat{\Pi}(0) = 0$, so the household's unique equilibrium payoff is indeed $\hat{\Pi}(0)$. For $w > 0$, the household can choose $(c_t, p_t) = ((1-\epsilon)c_t^*, c_t^*)$ in every $t \geq 0$ for $\epsilon > 0$ small. Vendor t strictly prefers to accept. As $\epsilon \downarrow 0$, the consumption sequence $\{(1-\epsilon)c_t^*\}_{t=0}^{\infty}$ gives the household a payoff that converges to $\hat{\Pi}(w)$. So the household must earn at least $\hat{\Pi}(w)$ in any equilibrium.

Turning to properties of $\hat{\Pi}(\cdot)$, we claim that $\hat{\Pi}(\cdot)$ is strictly increasing. Pick $0 \leq w < \tilde{w}$. Let $\{c_t^*\}_{t=0}^{\infty}$ be the sequence associated with w. If the initial wealth is $\tilde{w}$, it is feasible to choose $c_0 = c_0^* + \tilde{w} - w$ and $c_t = c_t^*$ for $t \geq 1$. Since $\hat{U}(\cdot)$ is strictly increasing, so too is $\hat{\Pi}(\cdot)$.

It remains to show that $\hat{\Pi}(\cdot)$ is continuous for all $w > 0$. If $w > 0$, then $\hat{C}(w) > 0$. For $\tilde{w}$ sufficiently close to w, setting $c_0 = \hat{C}(w) + (\tilde{w} - w)$ and $c_t = \hat{C}(w_t)$ for $t \geq 1$ is feasible. The household's payoffs converge to $\hat{\Pi}(w)$ as $\tilde{w} \to w$ under this perturbation, which means that $\lim_{\tilde{w} \uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$ and $\lim_{\tilde{w} \downarrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$. Since $\hat{\Pi}(\cdot)$ is increasing, we conclude that $\hat{\Pi}(\cdot)$ is continuous at every $w > 0$.

We now show that $\hat{\Pi}(\cdot)$ is continuous at $w = 0$. Consider $\lim_{w \downarrow 0} \hat{\Pi}(w)$. Since $R'(\bar{w}) = \frac{1}{\delta}$, the line tangent to $R(\cdot)$ at $\bar{w}$ is $\hat{R}(w) = R(\bar{w}) + \frac{w - \bar{w}}{\delta}$. Since $R(\cdot)$ is concave, $R(w) \leq \hat{R}(w)$ for all $w \geq 0$. Therefore, $\hat{\Pi}(w)$ is bounded from above by the household's maximum payoff if we replace $R(\cdot)$ with $\hat{R}(\cdot)$. For consumption path $\{c_t\}_{t=0}^{\infty}$ to be feasible under $\hat{R}(\cdot)$, it must satisfy

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t c_t \le (1-\delta)w_0 + \delta R(\bar{w}) - \bar{w}.$$

This means that the payoff of a household with initial wealth w_0 is at most

$$\hat{U}((1-\delta)w_0 + \delta R(\bar{w}) - \bar{w}).$$

Pick any small $\epsilon > 0$. There exists $T < \infty$ and sufficiently small $w_0 > 0$ such that

$$\delta^T \hat{U}\left((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}\right) < \frac{\epsilon}{2},$$

where $R^T(w_0)$ denotes the function that applies $R(\cdot)$ T-times to w_0.

Consider a hypothetical setting that is more favorable to the household: we allow the household to both consume *and* save her wealth until period T, after which she must play the original city game. The household's payoff from this hypothetical is strictly larger than $\hat{\Pi}(w_0)$ and is bounded from above by

$$(1-\delta)\sum_{t=0}^{T-1}\delta^t(\hat{U}(R^t(w_0)) + \delta^T\hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}).$$

As $w_0 \downarrow 0$, $R^T(w_0) \downarrow 0$, so $R^t(w_0) \downarrow 0$ for any $t < T$. Thus,

$$\hat{\Pi}(w_0) \le (1-\delta)\sum_{t=0}^{T-1}\delta^t\hat{U}(R^t(w_0)) + \delta^T\hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \epsilon.$$

This is true for any $\epsilon > 0$, so $\lim_{w\downarrow 0}\hat{\Pi}(w) = 0$.

Finally, consider any equilibrium in the city. If $w_0 = 0$, then $w_t = 0$ in any $t \ge 0$. If $w_0 > 0$, then we have shown that $c_t > 0$ in every $t \ge 0$, so $w_t > c_t > 0$. A standard argument (see below) implies the following Euler equation:

$$\hat{U}'(c_t) = \delta R'(w_t - c_t)\hat{U}'(c_{t+1}). \tag{3}$$

Together with $R'(\cdot) \ge \frac{1}{\delta}$ and $\hat{U}(\cdot)$ strictly concave, (3) implies $c_t \le c_{t+1}$, and strictly so if $w_t < \bar{w}$.

Next, we argue that $\hat{C}(\cdot)$ is strictly increasing in w. Let $\{c_t\}_{t=0}^{\infty}$ and $\{\tilde{c}_t\}_{t=0}^{\infty}$ be the equilibrium consumption sequences for $w > 0$ and $\tilde{w} > w$, respectively. Suppose $c_0 \ge \tilde{c}_0$, and let $\tau \ge 1$ be the first period such that $c_t < \tilde{c}_t$, which must exist because $\hat{\Pi}(\cdot)$ is strictly increasing. Then, $c_{\tau-1} \ge \tilde{c}_{\tau-1}$, $w_{\tau-1} - c_{\tau-1} < \tilde{w}_{\tau-1} - \tilde{c}_{\tau-1}$, and $c_\tau < \tilde{c}_\tau$, so at least one of $(c_{\tau-1}, w_{\tau-1}, c_\tau)$ and $(\tilde{c}_{\tau-1}, \tilde{w}_{\tau-1}, \tilde{c}_\tau)$ violates (3). Hence, $\hat{C}(w)$ is strictly increasing in w. Therefore, $c_{t+1} \geq c_t$ implies $w_{t+1} \geq w_t$, with strict inequalities if $w_t \leq \bar{w}$.

Since $(w_t)_{t=0}^{\infty}$ is monotone, it converges on $\mathbb{R}_+ \cup \{\infty\}$. Suppose $\lim_{t\to\infty} w_t \leq \bar{w}$. Since $c_t = w_t - R^{-1}(w_{t+1})$, $(c_t)_{t=0}^{\infty}$ converges as well. Then, $R'(w_t - c_t)$ converges to a number strictly above $\frac{1}{\delta}$. Hence, (3) is violated as $t \to \infty$. We conclude that $\lim_{t\to\infty} w_t > \bar{w}$. ■

A.2 Deriving the Euler Equation

Consider a household in the city, and let its optimal consumption and wealth sequence be $\{c_t^*, w_t^*\}_{t=0}^{\infty}$. We prove that if $w_0 > 0$, then

$$\hat{U}'(c_t^*) = \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$$

in every $t \geq 0$.

The proof of Proposition 1 says that $c_t^* > 0$, $c_{t+1}^* > 0$, and $w_t^* - c_t^* > 0$. Suppose that $\hat{U}'(c_t^*) > \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$. Then, we can perturb (c_t^*, c_{t+1}^*) to $(c_t^* + \epsilon, c_{t+1}^* - \chi(\epsilon))$, where $\chi(\epsilon)$ is chosen such that w_{t+2}^* remains the same as before the perturbation. In particular,

$$R(w_t^* - (c_t^* + \epsilon)) - (c_{t+1}^* - \chi(\epsilon)) = R(w_t^* - c_t^*) - c_{t+1}^*.$$

Hence, $\chi'(\epsilon) = R'(w_t^* - (c_t^* + \epsilon))$.

As $\epsilon \downarrow 0$, this perturbation strictly increases the household's payoff:

$$\begin{aligned}\lim_{\epsilon\downarrow 0}\left\{\hat{U}'(c_t^* + \epsilon) - \delta\hat{U}'(c_{t+1}^* - \chi(\epsilon))\chi'(\epsilon)\right\} &= \lim_{\epsilon\downarrow 0}\left\{\hat{U}'(c_t^* + \epsilon) - \delta\hat{U}'(c_{t+1}^* - \chi(\epsilon))R'(w_t^* - c_t^* - \epsilon)\right\} \\ &= \hat{U}'(c_t^*) + \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*) > 0.\end{aligned}$$

This contradicts the fact that (c_t^*, c_{t+1}^*) is optimal. Using a similar argument, we can show that $\hat{U}'(c_t^*) < \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$ is not possible either. ■

A.3 Proof of Lemma 4

We break the proof of this lemma into four steps.

A.3.1 Step 1: Locally Bounding the Slope of $\Pi^*(\cdot)$ from Below

We claim that for any $w \in [0, w^{se})$, there exists $\epsilon_w > 0$ such that for any $\epsilon \in (0, \epsilon_w)$,

$$\Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon.$$

First, suppose $\hat{\Pi}(w) \geq \Pi_c(w)$, and let $\{w_t, c_t\}_{t=0}^{\infty}$ be the wealth and consumption sequences if the household enters the city. The proof of Lemma 3 implies that for any $w_0 < w^{se}$, $R(w_0 - \bar{c}) < w_0$. Proposition 1 says that $\{w_t\}_{t=0}^{\infty}$ is increasing, so $c_0 < \bar{c}$. Hence, there exists $\epsilon_w > 0$ such that $U'(c_0 + \epsilon_w) > 1$. Since $\hat{U}'(c) \geq U'(c)$ for all $c > 0$, $\hat{U}'(c_0 + \epsilon_w) > 1$.

For any $\epsilon < \epsilon_w$, if $w_0 = w + \epsilon$, then the household can enter the city and choose $\hat{c}_0 = c_0 + \epsilon$, with $\hat{c}_t = c_t$ in all $t > 0$. We can bound $\Pi^*(w+\epsilon)$ from below by the payoff from this strategy,

$$\Pi^*(w + \epsilon) \geq (1 - \delta)\left(\hat{U}(c_0 + \epsilon) - \hat{U}(c_0)\right) + \hat{\Pi}(w) > (1 - \delta)\epsilon + \hat{\Pi}(w) = (1 - \delta)\epsilon + \Pi^*(w).$$

We conclude that $\Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon$, as desired.

Now, suppose $\hat{\Pi}(w) < \Pi_c(w)$. Let $\{w_t, c_t, f_t\}_{t=0}^{\infty}$ be the wealth, consumption, and favor sequence in a household-optimal equilibrium. There exists $\tau \geq 0$ such that $f_\tau > 0$ for the first time in period τ; otherwise, the household could implement the same consumption sequence in the city. Choose $\epsilon_w > 0$ to satisfy $\epsilon_w < \delta^\tau f_\tau$.

For $\epsilon \in (0, \epsilon_w)$ and initial wealth $w_0 = w + \epsilon$, consider the perturbed strategy such that $\hat{p}_t = p_t$, $\hat{c}_t = c_t$, and $\hat{f}_t = f_t$ in every period *except* τ. In period τ, $\hat{f}_\tau = f_\tau - \frac{\epsilon}{\delta^\tau}$ and $\hat{p}_\tau = p_\tau + \chi$, where χ is chosen so that $\hat{w}_{t+1} = w_{t+1}$. Then, $\hat{c}_\tau = c_\tau + \chi - \frac{\epsilon}{\delta^\tau}$. Based on the proof of Proposition 2, $w_t < w^{se}$ for all $t \leq \tau$. This observation together with $w^{se} \leq \bar{w}$ and Assumption 1, implies that we can choose a sufficiently small ϵ_w such that the marginal return from capital in every $t < \tau$ is strictly higher than $\frac{1}{\delta}$ even if the inital wealth is $w + \epsilon$ rather than w. Hence, $\chi > \frac{\epsilon}{\delta^\tau}$.

Under this perturbed strategy, (2) is satisfied in all $t < \tau$ because $f_t = 0$ in these periods; in $t = \tau$ because $\hat{f}_\tau < f_\tau$ and $\hat{w}_{\tau+1} = w_{\tau+1}$; and in $t > \tau$ because play is unchanged after τ. Moreover, $f_\tau - \frac{\epsilon}{\delta^\tau} > 0$ because $\epsilon < \epsilon_w$, and $\hat{f}_\tau + \hat{p}_\tau = \hat{c}_\tau$, so this strategy is feasible. Thus, it is an equilibrium. Consequently, $\Pi^*(w + \epsilon)$ is bounded from below by the household's payoff from this strategy,

$$\Pi^*(w + \epsilon) > (1 - \delta)\delta^\tau \frac{\epsilon}{\delta^\tau} + \Pi_c(w) = (1 - \delta)\epsilon + \Pi^*(w),$$

as desired.

A.3.2 Step 2: Moving from Local to Global Bound on Slope

Next, we show that for any $0 \leq w < w' < w^{se}$, $\Pi^*(w') - \Pi^*(w) > (1 - \delta)(w' - w)$.

Let

$$z(w) = \sup\{w'' | w < w'' \leq w^{se}, \text{ and } \forall w' \in (w, w''], \Pi^*(w') - \Pi^*(w) > (1 - \delta)(w' - w)\}.$$

By Step 1, $z(w) \geq w$ exists. Moreover,

$$\Pi^*(z(w)) - \Pi^*(w) \geq \lim_{\tilde{w} \uparrow z(w)} \Pi^*(\tilde{w}) - \Pi^*(w) \geq (1-\delta)(z(w) - w),$$

where the first inequality follows because $\Pi^*(\cdot)$ is increasing, and the second inequality follows by definition of $z(w)$.

Suppose that $z(w) < w^{se}$. By Step 1, there exists $\epsilon_{z(w)}$ such that for any $\epsilon < \epsilon_{z(w)}$,

$$\Pi^*(z(w) + \epsilon) - \Pi^*(z(w)) > (1-\delta)\epsilon.$$

Hence,

$$\Pi^*(z(w)+\epsilon) - \Pi^*(w) = \Pi^*(z(w)+\epsilon) - \Pi^*(z(w)) + \Pi^*(z(w)) - \Pi^*(w) > (1-\delta)\epsilon + (1-\delta)(z(w) - w).$$

This contradicts the definition of $z(w)$, so $z(w) \geq w^{se}$.

For any $w' < w^{se}$, $w' < z(w)$ and so $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w' - w)$, as desired.

A.3.3 Step 3: Investment is Increasing in Wealth.

Consider two wealth levels, $0 \leq w_L < w_H < w^{se}$, and suppose that $\Pi_c(w_L) > \hat{\Pi}(w_L)$ and $\Pi_c(w_H) > \hat{\Pi}(w_H)$. Given any household-optimal equilibria, let p_H, p_L be the respective period-0 payments under w_H, w_L. We prove that $w_H - p_H \geq w_L - p_L$. Define $I_k \equiv w_k - p_k$, $k \in \{L, H\}$. Towards contradiction, suppose that $I_H < I_L$.

We first show that $c_H > c_L + (w_H - w_L)$. Suppose instead that $c_H \leq c_L + (w_H - w_L)$. Since $I_H < I_L$, we have $p_H > p_L + (w_H - w_L)$. But then $f_H < f_L$, since

$$f_H = c_H - p_H < c_H - (p_L + (w_H - w_L)) \leq c_L + (w_H - w_L) - (p_L + (w_H - w_L)) = f_L.$$

Consider the following perturbation: $\hat{p}_H = p_L + (w_H - w_L) \in (p_L, p_H)$, $\hat{f}_H = f_H + p_H - \hat{p}_H \geq f_H$, and $\hat{c}_H = c_H$. Under this perturbation, $\hat{I}_H = w_H - \hat{p}_H = I_L$. Thus, to show that the perturbation satisfies (2), we need only show that $\hat{f}_H \leq f_L$. Indeed:

$$\hat{f}_H = f_H + p_H - (p_L + (w_H - w_L)) = c_H - (c_L - f_L) - (w_H - w_L) = f_L + c_H - (c_L + w_H - w_L) \leq f_L,$$

where the final inequality holds because $c_H \leq c_L + (w_H - w_L)$ by assumption. Thus, this perturbation is also an equilibrium.

We claim that a household with initial wealth w_H strictly prefers this equilibrium to the original equilibrium, which is true so long as

$$\begin{aligned} & (1-\delta)(U(c_H)-\hat{f}_H)+\delta\Pi^*(R(I_L)) > (1-\delta)(U(c_H)-f_H)+\delta\Pi^*(R(I_H)) \\ \iff & (1-\delta)(\hat{f}_H - f_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) \\ \iff & (1-\delta)(p_H - \hat{p}_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))). \end{aligned}$$

We know that $I_L = I_H + p_H - \hat{p}_H$. Since the household stays in the community, $I_H < I_L < w^{se}$, so $R'(I_H), R'(I_L) > \frac{1}{\delta}$. Thus,

$$R(I_L) - R(I_H) > \frac{1}{\delta}(I_L - I_H) = \frac{1}{\delta}(p_H - \hat{p}_H).$$

By Step 2, $\Pi^*(\cdot)$ increases at rate strictly greater than $(1-\delta)$, so we conclude

$$\delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) > \delta(1-\delta)\frac{1}{\delta}(p_H - \hat{p}_H),$$

as desired. Thus, if $I_H < I_L$, then $c_H > c_L + (w_H - w_L)$.

We are now ready to prove that $I_H < I_L$ contradicts household optimality. To do so, we consider two perturbations: one at w_L and one at w_H. At w_H, consider setting

$$\begin{aligned} \hat{c}_H &= c_L + (w_H - w_L) > c_L \geq 0, \\ \hat{p}_H &= p_L + w_H - w_L \in (p_L, w_H], \\ \hat{f}_H &= \hat{c}_H - \hat{p}_H = f_L. \end{aligned}$$

By construction, $w_H - \hat{p}_H = I_L$. Thus, $\hat{f}_H$ satisfies (2) because f_L does. Moreover, $\hat{p}_H + \hat{f}_H = \hat{c}_H$, so the neighbor is willing to accept. This perturbed strategy is therefore an equilibrium. For the original equilibrium to be household-optimal, we must therefore have

$$(1-\delta)(U(c_H)-f_H)+\delta\Pi^*(R(I_H)) \geq (1-\delta)(U(\hat{c}_H)-\hat{f}_H)+\delta\Pi^*(R(\hat{I}_H)). \tag{4}$$

At w_L, consider setting

$$\begin{aligned} \hat{c}_L &= c_H - (w_H - w_L) > c_L \geq 0, \\ \hat{p}_L &= p_H - (w_H - w_L) \in (p_L, w_L]\ , \\ \hat{f}_L &= \hat{c}_L - \hat{p}_L = f_H. \end{aligned}$$

By construction, $w_L - \hat{p}_L = I_H$. Thus, $\hat{f}_L$ satisfies (2) because f_H does. This perturbed strategy is again an equilibrium, so the original equilibrium is household-optimal only if

$$(1-\delta)(U(c_L)-f_L)+\delta\Pi^*(R(I_L)) \geq (1-\delta)(U(\hat{c}_L)-\hat{f}_L)+\delta\Pi^*(R(\hat{I}_L)). \tag{5}$$

Combining (4) and (5) and plugging in definitions, we have

$$U(c_H)-U(c_H-(w_H-w_L)) \geq U(c_L+(w_H-w_L))-U(c_L).$$

However, $c_H > c_L + w_H - w_L$ and $U(\cdot)$ is strictly concave, so this inequality cannot hold. Thus, if $I_H < I_L$, then at least one of the equilibria at w_H and w_L cannot be household-optimal.

A.3.4 Step 4: Establishing Monotonicity

We have shown that investment, $I(w)$, is increasing in w. Consider a household-optimal equilibrium with $w_1 \geq w_0$. Then, $I(w_1) \geq I(w_0)$, so $w_2 = R(I(w_1)) \geq R(I(w_0)) = w_1$. Thus, $w_2 \geq w_1$, and $w_{t+1} \geq w_t$ for all $t > 1$ by the same argument. Similarly, if $w_1 \leq w_0$, then $I(w_1) \leq I(w_0)$, $w_2 \leq w_1$, and $w_{t+1} \leq w_t$ in all $t \geq 0$. We conclude that $(w_t)_{t=0}^{\infty}$ is monotone in any household-optimal equilibrium. ■

B Online Appendix: Extensions to the Model

B.1 Reversible Exit

This appendix shows that underinvestment occurs even if the household can return to the community after leaving for the city. Formally, we modify the game in Section 2 so that at the start of every period while the household is in the city, it can return to the community. If it does, then it plays the community game until it again chooses to leave for the city. Payoffs and information structures are the same as in Section 2, and so neighbors observe all of the household's interactions with neighbors, while vendors observe only their own interactions.

We impose a slightly stronger version of Assumption 1 and also assume that the household's marginal utility of consumption is strictly higher in the city.

Assumption 2 *Define $\bar{c}_m$ as the solution to $\hat{U}'(\bar{c}_m) = 1$, and let $\hat{w}_m$ satisfy $R(\hat{w}_m - \bar{c}_m) = \hat{w}_m$. Then, $R'(\hat{w}_m) > \frac{1}{\delta}$. Moreover, for every $c > 0$, $\hat{U}'(c) > U'(c)$.*

Under this assumption, we can prove that underinvestment occurs even if exit is reversible.

Proposition 3 *Impose Assumption 2. There exists a* $w^{**} < \bar{w}$*, a* $w^{cc} < w^{**}$*, and a positive-measure set* $\mathcal{W} \subseteq [0, w^{**}]$ *with* $\sup \mathcal{W} = w^{**}$ *such that (i) if* $w_0 \notin \mathcal{W}$*, the household permanently exits the community, with* $\lim_{t\to\infty} w_t > \bar{w}$*, and (ii) if* $w_0 \in \mathcal{W}$*, the household is in the community for an infinite number of periods.*

If $w_0 \in \mathcal{W}$*, then for every* $t \geq 0$*,* $w_t < w^{**}$*. Moreover,* $w_{t+1} < w_t$ *whenever* $w_t \in (w^{cc}, w^{**}) \cap \mathcal{W}$*.*

B.1.1 Proof of Proposition 3

Much like the proof of Proposition 2, we break this proof into a sequence of lemmas. We begin by showing that the household's worst equilibrium payoff equals $\hat{\Pi}(\cdot)$, its worst equilibrium payoff from the game in Section 2.

Lemma 5 *For any initial wealth* $w \geq 0$*, the household's worst equilibrium payoff is* $\hat{\Pi}(\cdot)$*.*

Proof of Lemma 5: This proof is similar to the proof of Proposition 1. It is an equilibrium for $f_t = 0$ in every $t \geq 0$, in which case it is optimal for the household to permanently leave the community. In the city, vendor t accepts only if $p_t \geq c_t$. Therefore, $\hat{\Pi}(\cdot)$ gives the maximum equilibrium payoff if the household permanently leaves the community. But as in the proof of Proposition 1, the household cannot earn less than $\hat{\Pi}(\cdot)$, because vendor t must accept whenever $p_t > c_t$. □

Now, we turn to the household's maximum equilibrium payoff. Define $\Pi^{**}(w)$ as the maximum equilibrium payoff with initial wealth w. Define $\Pi^*_c(\cdot)$ identically to $\Pi_c(\cdot)$, except that $\Pi^*(\cdot)$ is replaced by $\Pi^{**}(\cdot)$. Define

$$\Pi^*_m(w) \equiv \max_{0 \leq c \leq w} \left\{ (1-\delta)\hat{U}(c) + \delta \Pi^{**}(R(w-c)) \right\}$$

as the household's maximum equilibrium payoff if it chooses the city in the current period. The key difference between this model and the baseline model is that $\Pi^*_m(\cdot)$ might entail the household returning to the community to take advantage of relational contracts in the future. Therefore, $\Pi^*_m(\cdot) \geq \hat{\Pi}(\cdot)$, since the latter entails staying in the city forever.

Lemma 6 *Both* $\Pi^*_c(\cdot)$ *and* $\Pi^*_m(\cdot)$ *are strictly increasing. For all* $w \geq 0$*,*

$$\Pi^{**}(w) \equiv \max \left\{ \Pi^*_c(w), \Pi^*_m(w) \right\}.$$

Proof of Lemma 6: As in Lemma 1, conditional on choosing the community in period 0, the household's maximum equilibrium payoff equals $\Pi^*_c(w_0)$. If the household instead chooses the city in period 0, then $f_0 = 0$ in any equilibrium. Thus, the household optimally sets $p_t = c_t$, so its maximum equilibrium continuation payoff equals $\Pi^{**}(R(w-c))$. We conclude that $\Pi^*_m(w)$ is the household's maximum equilibrium payoff conditional on choosing the city. It then immediately follows that $\Pi^{**}(w) \equiv \max\{\Pi^*_c(w), \Pi^*_m(w)\}$. Both $\Pi^*_c(\cdot)$ and $\Pi^*_m(\cdot)$ are strictly increasing by inspection. $\square$

Apart from some details of the proof, the next result is similar to Lemma 2.

Lemma 7 *If $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, then $\Pi^{**}(w_t) > \hat{\Pi}(w_t)$ in all $t \geq 0$ of any household-optimal equilibrium.*

Proof of Lemma 7: Suppose not, and let $\tau > 0$ be the first period such that $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. In period $\tau - 1$, (2) implies that $f_t = 0$ if the household stays in the community. Therefore, it is optimal for the household to leave the community in $\tau - 1$. But then it is optimal for the household to *permanently* leave the community in $\tau - 1$, since $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. So $\Pi^{**}(w_{\tau-1}) = \hat{\Pi}(w_{\tau-1})$, contradicting the definition of τ. $\square$

Lemma 8 *Suppose that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$. Then in every $t \geq 0$, (i) $\hat{U}'(c_t) \geq 1$, and (ii) there exists $\tau > t$ such that the household stays in the community in period τ.*

Proof of Lemma 8: Towards contradiction, suppose that there exists $t \geq 0$ such that $\hat{U}'(c_t) < 1$, so that *a fortiori*, $U'(c_t) < 1$. If $f_t > 0$, then we can decrease f_t and c_t by the same $\epsilon > 0$. This perturbation is also an equilibrium, and increases the household's period-t payoff at rate $1 - \hat{U}'(c_t) > 0$ as $\epsilon \to 0$. Thus, $f_t = 0$, which implies that $p_t = c_t > 0$.

By Lemma 7, $\Pi^{**}(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Therefore, there exists a $\tau > t$ such that $f_\tau > 0$, since otherwise the household could do no better than exiting the city permanently. Let τ be the *first* period after t such that $f_\tau > 0$. Note that the household must be in the community in period τ.

Consider the following perturbation: decrease p_t and c_t by $\epsilon > 0$, and increase p_τ and decrease f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, $\chi(\epsilon) \to 0$. Hence, this perturbation is feasible for small enough $\epsilon > 0$. It is an equilibrium, since (2) is trivially satisfied in all $t' \in [t, \tau - 1]$ because $f_{t'} = 0$ in those periods. This perturbation changes the household's period-t continuation payoff at rate no less than

$$-(1-\delta)\hat{U}'(c_t) + \delta^{\tau-t}(1-\delta)\frac{1}{\delta^{\tau-t}} > 0$$

as $\epsilon \to 0$. Thus, the original equilibrium could not have been household-optimal. $\square$

Next, we show that the household stays in the community for sufficiently low initial wealth levels.

Lemma 9 *The set*

$$\mathcal{W} \equiv \left\{ w | \Pi^{**}(w) > \hat{\Pi}(w) \right\}$$

has positive measure, with

$$w^{**} \equiv \sup \left\{ w | \Pi^{**}(w) > \hat{\Pi}(w) \right\} < \bar{w}.$$

Proof of Lemma 9: The proof that $\left\{ w | \Pi^{**}(w) > \hat{\Pi}(w) \right\}$ has positive measure is identical to the proof in Lemma 3, since the same constructions work at $w = 0$, $\Pi^{**}(\cdot)$ is increasing, and $\hat{\Pi}(\cdot)$ is continuous. If $w_0 \in \mathcal{W}$, then $c_t \leqslant \bar{c}_m$ in every t by Lemma 8, so $\Pi^{**}(w_0) \leqslant \hat{U}(\bar{c}_m)$. If w_0 is such that $R(w_0 - \bar{c}_m) > w_0$, then $\Pi^{**}(w_0) \geqslant \hat{\Pi}(w_0) > \hat{U}(\bar{c}_m)$. This implies that $R(w_0 - \bar{c}_m) \leqslant w_0$ for any $w_0 \in \mathcal{W}$. Hence, by Assumption 2, $w^{**} \leqslant \hat{w}_m < \bar{w}$.□

We are now in a position to prove Proposition 3. So far, the argument has hewn closely to the proof of Proposition 2. The rest of the proof marks a more substantial departure.

Lemma 9 shows that a positive-measure set $\mathcal{W}$ exists such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$ for all $w_0 \in \mathcal{W}$ and $\sup \mathcal{W} < \bar{w}$. For any $w_0 \notin \mathcal{W}$, $\Pi^{**}(w_0) = \hat{\Pi}(w_0)$ and so the household permanently exits the community and has a long-term wealth strictly above $\bar{w}$. Lemma 7 implies that for any $w_0 \in \mathcal{W}$, we can construct an infinite sequence of periods such that the household remains in the community for each period in that sequence. This proves the first part of Proposition 3.

Suppose $w_0 \in \mathcal{W}$. Then, Lemma 7 and the definition of w^{**} immediately imply that $w_t < w^{**}$ in every $t \geq 0$. It remains to identify a $w^{cc} < w^{**}$ such that if $w_t \in \mathcal{W} \cap (w^{cc}, w^{**})$, then $w_{t+1} < w_t$. Since $\Pi^{**}(\cdot)$ is increasing, (2) holds only if

$$f_t \leq \Delta(w_t) \equiv \frac{\delta}{1-\delta} (\Pi^{**}(w^{**}) - \hat{\Pi}(w_t)).$$

Proposition 1 implies that $\Delta(w_t)$ is strictly decreasing and continuous. Continuity implies that $\lim_{w \uparrow w^{**}} \hat{\Pi}(w) = \hat{\Pi}(w^{**})$. If $\Pi^{**}(w^{**}) > \hat{\Pi}(w^{**})$, then $\Pi^{**}(\cdot)$ increasing and $\hat{\Pi}(\cdot)$ continuous imply $\Pi^{**}(w) > \hat{\Pi}(w)$ just above w^{**}, contradicting the definition of w^{**}. Therefore, $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$ and $\Delta(w^{**}) = 0$.

For any $w \in [R^{-1}(w^{**}), w^{**}]$, define

$$G(w) \equiv \hat{U}(w - R^{-1}(w^{**})) - (U(w^{**} - R^{-1}(w) + \Delta(w)) + \Delta(w)).$$

Then, $G(\cdot)$ is strictly increasing, continuous, and strictly crosses 0 from below. Define $w^{cc1} \in (R^{-1}(w^{**}), w^{**})$ as the unique wealth such that $G(w^{cc1}) = 0$.

Consider a household with $w_t > w^{cc1}$ such that $\Pi^{**}(w_t) = \Pi^*_c(w_t) > \hat{\Pi}(w_t)$, with corresponding household-optimal choices (c_t, p_t, f_t). Towards contradiction, suppose $w_{t+1} > w^{cc1}$. Then, this household can exit permanently, choose $\hat{c}_t = \hat{p}_t = p_t = c_t - f_t$ and $\hat{f}_t = 0$, where $p_t \geq 0$ because $w^{cc1} \geq R^{-1}(w^{**})$. This deviation leaves w_{t+1} unchanged and results in continuation payoff $\hat{\Pi}(w_{t+1})$.

We argue that this deviation is profitable:

$$\begin{aligned}
(1-\delta)\hat{U}(\hat{c}_t) + \delta\hat{\Pi}(w_{t+1}) &\geq (1-\delta)\hat{U}(c_t - f_t) + \delta\hat{\Pi}(w^{cc1}) \\
&> (1-\delta)(U(c_t) + \Delta(w^{cc1})) + \delta\hat{\Pi}(w^{cc1}) \\
&= (1-\delta)(U(c_t) + \Delta(w^{cc1})) + \delta\Pi^{**}(w^{**}) - (1-\delta)\Delta(w^{cc1}) \\
&= (1-\delta)U(c_t) + \delta\Pi^{**}(w^{**}) \\
&\geq (1-\delta)U(c_t) + \delta\Pi^{**}(w_{t+1}) \\
&= \Pi^{**}(w_t).
\end{aligned}$$

Here, the first line follows from $\hat{c}_t = c_t - f_t$ and $w_{t+1} \geq w^{cc1}$; the second line is proven below; the third line from the definition of $\Delta(w^{cc1})$; and the fifth line because, by Lemma 7, $w_{t+1} \leq w^{**}$. The fourth and sixth lines are algebra.

The second line follows because for any $w_t > w^{cc1}$, $\hat{U}(c_t - f_t) > U(c_t) + \Delta(w^{cc1})$. To see this, note

$$w^{**} \geq w_{t+1} = R(w_t - p_t) > R(w^{cc1} - p_t).$$

Hence, $p_t > w^{cc1} - R^{-1}(w^{**})$. Similarly,

$$w^{cc1} < w_{t+1} = R(w_t - p_t) \leq R(w^{**} - p_t),$$

so $p_t < w^{**} - R^{-1}(w^{**})$. Therefore,

$$\begin{aligned}
\hat{U}(c_t - f_t) = \hat{U}(p_t) &> \hat{U}(w^{cc1} - R^{-1}(w^{**})) \\
&\geq U(w^{**} - R^{-1}(w^{cc1}) + \Delta(w^{cc1})) + \Delta(w^{cc1}) \\
&> U(p_t + \Delta(w^{cc1})) + \Delta(w^{cc1}) \\
&\geq U(p_t + \Delta(w_t)) + \Delta(w^{cc1}) \\
&\geq U(c_t) + \Delta(w^{cc1}).
\end{aligned}$$

Here, the first and third lines follow from $w^{**} - R^{-1}(w^{cc1}) > p_t > w^{cc1} - R^{-1}(w^{**})$; the second line from $G(w^{cc1}) = 0$; the fourth line from the fact that $w^{cc1} < w_t$ and $\Delta(\cdot)$ strictly decreasing; and the last line from $p_t + \Delta(w_t) \geq p_t + f_t = c_t$. The household therefore has a profitable deviation if $w_{t+1} > w^{cc1}$, so $w_{t+1} \leq w^{cc1}$ whenever $w_t \in (w^{cc1}, w^{**})$.

Define the function

$$F(w) \equiv (1-\delta)\hat{U}(w^{**} - R^{-1}(w)) + \delta\hat{\Pi}(w^{**}) - \hat{\Pi}(w)$$

on $w \in [0, w^{**}]$. Then $F(\cdot)$ is strictly decreasing and continuous, with

$$F(0) = (1-\delta)\hat{U}(w^{**}) + \delta\hat{\Pi}(w^{**}) > 0.$$

At $w_0 = w^{**}$, it is feasible for the household to permanently leave the community and consume $c_t = w^{**} - R^{-1}(w^{**})$ in each $t \geq 0$. However, doing so violates (Euler), since $R'(w^{**}) > \frac{1}{\delta}$. Therefore, this consumption path must be dominated by some other feasible consumption path once the household permanently leaves the community, which implies

$$\hat{U}(w^{**} - R^{-1}(w^{**})) < \hat{\Pi}(w^{**}).$$

Consequently,

$$F(w^{**}) = (1-\delta)\hat{U}(w^{**} - R^{-1}(w^{**})) - (1-\delta)\hat{\Pi}(w^{**}) < 0.$$

We conclude that there exists a unique $w^{cc2} \in (0, w^{**})$ such that $F(w^{cc2}) = 0$.

Suppose the household with $w_t \in \mathcal{W}$ and $w_t > w^{cc2}$ chooses to live in the city in t. Towards contradiction, suppose that $w_{t+1} \geq w_t$. Therefore, the household's payoff satisfies

$$\begin{aligned} \Pi^{**}(w_t) &\leq (1-\delta)\hat{U}(w^{**} - R^{-1}(w_t)) + \delta\Pi^{**}(w^{**}) \\ &= (1-\delta)\hat{U}(w^{**} - R^{-1}(w_t)) + \delta\hat{\Pi}(w^{**}) \\ &< \hat{\Pi}(w_t), \end{aligned}$$

where the first inequality follows because $f_t = 0$, so that $p_t = c_t = w_t - R^{-1}(w_{t+1}) \leq w^{**} - R^{-1}(w_t)$; the equality follows because $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$, and the final, strict inequality follows because $w_t > w^{cc2}$ and so $F(w_t) < 0$. But $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, proving a contradiction. So $w_{t+1} < w_t$.

Set $w^{cc} = \max\{w^{cc1}, w^{cc2}\}$. We have shown that $w_{t+1} < w_t$ whenever $w_t > w^{cc}$. This completes the proof. ■

B.2 Non-Linear Favors

B.2.1 Model and Statement of Result

In this appendix, we show that a (slightly weaker version of) Proposition 2 holds if the household's cost in providing f_t is convex.

Consider the **game with non-linear favors,** which is identical to the game in Section 2 except that the household's stage-game payoff is

$$\pi_t = \begin{cases} U(c_t d_t) - k(f_t) & \text{in the community} \\ \hat{U}(c_t, d_t) - k(f_t) & \text{in the city.} \end{cases}$$

Assume that $k(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable, and satisfies $k'(0) = 0$, $k(0) = 0$, and $\lim_{f \to \infty} k'(f) = \infty$. We assume $\hat{U}'(\cdot) > U'(\cdot)$. A special case of this game has $k = U^{-1}$, which corresponds to a setting in which the neighbors value f_t according to the same utility function as the household's consumption.

Define $f^*(c)$ as the unique solution to $U'(c) = k'(f^*(c))$ and c^* as the solution to $U(c^*) = \hat{U}(c^* - f^*(c^*))$. Note that c^* exists and is unique because $\hat{U}'(c) > U'(c)$, $f^*(\cdot)$ is decreasing, and $\lim_{c \to \infty} f^*(c) = 0$. We impose the following assumption.

Assumption 3 *Define $\tilde{c}(w)$ as the unique solution to $R(w - \tilde{c}(w)) = w$. Assume that*

$$\hat{U}(\tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w}))) - U(\tilde{c}(\bar{w})) > \frac{\delta}{1-\delta} U(c^*).$$

Suppose that the household has wealth $\bar{w}$, which is the wealth level at which strictly positive-return investments are exhausted. Assumption 3 ensures that this household's value from moving to the city and consuming $\tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w}))$ forever after is substantially greater than its utility from staying in the community and consuming $\tilde{c}(\bar{w})$ forever. Note that $\tilde{c}(w)$ is increasing because $R'(\cdot) \geq \frac{1}{\delta} > 1$.

We now prove our main result for the game with non-linear favors.

Proposition 4 *Impose Assumption 3 in the game with non-linear favors. Then, there exists a $w^{se} < \bar{w}$, a $w^{tr} \in [0, w^{se})$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{se})$, such that in any household-optimal equilibrium,*

1. ***Selection:*** *The household stays in the community forever if $w_0 \in \mathcal{W}$ and otherwise leaves immediately.*

2. ***Treatment:*** *If $w_0 \in \mathcal{W}$ and $w_t \geq w^{tr}$ on the equilibrium path, then $w_{t+1} < w_t$.*

B.2.2 Proof of Proposition 4

We focus only on those parts of the proof that differ substantially from the proof of Proposition 2.

The proof of Proposition 1 goes through without change, since $f_t = 0$ in every period of any equilibrium in the city. Similarly, once we substitute the cost function $k(f)$ into the household's payoff, the proofs of Lemmas 1 and 2 go through without change.

The next step of the proof, which shows that wealthy households leave the community and poorer households stay, requires a new argument.

Lemma 10 *The set* $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *has positive measure. Moreover,* $w^{se} \equiv \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *satisfies* $0 < w^{se} < \bar{w}$.

Proof of Lemma 10: The argument that $\Pi^*(0) > \hat{\Pi}(0) = 0$ goes through essentially without change. Thus, we need to show only that $w^{se} < \infty$.

Step 1: In any $t \geq 0$ of any household-optimal equilibrium, $f_t \leq f^*(c_t)$. If the household is in the city, $f_t = 0 \leq f^*(c_t)$. If it is in the community, then suppose that $f_t > f^*(c_t)$. Consider perturbing the equilibrium by decreasing c_t and f_t by $\epsilon > 0$. This perturbation is feasible as $\epsilon \to 0$, and moreover, increases the household's stage-game payoff at rate $k'(f_t) - U'(c_t) > k'(f^*(c_t)) - U'(c_t) = 0$. Thus, the original equilibrium was not household-optimal. This proves Step 1.

Step 2: For any $w \geq 0$, $\Pi^*(w) - \hat{\Pi}(w) \leq U(c^*)$. If $\Pi^*(w) = \hat{\Pi}(w)$, then the result is immediate. Suppose that $\Pi^*(w) > \hat{\Pi}(w)$. By Lemma 2, a household with $w_0 = w$ stays in the community in any $t \geq 0$ of any household-optimal equilibrium.

Fixing such an equilibrium, we can derive a lower bound on $\hat{\Pi}(w)$ using the following perturbation: the household leaves the community, chooses $\tilde{p}_t = p_t$ in each $t \geq 0$, and request consumption $\tilde{c}_t = c_t - f_t \geq 0$. This strategy is feasible, and the resulting payoff satisfies

$$\begin{aligned}\textstyle\sum_{t=0}^{\infty} \delta^t(1-\delta)\hat{U}(c_t - f_t) &= \textstyle\sum_{t|c_t \geq c^*} \delta^t(1-\delta)\hat{U}(c_t - f_t) + \sum_{t|c_t < c^*} \delta^t(1-\delta)\hat{U}(c_t - f_t) \\ &\geq \textstyle\sum_{t|c_t \geq c^*} \delta^t(1-\delta)\hat{U}(c_t - f^*(c_t)),\end{aligned}$$

where the inequality follows because $f_t \leq f^*(c_t)$ and $c_t - f_t \geq 0$. Therefore,

$$\begin{aligned}\Pi^*(w) - \hat{\Pi}(w) &\leq \textstyle\sum_{t=0}^{\infty} \delta^t(1-\delta)\left(U(c_t) - \hat{U}(c_t - f_t)\right) \\ &\leq \textstyle\sum_{t|c_t \geq c^*} \delta^t(1-\delta)\left(U(c_t) - \hat{U}(c_t - f^*(c_t))\right) + \sum_{t|c_t < c^*} \delta^t(1-\delta)U(c_t) \\ &\leq U(c^*),\end{aligned}$$

where the first two inequalities follow from our lower bound on $\hat{\Pi}(w)$, and the final inequality holds because $U(c_t) \leq \hat{U}(c_t - f^*(c_t))$ for all $c_t \geq c^*$, and $U(c_t) \leq U(c^*)$ for all $c_t < c^*$. This proves Step 2.

Step 3: There exists a w^{se} such that for all $w \geq w^{se}$, $\Pi^*(w) = \hat{\Pi}(w)$. Fix a wealth level such that $\Pi^*(w) > \hat{\Pi}(w)$ and $\hat{\Pi}(w) > U(c^*)$, and consider a household-optimal equilibrium with $w_0 = w$. By Lemma 2, the household must stay in the community forever. Therefore, there exists $t \geq 0$ such that $U(c_t) \geq \hat{\Pi}(w)$, so $c_t > c^*$.

In this period t, we must have $\Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Since the household can always leave the community, consume $c_t - f^*(c_t)$, and earn $\hat{\Pi}(w_{t+1})$, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ holds only if

$$(1-\delta)U(c_t) + \delta\Pi^*(w_{t+1}) > (1-\delta)\hat{U}(c_t - f^*(c_t)) + \delta\hat{\Pi}(w_{t+1})$$

or

$$\frac{\delta}{1-\delta}\left(\Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1})\right) > \hat{U}(c_t - f^*(c_t)) - U(c_t).$$

By Step 2, this inequality holds only if

$$\frac{\delta}{1-\delta}U(c^*) > \hat{U}(c_t - f^*(c_t)) - U(c_t). \tag{6}$$

Now, choose w^{se} such that $\hat{U}(\tilde{c}(w^{se}) - f^*(\tilde{c}(w^{se}))) - U(\tilde{c}(w^{se})) = \frac{\delta}{1-\delta}U(c^*)$; note that w^{se} exists and satisfies $w^{se} < \bar{w}$ by Assumption 3. For any $w \geq w^{se}$, (6) cannot hold, which means that in any household-optimal equilibrium, there exists a $t \geq 0$ such that $\Pi^*(w_t) = \hat{\Pi}(w_t)$. Lemma 2 then implies that for all $w \geq w^{se}$, $\Pi^*(w_0) = \hat{\Pi}(w_0)$. We conclude that $w^{se} < \bar{w}$, as desired. □

We can now complete the proof of Proposition 4. As in the proof of Proposition 2, **selection** follows from Lemmas 2 and 10. The argument for **treatment** is similar to the argument in Proposition 3, with one change. We must substitute $k(f_t)$ into the household's payoff. With this change, we can construct a similar w^{cc1} as in Proposition 3 so $w_{t+1} < w_t$ whenever $w_t > w^{cc1}$. We can therefore take $w^{tr} = w^{cc1}$ to prove Proposition 4. ■